NDARC — NASA Design and Analysis of Rotorcraft
Theoretical Basis and Architecture

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ABSTRACT

The theoretical basis and architecture of the conceptual design tool NDARC (NASA Design and Analysis of Rotorcraft) are described. The principal tasks of NDARC are to design (or size) a rotorcraft to satisfy specified design conditions and missions, and then analyze the performance of the aircraft for a set of off-design missions and point operating conditions. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. NDARC provides a capability to model general rotorcraft configurations, and estimate the performance and attributes of advanced rotor concepts. The software has been implemented with low-fidelity models, typical of the conceptual design environment. Incorporation of higher-fidelity models will be possible, as the architecture of the code accommodates configuration flexibility, a hierarchy of models, and ultimately multidisciplinary design, analysis and optimization.

INTRODUCTION·

The objectives of rotorcraft design work in a government laboratory are to support research and to support rotorcraft acquisition. Research activities require a robust design capability to aid in technology impact assessments and to provide system level context for research. At the applied research level, it is necessary to show how technology will impact future systems, and justify the levels of investment required to mature that technology to an engineering development stage. Design provides one avenue to accomplishing these objectives. The Department of Defense (DoD) acquisition phases requiring rotorcraft design work include concept exploration, concept decision, concept refinement, and technology development. During these acquisition phases, performing quantitative evaluation and independent synthesis of a wide array of aircraft designs is typically necessary, in order to provide the foundation for specification and requirement development.

Rotorcraft conceptual design consists of analysis, synthesis, and optimization to find the best aircraft meeting the required capabilities and performance. A conceptual design tool is used for synthesis and analysis of rotorcraft. These tools historically have been low fidelity for rapid application. Such sizing codes are built around the use of momentum theory for rotors, classical finite wing theory, a referred parameter engine model, and semi-empirical weight estimation techniques. The successful use of a low-fidelity tool requires careful consideration of model input parameters and judicious comparison with existing aircraft to avoid unjustified extrapolation of results.

The helicopter industry has proprietary conceptual design tools, including PRESTO (Bell Helicopter), RDM (Sikorsky Aircraft), and HESCOMP and VASCOMP (Boeing). Until now the tools available to the U.S. government have been characterized by out-of-date software and limited capabilities. Examples are

HESCOMP and VASCOMP (the versions developed by Boeing in the 1970s), and RC (developed by the U.S. Army AFDD in the 1990s).

NASA, with support from the U.S. Army, conducted in 2005 the NASA Heavy Lift Rotorcraft Systems Investigation (ref. 1), focused on the design and in-depth analysis of rotorcraft configurations that could satisfy the Vehicle Systems Program (VSP) technology goals. The VSP technology goals and mission were intended to identify enabling technology for civil application of heavy lift rotorcraft. The goals emphasized efficient cruise and hover, efficient structure, and low noise. The requirements included carrying 120 passengers over a 1200 nm range, 350 knots at 30,000 ft altitude. The configurations considered included the Large Civil Tiltrotor (LCTR), Large Civil Tandem Compound (LCTC), and Large Advancing Blade Concept (LABC). This project is an example of the role of a rotorcraft sizing code within a government laboratory. The design tool used was the AFDD RC code. The project illustrated the difficulties adapting or modifying a legacy code for configurations other than conventional helicopters and tiltrotors.

Since 2005, there have been numerous other joint NASA/U.S. Army investigations of advanced rotorcraft concepts, covering conventional tiltrotors and helicopters, slowed-rotor compound helicopters (ref. 2), a tilting-tandem concept, heavy-lift slowed-rotor tiltrotors (ref. 3), lift-offset rotor concepts (ref. 4), and a second generation large civil tiltrotor (LCTR2, ref. 5). These design projects have gone well beyond the conventional boundaries of the conceptual design process, combining high-fidelity analyses (including rotorcraft comprehensive analysis, computational fluid dynamics, and structural analysis) with the conceptual design tool. This approach has been required because of the increasing sophistication of the requirements and the technology, and the increased level of certainty needed to differentiate between system concepts.

Based on this experience, a new conceptual design tool has been developed to support future needs of the NASA Subsonic Rotary Wing project and the U.S. Army AFDD Advanced Design Office: NASA Design and Analysis of Rotorcraft (NDARC). The software development started in January 2007, and the initial code release occurred in May 2009. This paper summarizes the NDARC theoretical basis and architecture; the complete description is in reference 6. A companion paper (ref. 7) presents validation and demonstration results from the NDARC development.

REQUIREMENTS

Based on the recent experience of NASA and AFDD at Ames Research Center, the following requirements were defined for the new conceptual design tool.

The principal tasks of the tool are to design rotorcraft to meet specified requirements, and then analyze the performance of the aircraft for a set of flight conditions and missions. Multiple and flexible design requirements, from specific flight conditions and various missions, must be accommodated in the sizing task. The aircraft performance analysis must cover the entire spectrum of the aircraft capabilities, and component performance and engine models must cover all operating conditions. A general and flexible capability to define conditions and missions is required.

For government applications and to support research, it is important to have the capability to rapidly model general rotorcraft configurations, including estimates of the performance and attributes of advanced rotor concepts and the capability to model the impact of new technology at the system and component level. In such an environment, software extensions and modifications will be routinely required to meet the unique requirements of individual projects. Complete and thorough documentation of the theory and its software implementation is essential, to support development and maintenance and to enable effective use and modification.

The code architecture must accommodate configuration flexibility and alternate models, including a hierarchy of model fidelity. Although initially implemented with low-fidelity models, typical of the conceptual design environment, ultimately the architecture must allow multidisciplinary design, analysis, and optimization.

Definition and development of the NDARC requirements benefited substantially from the experiences and computer codes of the preliminary design team of the U.S. Army Aeroflightdynamics Directorate (AFDD) at Ames Research Center. This background is described in reference 6. In the early 1990s, the RC code (for RotorCraft) emerged from the AFDD efforts, with RC97 a major version that unified tiltrotor and helicopter analyses. NDARC is entirely new software, built on a new architecture for the design and analysis of rotorcraft. From the RC theoretical basis, the equations of the parametric weight equations and the Referred Parameter Turboshaft Engine Model were used with only minor changes. Use was also made of some RC component aerodynamic models and rotor performance models. The current users

of RC, informed by past and recent applications, contributed significantly to the requirements definition.

NDARC TASKS

The NDARC code performs design and analysis tasks. The design task sizes the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 1 illustrates the tasks. The principal tasks (sizing, mission analysis, flight performance analysis) are shown in the figure as boxes with heavy borders. Black arrows show control of subordinate tasks.
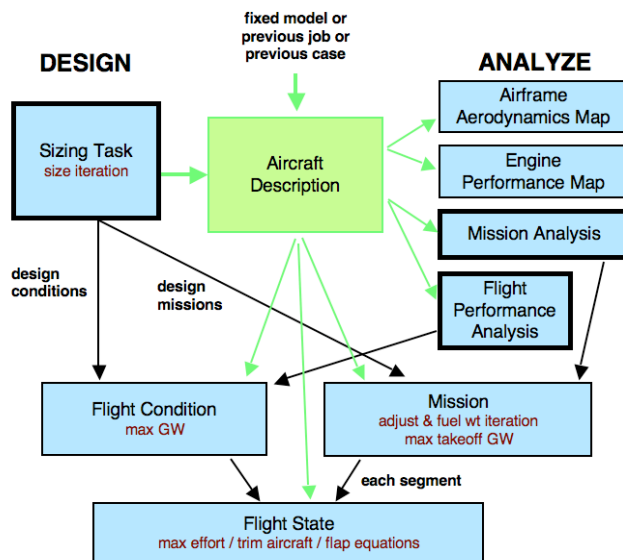

Figure 1. Outline of NDARC tasks.

The aircraft description (figure 1) consists of all the information, input and derived, that defines the aircraft. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by green arrows).

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight

generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments, for which time, distance, and fuel burn are evaluated. For the sizing task, certain missions are designated for engine sizing, for design gross weight calculations, for transmission sizing, and for fuel tank sizing. The engines are sized to meet the power requirement of each segment. The mission parameters include mission takeoff gross weight and useful load. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on time or distance (if adjustable), or on fuel weight.

Flight conditions are specified for the sizing task and for the flight performance analysis. For the sizing task, certain flight conditions are designated for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, and for antitorque or auxiliary-thrust rotor sizing. The engines are sized to meet the power requirement of each condition. The flight condition parameters include gross weight and useful load.

For flight conditions and mission takeoff, the gross weight can be maximized such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified by identifying a target (such as best endurance, best range, or power required equal power available) to be achieved by adjusting a variable (such as speed, rate of climb, or altitude).

The aircraft must be trimmed by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states, hence for each mission segment and flight condition.

Evaluating the rotor hub forces and blade pitch angles may require solution of the blade flap equations of motion.

The following sections describe the NDARC tasks in more detail.

AIRCRAFT DESCRIPTION

Decomposition of the aircraft system into fundamental components is critical to achieving the ability to rapidly model a wide array of rotorcraft concepts. Thus the aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single main-rotor and tail-rotor helicopter; tandem helicopter; coaxial helicopter; and tiltrotors. Novel and advanced concepts typically are modeled by starting with one of these conventional configurations. For example, compound rotorcraft can be constructed by adding wings and propellers.

The following components form the aircraft.

a) Systems: The systems component contains weight information (fixed useful load, vibration, contingency, systems and equipment) for the aircraft.

b) Fuselage: There is one fuselage for the aircraft.

c) Landing Gear: There is one landing gear for the aircraft.

d) Rotors: The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the component can model tail rotors, propellers, proprotors, and ducted fans. A rotor is designated a main rotor, tail rotor, or propeller; and can be tilting, ducted, and/or antitorque. The rotor power required is evaluated using the energy method, as a sum of induced, profile, and parasite power. The power components are calculated in terms of an induced power factor and a mean drag coefficient. The power models account for the influence of speed, thrust, compressibility, stall, lift offset, and the induced interference between twin rotors. The calibration of these induced and profile power models reflects the level of technology being considered. Blade element theory is used to calculate rotor hub forces and moments and to solve for blade pitch control or flapping.

e) Forces: The force component is a simple model for a lift, propulsion, or control subsystem, including a weight and fuel flow description.

f) Wings: The aircraft can have one or more wings, or no wings.

g) Tails: The aircraft can have one or more horizontal or vertical tail surfaces, or no tails.

h) Fuel Tank: There is one fuel tank component for the aircraft. There are one or more sizes of auxiliary fuel tanks.

i) Propulsion Groups: There are one or more propulsion groups. Each propulsion group is a set of rotors and engine groups, connected by a drive system. The components define the power required, and the engine groups define the power available. There are one or more drive states, with a set of gear ratios for each state. The power required equals the sum of component power, transmission losses, and accessory losses. There are drive system torque limits, and rotor and engine shaft ratings.

j) Engine Groups: Each engine group has one or more engines of the same type. For each engine type an engine model is defined.

The engine model describes a particular engine, used in one or more engine groups. The engine performance information includes mass flow, fuel flow, jet thrust, and momentum drag at the required power. A Referred Parameter Turboshaft Engine Model (RPTEM) enables the aircraft performance analysis to cover the entire spectrum of operation. This model uses curve fits of referred performance from an engine deck, including the effect of turbine speed. The effects of size (a scaling model, based on mass flow) and technology (specific power and specific fuel consumption) are included in the engine model.

The ability to define the aircraft control structure through input is a key feature for configuration generality. Aircraft controls are connected to component controls. Aircraft controls consist of pilot's controls (for trim), configuration variables (e.g. tilt of nacelle/pylon, engine, rotor shaft), and direct connections to component controls. There can be one or more control states, with different connections to components (for example, to model the controls of a tiltrotor in helicopter mode and airplane mode flight). There are default control connections for each configuration.

Weights are calculated or input for all components and subsystems. Calculated weights are obtained from parametric equations based on weights of existing turbine

powered helicopters and tiltrotors (and some fixed wing aircraft component weights). Multiplicative technology factors can be used for the weights of all elements of the aircraft, to either match measured weights or account for advanced technology.

SIZING TASK

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight W_D or weight empty W_E, rotor radius R, and engine power available P_{eng}. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized:

a) Engine power sized: Determine P_{eng}, for fixed R. The engine power is the maximum of the power required for all designated sizing flight conditions and sizing missions (typically including vertical flight, forward flight, and one-engine inoperative). Hence the engine power is changed by the ratio P_{reqPG}/P_{avPG} (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). This is the most commonly used approach for the sizing task.

b) Rotor radius sized: Determine R for input P_{eng}. The maximum power required for all designated sizing flight conditions and sizing missions is calculated, and then the rotor radius determined such that power required equals the input power available. The change in radius is estimated as $R = R_{old}\sqrt{P_{reqPG}/P_{avPG}}$ (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). For multi-rotor aircraft, the radius can be fixed rather than sized for some rotors.

Alternatively, P_{eng} and R can be input rather than sized. Several aircraft parameters can be determined by a subset of the design conditions and missions:

a) Design gross weight W_D: maximum gross weight from designated conditions and missions (for which gross weight is not fixed).

b) Maximum takeoff gross weight W_{MTO}: maximum gross weight from designated conditions (for which gross weight is not fixed).

c) Drive system torque limit $P_{DS\,limit}$: maximum torque from designated conditions and missions (for each propulsion group; specified as power limit at reference rotor speed).

d) Fuel tank capacity $W_{fuel-cap}$: maximum fuel weight from designated missions (without auxiliary tanks).

e) Antitorque or auxiliary-thrust rotor design thrust T_{design}: maximum rotor thrust from designated conditions.

Alternatively, these parameters can be fixed at input values. The design gross weight W_D can be fixed. The weight empty can be fixed (achieved by adjusting the contingency weight).

A successive substitution method is used for the sizing iteration. Relaxation is applied to P_{eng} or R, W_D, W_{MTO}, $P_{DS\,limit}$, $W_{fuel-cap}$, and T_{design}. Convergence is tested in terms of these parameters, and the aircraft weight empty W_E. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, for each propulsion group. The inner loop is an iteration on parameters: W_D, W_{MTO}, $P_{DS\,limit}$, $W_{fuel-cap}$, and T_{design}. Either loop can be absent, depending on the definition of the size task.

For each flight condition and for each mission, the gross weight and useful load are specified. The gross weight can be input, or maximized, or fallout. For flight conditions, the payload or fuel weight can be specified, and the other calculated; or both payload and fuel weight specified, with gross weight fallout. For missions, the payload or fuel weight can be specified, the other fallout, and then time or distance of mission segments adjusted; or fuel weight calculated from mission, and payload fallout; or both payload and fuel weight specified (or payload specified and fuel weight calculated from mission), with gross weight fallout.

Sizing flight conditions typically include takeoff (hover or specified vertical rate-of-climb), one-engine-inoperative (OEI), cruise or dash, perhaps transmission, and perhaps mission midpoint hover. Sizing missions typically include a design mission and a mission to determine fuel tank capacity.

Component Sizing

Numerous choices are implemented for identification of independent (fixed) and dependent (fallout) design

parameters, with parameter variation facilitated by automating dependencies.

Engine Power. The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines is specified for each engine group. If the sizing task determines the engine power for a propulsion group, the power of at least one engine group is found (including the first engine group). The power of the remaining engine groups is fixed. The sized power (propulsion group power required minus fixed engine group power) is apportioned among the sized engine groups according to input factors.

Main Rotor. The main rotor size is defined by the radius R or disk loading W/A, thrust-weighted solidity σ, hover tip speed V_{tip}, and blade loading $C_W/\sigma = W/\rho A V_{\text{tip}}^2 \sigma$. With more than one main rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density ρ for C_W/σ is obtained from a specified takeoff condition.

If the rotor radius is fixed for the sizing task, three of R (or W/A), C_W/σ, V_{tip}, and σ are input; and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor.

If the sizing task determines the rotor radius (R and W/A), then two of C_W/σ, V_{tip}, and σ are input; and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others. The radii of all sized rotors are adjusted by the same factor.

Antitorque or Auxiliary-thrust Rotor. For antitorque and auxiliary-thrust rotors, three of R (or W/A), C_W/σ, V_{tip}, and σ are input; and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. The disk loading and blade loading are based on $f_T T_{\text{design}}$, where f_T is an input factor and T_{design} is the maximum thrust from designated design conditions. Optionally the tail rotor radius can be scaled with the main rotor radius and the disk loading.

Wing. The wing size is defined by the wing area S or wing loading W/S; span (perhaps calculated from other geometry), chord, and aspect ratio. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$.

Two of the following parameters are input: area (or wing loading), span, chord, and aspect ratio; the other parameters are derived. Optionally the span can be calculated from the rotor radius, fuselage width, and clearance (typically used for tiltrotors). Optionally the span can be calculated from a specified ratio to the span of another wing.

Tail. The tail size is determined by the area or tail volume, span, chord, and aspect ratio.

Fuel Tank. The fuel tank capacity $W_{\text{fuel-cap}}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel-miss}}$ (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} W_{\text{fuel-miss}}, W_{\text{fuel-miss}} + W_{\text{reserve}})$$

where $f_{\text{fuel-cap}} \geq 1$ is an input factor. Alternatively, the fuel tank capacity can be input.

Weights. The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input, or specified as $d + fW$, for input increment d and fraction f. This convention allows the weights to be input directly ($f = 0$), or scaled with W_D. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity). For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition.

Drive System Rating. The drive system rating is defined as a power limit, $P_{DS\,\text{limit}}$. The rating is properly a torque limit, $Q_{DS\,\text{limit}} = P_{DS\,\text{limit}}/\Omega$, but is expressed as a power limit for clarity. The drive system rating can be (a) input; (b) calculated from the engine takeoff power rating; (c) calculated from the power available at the transmission sizing flight conditions; or (d) calculated from the power required at the transmission sizing flight conditions. The drive system rating is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft and of each engine group. The engine shaft rating is calculated as for the drive system rating, without the sum over engine groups. The rotor shaft rating is either input, or calculated from the rotor power required at the transmission sizing flight conditions.

ANALYSIS TASKS

For the mission analysis, the fuel weight or payload weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits. Missions can be fixed or adjustable.

For each performance flight condition, the power required is calculated or maximum gross weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits.

For the airframe aerodynamics map, the aerodynamics loads can be calculated for a specified range of angle-of-attack, sideslip angle, and control angles. For the engine map, the engine performance can be calculated for a specified range of power, altitude, and speed.

FLIGHT CONDITON DEFINITION

Flight conditions are specified for the sizing task and for the flight performance analysis. For each condition, a flight state is also defined. For the sizing task, certain flight conditions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, or for rotor thrust sizing. The flight condition parameters include gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method.

a) Design gross weight, W_D (calculated or input).
b) Structural design gross weight W_{SD}, or maximum takeoff weight W_{MTO} (which may depend on W_D).
c) Function $d + fW$ of W_D, W_{SD}, or W_{MTO}.
d) Input.
e) Gross weight from specified mission segment (such as midpoint hover).
f) Gross weight maximized, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$; default $d = 0$ and $f = 1$ gives zero power margin.
g) Gross weight maximized, such that transmission torque equals limit: zero torque margin.
h) Gross weight maximized, such that power required equals specified power or transmission torque equals limit (most restrictive).
i) Gross weight fallout from input payload and fuel weights: $W_G = W_O + W_{pay} + W_{fuel}$.

Only the last four options are available for W_D design conditions in the sizing task. The gross weight can be obtained from a mission segment only for the sizing task. Optionally the altitude can be obtained from the specified midpoint mission segment. The secant method or the method of false position is used to solve for the maximum gross weight.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification.

a) Input payload weight W_{pay}, fuel weight fallout:
$W_{fuel} = W_G - W_O - W_{pay}$.
b) Input fuel weight W_{fuel}, payload weight fallout:
$W_{pay} = W_G - W_O - W_{fuel}$.
c) Input payload and fuel weights, gross weight fallout:
$W_G = W_O + W_{pay} + W_{fuel}$ (must match gross weight option).

The input fuel weight is specified as a fraction of primary tank capacity, plus auxiliary tanks. For fallout fuel weight, the number of auxiliary tanks $N_{auxtank}$ is adjusted (optionally only increased). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{fuel} = W_G - W_O - W_{pay}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight adjusted instead. There can be increments to the fixed useful load, including crew weight increment, furnishings weight increment, and installed folding, wing extension, and other kits. These increments are reflected in the fallout weight.

MISSION DEFINITION

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments. A flight state is defined for each mission segment. For the sizing task, certain missions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, or for fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method.

a) Design gross weight, W_D (calculated or input).
b) Structural design gross weight W_{SD}, or maximum takeoff weight W_{MTO} (which may depend on W_D).
c) Function $d + fW$ of W_D, W_{SD}, or W_{MTO}.
d) Input.
e) Gross weight from specified mission segment (such as midpoint hover).
f) Gross weight maximized at specified mission segments, such that power required equals specified power:
$P_{reqPG} = fP_{avPG} + d$; default $d = 0$ and $f = 1$ gives zero power margin.
g) Gross weight maximized at specified mission segments, such that transmission torque equals limit: zero torque margin.
h) Gross weight maximized at specified mission segments, such that power required equals specified power or transmission torque equals limit (most restrictive).
i) Gross weight fallout from input payload and fuel weights: $W_G = W_O + W_{pay} + W_{fuel}$.

j) Gross weight fallout from input payload weight and calculated mission fuel weights: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

If maximum gross weight is specified for more than one mission segment, then the minimum takeoff gross weight increment is used; so the power margin is zero for the critical segment, and positive for other designated segments. Only the last five options are available for W_D design conditions in the sizing task. The secant method or the method of false position is used to solve for the maximum gross weight.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification.

a) Input payload weight W_{pay}, fuel weight fallout:
$W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$.
b) Input fuel weight W_{fuel}, initial payload weight fallout:
$W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.
c) Calculated mission fuel weight, initial payload weight fallout: $W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.
d) Input payload and fuel weights, takeoff gross weight fallout: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$ (must match gross weight option).
h) Input payload weight and calculated mission fuel weight, takeoff gross weight fallout:
$W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$ (must match gross weight option).

The input fuel weight is specified as a fraction of primary tank capacity, plus auxiliary tanks. If the fuel weight is not calculated from the mission, then the mission is adjusted. There can be increments to the fixed useful load, including installed folding kits; other increments are specified for individual mission segments.

The takeoff gross weight is evaluated at the start of the mission, perhaps maximized for zero power margin at a specified mission segment (either takeoff conditions or mid point). Then the aircraft is flown for all segments. For calculated mission fuel weight, the fuel weight at takeoff is adjusted to equal the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on mission fuel weight. Range credit segments (defined below) can also require an iteration. A successive substitution method is used if an iteration is required. The iteration to maximize takeoff gross weight could be an outer loop around the mission iteration, but instead it is executed as part of the mission iteration. At the specified mission segment, the gross weight is maximized for zero power margin, and the resulting gross weight increment added to the takeoff gross weight for the next mission iteration. Thus takeoff gross weight is also a variable of the mission iteration.

Each mission consists of a specified number of mission segments. The following segment types can be specified.

a) Taxi or warm-up (fuel burned but no distance added to range).
b) Distance: fly segment for specified distance (calculate time).
c) Time: fly segment for specified time (calculate distance).
d) Hold: fly segment for specified time (loiter, so fuel burned but no distance added to range).
e) Climb: climb or descend from present altitude to next segment altitude (calculate time and distance).
f) Spiral: climb or descend from present altitude to next segment altitude, fuel burned but no distance added to range.

For each mission segment a payload weight can be specified; or a payload weight change can be specified, as an increment from the initial payload or as a fraction of the initial payload.

The number of auxiliary fuel tanks can change with each mission segment: N_{auxtank} is adjusted based on the fuel weight (optionally only increased relative to the input number at takeoff, optionally fixed during mission). For input fuel weight, N_{auxtank} is specified at takeoff. For fallout fuel weight, the takeoff fuel weight is adjusted for the auxiliary fuel tank weight given N_{auxtank} (fixed $W_G - W_{\text{pay}} = W_O + W_{\text{fuel}}$). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the takeoff payload weight adjusted instead. For fuel tank design missions, $N_{\text{auxtank}} = 0$ and fuel tank capacity is determined from W_{fuel}. Optionally the aircraft can refuel (either on the ground or in the air) at the start of a mission segment, by either filling all tanks to capacity or adding a specified fuel weight. Optionally fuel can be dropped at the start of a mission segment. There can be adjustments to the fixed useful load, including crew weight increment; furnishings weight increment; and installed wing extension and other kits.

For calculation of the time or distance in a mission segment, a headwind or tailwind can be specified. The wind velocity is a linear function of altitude h: $V_w = \pm \max(0, d_{\text{wind}} + f_{\text{wind}} h)$, with the plus sign for a headwind and the minus sign for a tailwind.

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at speed for best endurance. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{burn}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{burn}} + f_{\text{res}}W_{\text{fuel-cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used. Time and distance in reserve segments are not included in endurance and range.

To facilitate specification of range, range calculated for group of segments (typically climb and descent segments) can be credited to a designated distance segment. For mission analysis, missions can be fixed or adjustable. In an adjustable mission, the fuel is input, so the time or distance in specified segments is adjusted based on the calculated fuel burned. If more than one segment is adjusted, all must be distance or all must be time or hold. Each segment can have only one special designation: reserve, adjustable, or range credit.

A segment with a large distance, time, or altitude change can be split into several segments, for more accurate calculation of the performance and fuel burned. The number of segments can be input, or calculated from an input distance, time, or altitude increment.

Table 1 summarizes the time T and distance D calculations for each segment. The segment fuel burned is $W_{\text{burn}} = T\dot{w}$, where $\dot{w}$ is the fuel flow. The horizontal velocity is V_h, the vertical velocity (climb or descent) is V_c. The altitude at the start of the segment is h, and at the end of the segment (start of next segment) h_{end}. The wind speed is V_w, the ground speed is $V_h - V_w$.

Table 1. Mission segment calculations.

segment kind	time T	distance D
taxi	input	0
distance	$D/(V_h - V_w)$	input
time	input	$T(V_h - V_w)$
hold	input	0
climb	$(h - h_{\text{end}})/V_c$	$T(V_h - V_w)$
spiral	$(h - h_{\text{end}})/V_c$	0
adjusted		
distance	$T + dD/(V_h - V_w)$	$D + dD = D + f\Delta D$
time	$T + dT = T + f\Delta T$	$D + dT(V_h - V_w)$
hold	$T + dT = T + f\Delta T$	0

In an adjusted mission, the distances or times are changed at the end of the mission such that the sum of the fuel burned increments will equal the difference between takeoff fuel weight (plus any added fuel) and the calculated mission fuel:

$$\Sigma\, dW_{\text{burn}} = \Sigma\, \dot{w}\, dT = \Sigma\, \dot{w}\, dD/(V_h - V_w) = \Delta W_{\text{fuel}}$$

The increments are apportioned among the adjusted segments by the factor f, determined from the ratio of the input distances or times: $dD = f\Delta D$ or $dT = f\Delta T$. Hence $\Delta D = \Delta W_{\text{fuel}}/(\Sigma f\dot{w}\, dD/(V_h - V_w))$ or $\Delta T = \Delta W_{\text{fuel}}/(\Sigma f\dot{w})$.

For a segment that is a source of range credit, the range increment is set to zero, and the distance D is added to D_{other} of the destination segment. For the destination segment, the range contribution remains fixed at the input value D_{input}, but the time and hence fuel burned are calculated from $(D_{\text{input}} - D_{\text{other}})$.

The segment time, distance, and fuel burned are evaluated by integrating over the segment duration. This integration can be performed by using the horizontal velocity, climb velocity, and fuel flow obtained for the flight state with the gross weight and altitude at the start of the segment; at the middle of the segment; or the average of the segment start and segment end values (trapezoidal integration). The gross weight at the segment middle equals the gross weight at the segment start, less half the segment fuel burned (obtained from the previous mission iteration). The gross weight at the segment end equals the gross weight at the segment start, less the segment fuel burned. With trapezoidal integration, for the output the flight state is finally evaluated at the segment middle.

The mission endurance (block time), range, and fuel burned are $E = \Sigma T$, $R = \Sigma\, dR$, $W_{\text{burn}} = \Sigma\, dW_{\text{burn}}$ (sum over all non-reserve segments). The reserve fuel from mission segments is $W_{\text{res}} = \Sigma\, dW_{\text{burn}}$ (sum over all reserve segments). Optionally the reserve fuel is the maximum of that from mission segments and the fraction $f_{\text{res}}W_{\text{burn}}$, or the fraction $f_{\text{res}}W_{\text{fuel-cap}}$. The calculated mission fuel is then $W_{\text{fuel}} = W_{\text{burn}} + W_{\text{res}}$.

Takeoff Distance

The takeoff distance can be calculated, either as ground run plus climb to clear an obstacle, or accelerate-stop distance in case of engine failure. The obstacle height h_O is typically 35 ft for commercial transport aircraft, or 50 ft for military aircraft and general aviation. This calculation allows determination of the balanced field length: engine failure at critical speed, such that the distance to clear the obstacle equals the distance to stop. Landing and VTOL

takeoff calculations are not implemented, as these are best solved as an optimal control problem.

The takeoff distance consists of a ground run, from zero ground speed to liftoff speed V_{LO}, perhaps including engine failure at speed V_{EF}; then rotation, transition, and climb; or decelerate to stop. Figure 2 describes the elements of the takeoff distance and the accelerate-stop distance, with the associated speeds. The ground is at angle γ relative to the horizontal (inertial axes). The takeoff profile is defined in terms of ground speed or climb speed, input as calibrated airspeed (CAS). The aircraft speed relative to the air is obtained from the ground speed, wind, and ground slope. The aircraft acceleration as a function of ground speed is integrated to obtain the ground distance, as well as the time, height, and fuel burned. Usually the speed increases from the start to liftoff (or engine failure), but the calculated acceleration depends on the flight state specification.
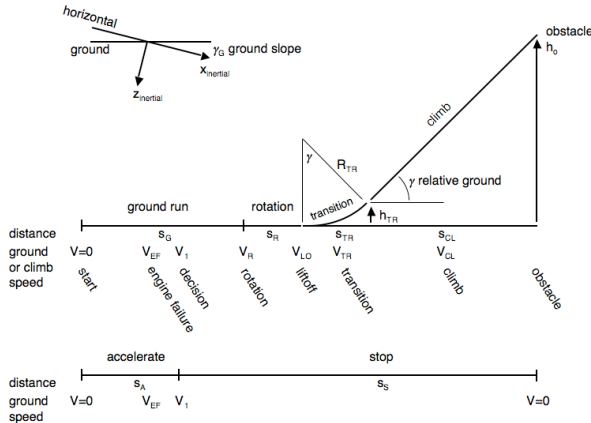

Figure 2. Takeoff distance and accelerate-stop distance elements.

The takeoff profile consists of a set of mission segments. The first segment is at the start of the takeoff, $V = 0$. Subsequent segments correspond to the ends of the integration intervals. The last segment has the aircraft at the required obstacle height, or stopped on the ground. Takeoff segments contribute to the mission fuel burned, but do not contribute to the mission time, distance, or range. The takeoff distance calculation is performed for a set of adjacent segments, the first segment specified as the takeoff start, and the last segment identified as before a non-takeoff segment or before another takeoff start. The takeoff distance is calculated if a liftoff segment (with V_{LO}) is specified; otherwise the accelerate-stop distance is

calculated. The mission segments for takeoff calculations can include ground run, engine failure, liftoff, rotation, transition, and climb or brake segments.

Each takeoff segment requires that the flight state specify the appropriate configuration (including atmosphere, in-ground-effect, gear down or retracted, power rating, nacelle tilt, flap setting), trim option (for ground run, fixed attitude with longitudinal force trimmed using collective; for brake, fixed attitude; for climb, force and moment trim), and maximum effort (for ground run, maximum longitudinal acceleration; for climb, best rate or best angle). In particular, the number of inoperative engines for a segment is part of the flight state specification, regardless of whether there is an engine failure segment defined. The engine failure segment (if present) serves to implement a delay in decision after failure: for a time t_1 after engine failure, the engine rating, power fraction, and friction of the engine failure segment are used. For the ground run and brake, the aircraft acceleration as a function of ground speed is integrated to obtain the segment time and ground distance:

$$s = \int v \, dt = \int d(v^2)/2a$$

(conventional approach for takeoff distance calculations), with appropriate ground friction forces include in the acceleration.

FLIGHT STATE

A flight state is defined for each flight condition and for each mission segment. The following parameters are required.

a) Speed: flight speed and rate of climb; including hover, vertical climb, forward flight, rearward or sideward flight.
b) Aircraft motion; including pitch and roll angles; turn, pullup, or linear acceleration.
c) Altitude: for mission segment, optionally input or from last mission segment; climb segment end altitude from next segment.
d) Atmosphere.
e) Height of landing gear above ground level. Landing gear state (extended or retracted).
f) Aircraft control state: input; or conversion schedule.
g) Aircraft control values.
h) Aircraft center-of-gravity position.

For each propulsion group, the following parameters are required:

i) Drive system state.
j) Rotor tip speed for primary rotor: input; reference; conversion schedule or function of speed; default for

hover, cruise, maneuver, one-engine inoperative, or transmission sizing condition; from input C_T/σ or μ or M_{at}.

And for each engine group of each propulsion group:

k) Number of inoperative engines.
l) Infrared suppressor state: off (hot exhaust) or on (suppressed exhaust).
m) Engine rating, and fraction of rated engine power available.

Aircraft and rotor performance parameters for each flight state:

o) Aircraft drag: forward flight drag increment, accounting for payload aerodynamics.
p) Rotor performance: induced power factor κ and profile power mean c_d (rotor table performance method only).
q) Aircraft trim state and trim targets.

Maximum Effort
The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. For the maximum effort, a quantity and variable are specified. The available maximum-effort quantities include:

a) Best endurance: maximum $1/\dot{w}$.
b) Best range: 99% maximum $V/\dot{w}$ (high side); or low side; or 100%.
c) Best climb or descent rate: maximum V_z or $1/P$.
d) Best climb or descent angle: maximum V_z/V_h or V/P.
e) Ceiling: maximum altitude.
f) Power limit: zero power margin,
$\min(P_{avPG} - P_{reqPG}) = 0$ (minimum over all propulsion groups).
g) Torque limit: zero torque margin, $\min(Q_{limit} - Q_{req}) = 0$ (minimum over all propulsion groups, engine groups, and rotors).
h) Power limit or torque limit (most restrictive).
i) Wing stall: zero wing lift margin, $C_{L\max} - C_L = 0$.
j) Rotor stall: zero rotor thrust margin,
$(C_T/\sigma)_{\max} - C_T/\sigma = 0$ (steady or transient limit).

Here $\dot{w}$ is the aircraft fuel flow, and P the aircraft power. The available maximum-effort variables include:

a) Horizontal velocity V_h or vertical rate of climb V_z (times an input factor).
b) Aircraft altitude.
c) Aircraft angular rate, $\dot{\theta}$ (pull-up) or $\dot{\psi}$ (turn).
d) Aircraft linear acceleration (airframe, inertial, or ground axes).

If the variable is velocity, first the velocity is found for the specified maximum effort; then the performance is evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized (slope zero), then the variable is found such that $V/\dot{w}$ equals 99% of that maximum. Two maximum effort quantity/variable pairs can be specified, solved in nested iterations. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden section method.

Given the gross weight and useful load (from the flight condition or mission specification), the performance is calculated for this flight state. The calculated state information includes weight, speed and velocity orientation (climb and sideslip), aircraft Euler angles, rotor tip speeds, and aircraft controls.

The aircraft weight statement spefices the fixed useful load and operating weight for the design configuration. For each flight state, the fixed useful load may be different than the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or furnishings weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

Environment and Atmosphere
The aerodynamic environment is defined by the speed of sound c_s, and density ρ, and kinematic viscosity $\nu = \mu/\rho$ of the air (or other fluid). These quantities can be obtained from the standard day (International Standard Atmosphere), or input directly.

The International Standard Atmosphere (ISA) is a model for the variation with altitude of pressure, temperature, density, and viscosity, published as International Standard ISO 2533 by the International Organization for Standardization (ISO) (ref. 8). The ISA is intended for use in calculations and design of flying vehicles, to present the test results of flying vehicles and their components under identical conditions, and to allow unification in the field of development and calibration of instruments. The ISA is defined up to 80 km geopotential altitude and is identical

to the ICAO Standard Atmosphere up to 32 km. The ISA consists of a series of altitude ranges with constant lapse rate (linear temperature change with altitude).

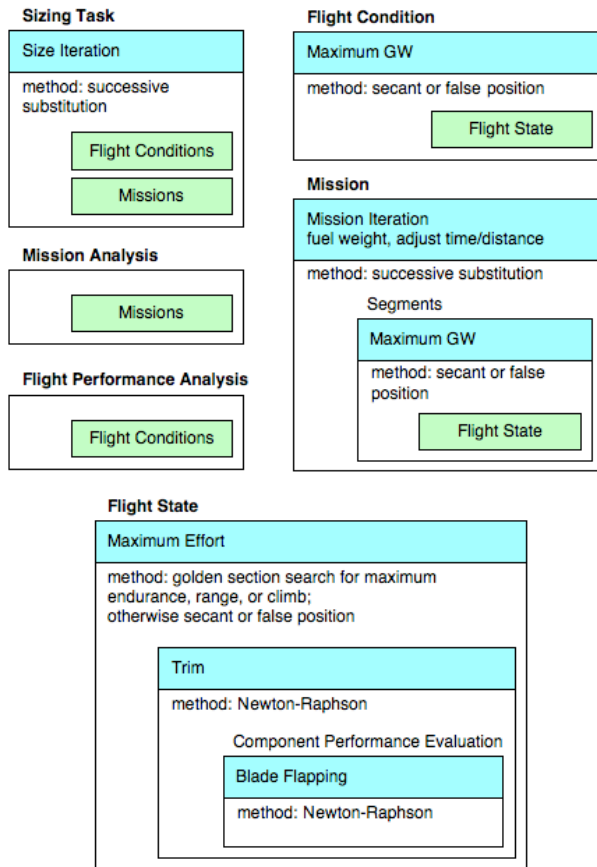The gravitational acceleration g can have the standard value or an input value.

Sizing Task

Size Iteration
method: successive substitution

> Flight Conditions
> Missions

Mission Analysis

> Missions

Flight Performance Analysis

> Flight Conditions

Flight Condition

Maximum GW
method: secant or false position

> Flight State

Mission

Mission Iteration fuel weight, adjust time/distance
method: successive substitution

Segments

Maximum GW
method: secant or false position

> Flight State

Flight State

Maximum Effort
method: golden section search for maximum endurance, range, or climb; otherwise secant or false position

Trim
method: Newton-Raphson

Component Performance Evaluation

Blade Flapping
method: Newton-Raphson

Figure 3. Design and analysis tasks, with nested loops and solution methods.

SOLUTION PROCEDURE

The NDARC code performs design and analysis tasks. Figure 1 illustrates the tasks. The principal tasks (sizing, mission analysis, flight performance analysis) are shown in the figure as boxes with dark borders. Black arrows show control of subordinate tasks. The aircraft description consists of all the information, input and derived, that defines the aircraft. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by green arrows in figure 1).

The nested iteration loops involved in the solution process are indicated by the subtitles in the boxes of figure 1, and illustrated in more detail in figure 3. The flight state solution involves up to three loops. The innermost loop is the solution of the blade flap equations of motion, needed for an accurate evaluation of the rotor hub forces. The next loop is the trim solution, which is required for most flight states. The flight state optionally has one or two maximum effort iterations. The flight state solution is executed for each flight condition and for each mission segment. A flight condition solution or any mission segment solution can optionally maximize the aircraft gross weight. The mission usually requires an iterative solution, for fuel weight or for adjustable segment time or distance. Thus each flight condition solution involves up to four nested iterations: maximum gross weight (outer), maximum effort, trim, and blade motion (inner). Each mission solution involves up to five nested iterations: mission (outer), then for each segment maximum gross weight, maximum effort, trim, and blade motion (inner). Finally, the design task introduces a sizing iteration, which is the outermost loop of the process. The solution method for each iteration is indicated in figure 3. Details of the solution methods and their implementation are given in reference 6.
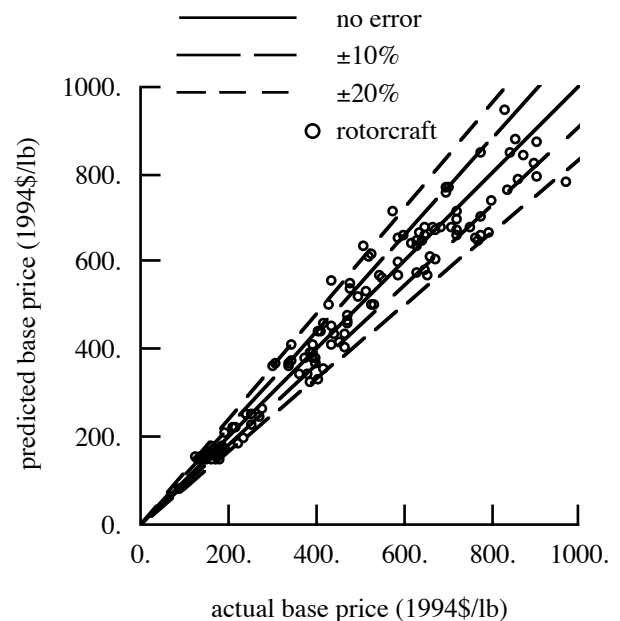
Figure 4. Statistical estimation of rotorcraft flyaway cost.

COST

Costs are estimated using statistical models based on historical aircraft price and maintenance cost data, with appropriate factors to account for technology impact and inflation. The aircraft flyaway cost consists of airframe, mission equipment package, and flight control electronics costs. The direct operating cost plus interest (DOC+I, in cents per available seat mile) is the sum of maintenance cost, flight crew salary and expenses, fuel and oil cost, depreciation, insurance cost, and finance cost. Inflation factors can be input, or internal factors used (either DoD deflators or CPI factors).

The CTM rotorcraft cost model (refs. 9 to 11) gives an estimate of aircraft flyaway cost and direct operating cost plus interest. The statistical equation for airframe cost predicts the price of 123 out of 128 rotorcraft within 20% (figure 4). The fuel burn, block time, and block range are obtained from a designated mission.

AIRCRAFT MODEL

The aircraft disk loading is the ratio of the design gross weight and a reference rotor area: $DL = W_D / A_{ref}$. The reference area is a sum of specified fractions of the rotor areas, $A_{ref} = \Sigma f_A A$. Typically A_{ref} is the projected area of the lifting rotors, where the lifting rotors are all those not designated antitorque or auxiliary-thrust. The disk loading of a rotor is the ratio of a specified fraction of the design gross weight and the rotor area: $DL_{rotor} = T / A = f_W W_D / A$. For rotors designated antitorque or auxiliary-thrust, the disk loading is calculated from the design rotor thrust: $DL_{rotor} = T_{design} / A$. For coaxial rotors, the default reference area is the area of one rotor. For tandem rotors, the default reference area is the projected area: $A_{ref} = (2 - m)A$ where mA is the overlap area ($m = 0$ for no overlap, $m = 1$ for coaxial).

The aircraft wing loading is the ratio of the design gross weight and a reference wing area: $WL = W_D / S_{ref}$. The reference area is a sum of the wing areas, $S_{ref} = \Sigma S$. The wing loading of an individual wing is the ratio of a specified fraction of the design gross weight and the wing area: $WL_{wing} = W / S = f_W W_D / S$.

Aircraft Controls

A set of aircraft controls c_{AC} are defined, and connected to the component controls. The connection to the component control c is typically of the form $c = STc_{AC} + c_0$, where T is an input matrix and c_0 the component control for zero aircraft control. The factor S

is available for internal scaling of the matrix. Figure 5 illustrates the control relationships.

The connection (matrix T) is defined for several control system states, allowing change of control configuration with flight state. An example is control of a tiltrotor in helicopter mode and airplane mode flight. The control state and initial control values are specified for each flight state. Default control matrices are defined based on the configuration: helicopter, tandem, coaxial, or tiltrotor.
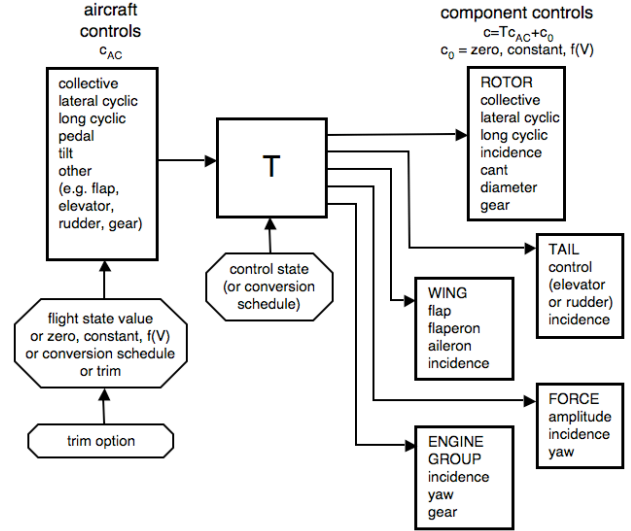

Figure 5. Aircraft and component controls

Typical (default) aircraft controls are the pilot's controls: collective stick, lateral and longitudinal cyclic sticks, pedal, and tilt. Units and sign convention of the pilot's controls are contained in the matrix T. For the single main-rotor and tail-rotor configuration, it is often convenient for the collective and cyclic stick motion to equal the collective and cyclic pitch input of the main rotor, and the pedal motion to equal the collective pitch input of the tail rotor. The aircraft controls should be scaled to approximately the same amplitude (to facilitate control increments in the trim solution procedure), by appropriate definition of the T matrix and scale factor S.

These aircraft controls are available for trim of the aircraft. Any aircraft controls not selected for trim will remain fixed at the values specified for the flight state. Thus by defining additional aircraft controls, component controls can be specified as required for a flight state.

The tilt control variable α_{tilt} is intended for nacelle tilt angle or conversion control, particularly for tiltrotors.

Typically this variable will be connected to nacelle/pylon, engine, rotor shaft, and even wings. The convention is $\alpha_{tilt} = 0$ for cruise, and $\alpha_{tilt} = 90$ deg for helicopter mode.

An optional control conversion schedule is defined in terms of conversion speeds: hover and helicopter mode for speeds below V_{Chover}, cruise mode for speeds above $V_{Ccruise}$, and conversion mode between. The nacelle angle is $\alpha_{tilt} = 90$ in helicopter mode, $\alpha_{tilt} = 0$ in airplane mode, and varies linearly with speed in conversion mode. The tip speed is $V_{tip\text{-}hover}$ in helicopter and conversion mode, and $V_{tip\text{-}cruise}$ in airplane mode. Control states and drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

The flight speed used for control scheduling is usually the calibrated airspeed (CAS), hence variation with dynamic pressure. Velocity schedules are used for conversion, controls and motion, rotor tip speed, landing gear retraction, and trim targets. Optionally these velocity schedules use either calibrated airspeed V_{cal} or the true airspeed V.

Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. In steady flight (including hover, level flight, climb and descent, and turns), equilibrium implies zero net force and moment on the aircraft. There can be additional quantities that at equilibrium must equal target values. In practice, the trim solution can deal with a subset of these quantities. Usually it is at least necessary to achieve equilibrium in the aircraft lift and drag forces, as well as in yaw moment for torque balance. The basic purpose of the trim solution is to determine the component states, including aircraft drag and rotor thrust, sufficient to evaluate the aircraft performance.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. The available trim quantities include:

a) aircraft total force and moment;
b) aircraft load factor;
c) propulsion group power, or power margin $P_{avPG} - P_{reqPG}$, or torque margin $P_{DSlimit} - P_{reqPG}$;

d) rotor force (lift, vertical, or propulsive);
e) rotor thrust C_T/σ, or thrust margin $(C_T/\sigma)_{max} - C_T/\sigma$;
f) rotor flapping β_c, β_s; or rotor hub moment, roll and pitch;
g) rotor torque;
h) wing force; wing lift coefficient C_L, or wing lift margin $C_{L\max} - C_L$;
i) tail force.

Targets for aircraft total force and total moment (including inertial loads in turns) are always zero. The available trim variables include:

a) aircraft controls;
b) aircraft orientation, θ (pitch) and ϕ (roll);
c) aircraft horizontal velocity V_h;
d) aircraft vertical rate of climb V_c;
e) aircraft sideslip angle β;
f) aircraft angular rate, $\dot{\theta}$ (pullup) and $\dot{\psi}$ (turn).

The aircraft orientation variables are the Euler angles of the body axes relative to inertial axes. The aircraft controls (appropriately scaled) are connected to the component controls.

A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation.

Geometry

The aircraft coordinate system has the x-axis forward, y-axis to the right, and z-axis down, measured from the center-of-gravity (figure 6). These aircraft axes are body axes (x is not aligned with the wind), the orientation determined by the convention used for the input geometry. The center-of-gravity is the appropriate origin for describing the motion of the aircraft and summing the forces and moments acting on the aircraft.
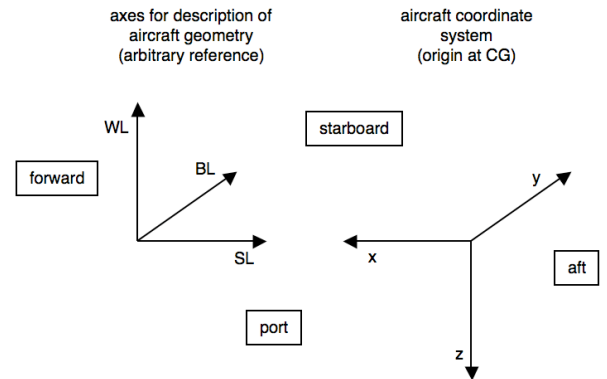

Figure 6. Description of aircraft geometry.

Layout of the geometry is typically in terms of station line (SL, positive aft), buttline (BL, positive right), and waterline (WL, positive up), measured relative to some arbitrary origin (figure 6). The x, y, and z axes are parallel to the SL, BL, and WL directions. One or more locations are defined for each component of the aircraft. Each component will at least have a location that is the point where component forces and moments act on the aircraft. Each location is input in fixed or scaled form. The fixed form input is SL/BL/WL (dimensional). The scaled form input is x/L (positive aft), y/L (positive right), and z/L (positive up), from a reference point. The reference length L is the rotor radius or wing span of a designated component, or the fuselage length. The reference point can optionally be input, or the location (hub) of a designated rotor, or the location (center of action) of a designated wing component, or the location (center of action) of the fuselage, or the location of the center-of-gravity. Fixed input can be used for the entire aircraft, or just for certain components.

From this fixed or scaled input and the current aircraft size, the actual geometry (x, y, z) can be calculated for each location. There are also options to calculate geometry from other parameters (such as tiltrotor span from rotor radius and clearance). This calculated geometry has the sign convention of the aircraft axes (x positive forward, y positive right, z positive down), but has the origin at the reference point (which may or may not be the center-of-gravity).

The location of the aircraft center-of-gravity is specified for a baseline configuration. With tilting rotors, this location is in helicopter mode. For each flight state the aircraft center-of-gravity is calculated, from the baseline location plus any shift due to nacelle tilt, plus an input center-of-gravity increment. Alternatively, the aircraft center-of-gravity location for the flight state can be input. Any change of the center-of-gravity position with fuel burn during a mission is not automatically calculated, but could be accounted for using the flight state input.

Aircraft Motion

The aircraft velocity and orientation are defined by the following parameters: flight speed V; turn rate; orientation of the body frame relative to inertial axes (Euler angles); and orientation of the velocity frame relative to inertial axes (flight path angles).

Aircraft conventions are followed for the direction and orientation of axes: the z-axis is down, the x-axis forward, and the y-axis to the right; and a yaw-pitch-roll

sequence is used for the Euler angles. However, the airframe axes are body axes (fixed to the airframe, regardless of the flight direction) rather than wind axes (which have the x-axis in the direction of the flight speed).

The orientation of the body frame F relative to inertial axes is defined by yaw ψ_F, pitch θ_F, and roll ϕ_F Euler angles, which are rotations about the z, y, and x axes respectively. So yaw is positive to the right, pitch is positive nose up, and roll is positive to the right. The flight path is specified by the velocity V, in the positive x-axis direction of the velocity axes. The orientation of the velocity axes relative to inertial axes is defined by yaw ψ_V (sideslip) and pitch θ_V (climb) angles. Sideslip is positive for the aircraft moving to the right, and climb is positive for the aircraft moving up. The horizontal velocity component is $V_h = V\cos\theta_V$. In straight flight, all these angles are constant. In turning flight at a constant yaw rate, the yaw angle is $\psi_F = \dot\psi_F t$; the turn radius is $R_T = V_h / \dot\psi_F$; and the nominal bank angle and load factor are $\tan\phi_F = \sqrt{n^2 - 1} = \dot\psi_F V_h / g$. Then the forward, sideward, and climb velocities are $V_f = V_h \cos\psi_V$, $V_s = V_h \sin\psi_V$, $V_c = V\sin\theta_V = V_h \tan\theta_V$. The aircraft velocity relative to the air is v_{AC}^F. The aircraft angular velocity ω_{AC}^F is obtained from the Euler angle rates. For steady state flight, $\dot\theta_F = \dot\phi_F = 0$; $\dot\psi_F$ is nonzero in a turn.

Accelerated flight is also considered, in terms of linear acceleration $a_{AC}^F = gn_L$ and pitch rate $\dot\theta_F$. The nominal pullup load factor is $n = 1 + \dot\theta_F V_h / g$. For accelerated flight, the instantaneous equilibrium of the forces and moments on the aircraft is evaluated for specified acceleration and angular velocity; the equations of motion are not integrated to define a maneuver. Note that the fuselage and wing aerodynamic models do not include all roll and yaw moment terms needed for general unsteady flight (notably derivatives L_v, L_p, L_r, N_v, N_p, N_r are absent).

The aircraft pitch and roll angles are available for trim of the aircraft. Any motion not selected for trim will remain fixed at the values specified for the flight state.

Loads and Performance

For each component, the power required and the net forces and moments acting on the aircraft can be calculated. The aerodynamic forces F and moments M are typically calculated in wind axes and then resolved into body axes, relative to the origin of the body axes (the aircraft center-of-gravity). The power and loads of all components are summed to obtain the aircraft power and loads. Typically

the trim solution drives the net forces and moments on the aircraft to zero.

The aircraft equations of motion, in body axes F with origin at the aircraft center-of-gravity, are the equations of force and moment equilibrium:

$$m(\dot{v} + \omega \times v) = F + F_{\text{grav}}$$

$$I\dot{\omega} + \omega \times I\omega = M$$

where $m = W/g$ is the aircraft mass; the gravitational force is $F_{\text{grav}} = mg$ (in F axes). For steady flight, $\dot{\omega} = \dot{v} = 0$, and $\omega = R(0 \ 0 \ \dot{\psi}_F)^T$ is nonzero only in turns. For accelerated flight, $\dot{v} = 0$ can be nonzero, and $\omega = R(0 \ \dot{\theta}_F \ \dot{\psi}_F)^T$. The forces and moments are the sum of loads from all components of the aircraft: fuselage, rotor, force, wing, tail, engine, and tank. A particular component can have more than one source of loads; for example, the rotor component produces hub forces and moments, but also includes hub and pylon drag.

The component power required P_{comp} is evaluated for all components (rotors) of the propulsion group. The total power required for the propulsion group P_{reqPG} is obtained by adding the transmission losses and accessory power. The power required for the propulsion group must be distributed to the engine groups. From the power required the fuel flow is calculated. The fuel flow of the propulsion group is obtained from the sum over the engine groups. The total fuel flow is the sum from all components of the aircraft: $\dot{w} = \Sigma \dot{w}_{\text{reqEG}} + \Sigma \dot{w}_{\text{force}}$

Aerodynamics

Each component has a position z^F in aircraft axes F, relative to the reference point, and orientation of component axes B relative to aircraft axes given by a rotation matrix. It is expected that the component axes are (roughly) x forward and z down (or in negative lift direction). Acting at the component are interference velocities v_{int} (velocity of air, in F axes) from all other components. Then the total component velocity relative to the air is $v^F = v_{AC}^F + \omega_{AC}^F \times \Delta z^F - \Sigma v_{\text{int}}^F$, where $\Delta z^F = z^F - z_{cg}^F$. The aerodynamic environment is defined in the component axes: velocity magnitude $v = |v^B|$, dynamic pressure $q = \frac{1}{2}\rho v^2$, angle-of-attack α, and sideslip angle β. From v, q, α, and β, the component aerodynamic model calculates the loads in wind axes: drag, side force, and lift (D, Y, and L) and the roll, pitch, and yaw moments; and from wind axis loads the aerodynamic loads in aircraft axes acting at the center-of-gravity are calculated.

The component aerodynamic models are based on equations intended to cover the principal operating conditions, including small angle-of-attack and sideslip with a representation of stall and maximum lift, vertical flight, sideward flight, and rearward flight.

In hover and low speed (below 10 knots), the download is calculated: the downward component of the aerodynamic force in inertial axes. Download can be expressed as a fraction of the total rotor vertical force, or as a fraction of gross weight.

Drag

Each component can contribute drag to the aircraft. A fixed drag can be specified, as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on an appropriate area S. There may also be other ways to define a scaled drag value. For fixed drag, the coefficient is $C_D = (D/q)/S$ (the aerodynamic model is formulated in terms of drag coefficient). For scaled drag, the drag area is $D/q = SC_D$. For all components, the drag $(D/q)_{\text{comp}}$ or $C_{D\text{comp}}$ is defined for forward flight or cruise; typically this is the minimum drag value. For some components, the vertical drag ($(D/q)_{V\text{comp}}$ or $C_{DV\text{comp}}$) or sideward drag ($(D/q)_{S\text{comp}}$ or $C_{DS\text{comp}}$) is defined. For some components, the aerodynamic model includes drag due to lift, angle-of-attack, or stall.

Table 2. Component contributions to drag.

component	drag contribution	reference area
fuselage	fuselage	fuselage wetted area
	fuselage vertical	fuselage projected area
	fixtures and fittings	fuselage wetted area
	rotor-body interf	fuselage wetted area
	contingency	—
	payload increment	—
gear	landing gear	—
rotor	hub, hub vertical	rotor disk area
	pylon, pylon vert	pylon wetted area
	spinner	spinner wetted area
wing	wing, wing vert	wing planform area
	wing-body interf	wing planform area
tail	tail, tail vertical	tail planform area
engine	nacelle, nac. vert	nacelle wetted area
	momentum drag	—
fuel tank	auxiliary tank	—

Table 2 summarizes the component contributions to drag, and the corresponding reference areas. If no reference area is indicated, then the input is only drag area D/q. An appropriate drag reference area is defined for each component; the reference area is either input or calculated. Wetted area is calculated for each component, even if it is not the reference area. The component wetted areas are summed to obtain the aircraft wetted area. Some of the weight models also require the wetted area.

The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

Optionally the aircraft drag can be fixed. The quantity specified is the sum (over all components) of the drag area D/q (minimum drag, excluding drag due to lift and angle-of-attack), without accounting for interference effects on dynamic pressure. The input parameter can be D/q; or the drag can be scaled, specified as a drag coefficient based on the rotor disk area, so $D/q = A_{\text{ref}}C_D$ (A_{ref} is the reference rotor disk area); or the drag can be estimated based on the gross weight, $D/q = k(W_{MTO}/1000)^{2/3}$ (W_{MTO} is the maximum take-off gross weight). Based on historical data, the drag coefficient $C_D = 0.02$ for old helicopters and $C_D = 0.008$ for current low drag helicopters. Based on historical data, $k = 9$ for old helicopters, $k = 2.5$ for current low drag helicopters, $k = 1.6$ for current tiltrotors, and $k = 1.4$ for turboprop aircraft (k in English units). If the aircraft drag is input, then the fuselage contingency drag is adjusted so the total aircraft D/q equals the input value.

Optionally the aircraft vertical drag (download fraction) can be fixed. The quantity specified is the sum over all components of the vertical drag area $(D/q)_V$. The input parameter can be $(D/q)_V$, or $k = (D/q)_V / A_{\text{ref}}$ (A_{ref} is reference rotor disk area). Approximating the dynamic pressure in the wake as $q = \frac{1}{2}\rho(2v_h)^2 = T/A_{\text{ref}}$, the download fraction is $DL/T = q(D/q)_V / T = k$. If the aircraft vertical drag is input, then the fuselage contingency vertical drag is adjusted so the total aircraft $(D/q)_V$ equals the input value.

The nominal drag areas of the components and the aircraft are part of the aircraft description and are used when the aircraft drag is fixed. While vertical drag parameters are part of the aerodynamic model for the hub, pylon, and

nacelle, aerodynamic interference at the rotor and at the engine group is not considered, so these terms do not contribute to download. In the context of download, only the fuselage, wing, tail, and contingency contribute to the nominal vertical drag.

From the input and the current aircraft size, the drag areas D/q and coefficients C_D are calculated. The aerodynamic analysis is usually in terms of coefficients. If the aircraft drag is fixed for the aircraft model, then the fuselage contingency drag is set:

$$(D/q)_{\text{cont}} = (D/q)_{\text{fixed}} - \Sigma (D/q)_{\text{comp}}$$

and similarly for fixed vertical drag. Note that this adjustment ignores changes caused by interference in the dynamic pressure and the velocity direction, which will affect the actual component drag.

The component aerodynamic model calculates the drag, typically from a drag coefficient C_D, a reference area, and the air velocity of the component. The drag force is then $D = \Sigma q_{\text{comp}}S_{\text{ref}}C_D$, where the dynamic pressure q_{comp} includes interference. From the aerodynamic forces and moments in wind axes, the total force and moment in body axes are calculated. For reference, the aircraft total drag and total drag area are $D_{AC} = \Sigma e_d^T F_d^F$ and $(D/q)_{AC} = D_{AC}/q$; where the aircraft velocity (without interference) gives the direction $e_d = v_{AC}^F / |v_{AC}^F|$ and dynamic pressure $q = \frac{1}{2}\rho |v_{AC}^F|^2$. Here F_d^F is the component force in body axes that is produced only by the component drag. An overall skin friction drag coefficient is then $C_{D\,AC} = (D/q)_{AC\,\text{wet}} / S_{AC}$, based on the aircraft wetted area $S_{AC} = \Sigma S_{\text{wet}}$ and excluding drag terms not associated with skin friction (specifically landing gear, rotor hub, and contingency drag).

Performance Indices

The following performance indices are calculated for the aircraft. The aircraft hover figure of merit is $M = W\sqrt{W/2\rho A_{\text{ref}}}/P$. The aircraft effective drag is $D_e = P/V$, hence the effective lift-to-drag ratio is $L/D_e = WV/P$. Isolated rotor performance indices are described below.

SYSTEMS

The systems component contains weight information (fixed useful load, vibration, contingency, systems and equipment).

FUSELAGE

There is one fuselage component for the aircraft. There are several options for calculating the fuselage length, wetted area (reference area for drag coefficients), and projected area (reference area for vertical drag). The fuselage has a position z^F, where the aerodynamic forces act; and the component axes are aligned with the aircraft axes. The fuselage has no control variables.

The aerodynamic velocity of the fuselage relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack, sideslip angle, and dynamic pressure are calculated from v^B. The drag area or drag coefficient is defined for forward flight, vertical flight, and sideward flight. For small angle-of-attack, the drag increases proportional to $(1 + K |\alpha|^X)$, using input factor K and exponent X. In addition, the forward flight drag area or drag coefficient is defined for fixtures and fittings, and for rotor-body interference. The drag force is

$$D = qS_{\text{wet}}(C_D + C_{D\,\text{fit}} + \Sigma C_{D\,\text{rb}}) + q((D/q)_{\text{pay}} + (D/q)_{\text{cont}})$$

including the drag area of the payload (specified for the flight state) and the contingency drag area.

The fuselage lift, pitch moment, side force, and yaw moment are defined in fixed form (e.g. L/q) or scaled form (e.g. C_L). Maximum lift coefficient and maximum side force coefficient are defined. The fuselage roll moment is zero.

LANDING GEAR

There is one landing gear component for the aircraft. The landing gear can be located on the body or on the wing. The landing gear can be fixed or retractable; a gear retraction speed is specified (CAS), or the landing gear state can be specified in the flight state.

The height of the bottom of the landing gear above ground level, h_{LG}, is specified in the flight state. The landing gear position z^F is a distance d_{LG} above the bottom of the gear. A drag area is specified for landing gear extended.

ROTOR

The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the rotor component can model tail rotors, propellers, proprotors, ducted fans, thrust vectoring rotors, and auxiliary-thrust rotors. The principal configuration designation (main rotor, tail rotor, or propeller) determines where the weights are put in the weight statement (rotor group, empennage group, or propulsion group), and each configuration can possibly have a separate performance or weight model. Antitorque rotors and auxiliary-thrust rotors can be identified, for special sizing options. Other configuration features are variable diameter and ducted fan.

Multi-rotor systems (such as coaxial or tandem configuration) are modeled as a set of separate rotors, in order to accommodate the description of the position, orientation, controls, and loads. The performance calculation for twin rotor systems can include the mutual influence of the induced velocity on the power.

Drive System

The drive system defines gear ratios for all the components that it connects (rotors and engine groups). There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). For the primary rotor, a reference tip speed $V_{\text{tip-ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1.

The flight state specification of the tip speed can be an input value; the reference tip speed; a function of flight speed or a conversion schedule; or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

Geometry

The rotor rotation direction is described by the parameter r: $r = 1$ for counter-clockwise rotation, $r = -1$ for clockwise rotation (as viewed from the positive thrust side of the rotor). The rotor solidity and blade mean chord are related by $\sigma = Nc/\pi R$. Generally thrust-weighted values are used, but geometric values are also required by the analysis. The mean chord is the average of the chord over the rotor blade span, from root cutout to tip. The thrust-weighted chord is the average of the chord over the rotor blade span, from root cutout to tip, weighted by r^2. A general blade chord distribution is specified as $c(r) = c_{\text{ref}}\hat{c}(r)$, where c_{ref} is the thrust-weighted chord. Linear taper is specified in terms of a taper ratio $t = c_{\text{tip}}/c_{\text{root}}$, or in terms of the ratio of thrust-weighted and geometric chords, $f = \sigma_t/\sigma_g = c_{.75R}/c_{.50R}$.

The rotor hub is at position z_{hub}^F. A component of the position can be calculated, superseding the location input. The calculated geometry depends on the configuration. For a coaxial rotor, the hub locations can be calculated

from the input separation, and the input location midway between the hubs. For a tandem rotor, the hub locations can be calculated from the input overlap, and the input location midway between the hubs. For a tail rotor, the longitudinal position can be calculated from the main rotor radius, tail rotor radius, and tail-rotor/main-rotor clearance. For a tiltrotor, the lateral position can be calculated from the rotor radius (cruise value for variable diameter rotor), fuselage/rotor clearance, and fuselage width (with the pivot, pylon, and nacelle center-of-gravity lateral positions adjusted to keep the same relative position to the hub). Alternatively for a tiltrotor, the lateral position can be calculated from the wing span so the rotors are at the wing tips; or from a designated wing panel edge.

The reference areas for the component drag coefficients are the rotor disk area (for hub drag), pylon wetted area S_{pylon}, and spinner wetted area S_{spin}. The pylon wetted area is input; or calculated from the drive system (gear box and rotor shaft) weight, or from the drive system plus engine system (engine, exhaust, and accessories) weight: $S_{\text{pylon}} = k(w/N_{\text{rotor}})^{2/3}$, where $w = W_{gbrs}$ or $w = W_{gbrs} + \Sigma W_{ES}$. The spinner wetted area is input, or calculated from the spinner frontal area: $S_{\text{spin}} = f(\pi R_{\text{spin}}^2)$.

Control and Loads

The rotor controls consist of collective, lateral cyclic, longitudinal cyclic, and perhaps shaft incidence (tilt) and cant angles. Rotor cyclic control can be defined in terms of tip-path plane or no-feathering plane command. The collective control variable is the rotor thrust amplitude or the collective pitch angle.

The relationship between tip-path plane tilt and hub moment is $M = (N/2)I_b\Omega^2(\nu^2-1)\beta = K_{\text{hub}}\beta$, where N is the number of blades, Ω the rotor speed, and ν the dimensionless fundamental flap frequency. The flap moment of inertia I_b is obtained from the input Lock number: $\gamma = \rho acR^4/I_b$, for SLS density ρ and lift curve slope $a = 5.7$. The flap frequency and Lock number are specified for hover radius and rotational speed. The flap frequency and hub stiffness are required for the radius and rotational speed of the flight state. For a hingeless rotor, it is assumed that the blade flap spring $K_{\text{flap}} = I_b\Omega^2(\nu^2-1)$ is constant. For an articulated rotor, it is assumed that the hinge offset $e = Rx/(1+x)$, $x = (2/3)(\nu^2-1)$ is constant.

Optionally the rotor can have a variable diameter. The rotor diameter is treated as a component control. As a control, it can be connected to an aircraft control and thus set for each flight state. The basic variation can be specified based on the conversion schedule, or input as a function of flight speed. During the diameter change, the chord, chord radial distribution, and blade weight are assumed fixed; hence solidity scales as $\sigma \propto 1/R$, blade flap moment of inertia as $I_b \propto R^2$, and Lock number as $\gamma \propto R^2$.

Control Variables

The collective control variable is direct command of rotor thrust magnitude T or C_T/σ (in shaft axes), from which the collective pitch angle can be calculated; or rotor collective pitch angle $\theta_{0.75}$, from which the thrust and inflow can be calculated.

Shaft tilt control variables are incidence (tilt) and cant angles, acting at a pivot location.

Tip-path plane command is direct control of the tip-path plane tilt. This control mode requires calculation of rotor cyclic pitch angles from the flapping. The control variables are longitudinal tilt β_c (positive forward) and lateral tilt β_s (positive toward retreating side). Alternatively, the cyclic control can be specified in terms of hub moment or lift offset, if the blade flap frequency is greater than 1/rev.

No-feathering plane command is control of rotor cyclic pitch angles, usually producing tilt of the thrust vector. This control mode requires calculation of rotor tip-path plane tilt from the cyclic control, including the influence of inflow. The control variables are longitudinal cyclic pitch angle θ_s (positive aft) and lateral cyclic pitch angle θ_c (positive toward retreating side).

Aircraft Controls

Each rotor control can be connected to the aircraft controls: $c = STc_{AC} + c_0$. The factor S can be introduced to automatically scale the collective matrix: $S = a/6 = 1/60$ if the collective control variable is C_T/σ; $S = \rho V_{\text{tip}}^2 A_{\text{blade}}(a/6)$ if the collective control variable is rotor thrust T; $S = 1$ if the collective control variable is pitch angle $\theta_{0.75}$. For cyclic matrices, $S = 1$ with no-feathering plane command, and $S = -1$ for tip-path plane command.

Rotor Axes and Shaft Tilt

The rotor hub is at position z_{hub}^F, where the rotor forces and moments act; the orientation of the rotor shaft axes relative to the aircraft axes is given by the rotation matrix C^{SF}. The pivot is at position z_{pivot}^F. The hub or shaft axes S have origin at the hub center; the z-axis is the shaft, positive in the positive thrust direction; and the x-axis

usually downstream. The rotor orientation is specified by selecting a nominal direction in body axes (positive or negative x, y, or z-axis) for the positive thrust direction; the other two axes are then the axes of shaft control. For a main rotor the nominal direction would be the negative z-axis; for a tail rotor the nominal direction would be the $\pm y$-axis (depending on the direction of rotation of the main rotor); for a propeller the nominal direction would be the positive x-axis. The hub and pivot axes have a fixed orientation relative to the body axes: hub incidence and cant, pivot dihedral, pitch, and sweep. The shaft angle control consists of incidence and cant about the pivot axes.

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The rotor shaft incidence angle is then connected to α_{tilt} by defining the control matrix appropriately. From the pivot location z_{pivot}^F and the hub location for the reference shaft control $z_{\text{hub}0}^F$, the hub location, pylon location, and nacelle center-of-gravity location are calculated for given shaft control. The shift in the aircraft center-of-gravity produced by nacelle tilt is also calculated.

Hub Loads

The rotor controls give the thrust magnitude and the tip-path plane tilt angles β_c and β_s, either directly or from the collective and cyclic pitch. The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z, x, and y-axis directions respectively). The hub pitch and roll moment are proportional to the flap angles. The hub torque is obtained from the shaft power P_{shaft} and rotor speed Ω. The force and moment acting on the hub, in shaft axes, are then:

$$F^S = \begin{pmatrix} H \\ Y \\ T \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f_B T \end{pmatrix}$$

$$M^S = \begin{pmatrix} M_x \\ M_y \\ -rQ \end{pmatrix} = \begin{pmatrix} K_{\text{hub}}(r\beta_s) \\ K_{\text{hub}}(-\beta_c) \\ -rP_{\text{shaft}}/\Omega \end{pmatrix}$$

The force includes a term equal to the rotor thrust times an input blockage factor $f_B = \Delta T / T$. This term accounts for blockage or download, as an alternative to including the drag of the fuselage or a lifting surface in the aircraft trim. For example, f_B can model the tail rotor blockage caused by operation near the vertical tail.

The rotor loads in aircraft axes acting at the center-of-gravity (F^F and M^F) are then calculated from the shaft axis loads (F^S and M^S) and $\Delta z = z_{\text{hub}}^F - z_{\text{cg}}^F$.

The wind axes lift L and drag X are calculated from the net rotor hub force F^F and the rotor velocity v^F. The velocity relative to the air gives the propulsive force direction $e_P = v^F / |v^F|$ (no interference), and the velocity magnitude $V = |v^F|$. The lift and drag components of the force are $XV = -(v^F)^T F^F$ and $L^2 = |F^F|^2 - X^2$.

Aerodynamics

The rotor velocity relative to the air is $v^F = v_{AC}^F + \omega_{AC}^F \times \Delta z^F$ in aircraft axes. The velocities in shaft axes are

$$v^S = C^{SF} v^F = \Omega R \begin{pmatrix} -\mu_x \\ r\mu_y \\ \mu_z \end{pmatrix} \qquad \omega^S = C^{SF} \omega_{AC}^F = \Omega \begin{pmatrix} r\dot{\alpha}_x \\ \dot{\alpha}_y \\ r\dot{\alpha}_z \end{pmatrix}$$

where ΩR is the rotor tip speed. The advance ratio, inflow ratio, and shaft angle-of-attack are defined as $\mu = (\mu_x^2 + \mu_y^2)^{1/2}$, $\lambda = \lambda_i + \mu_z$, and $\alpha = \tan^{-1}(\mu_z / \mu)$. The rotor thrust coefficient is defined as $C_T = T / \rho A (\Omega R)^2$. The dimensionless ideal induced velocity λ_i is calculated from μ, μ_z, and C_T; the dimensional velocity is $v_i = \Omega R \lambda_i$. The ideal induced power is then $P_{\text{ideal}} = T v_i$. Note that for these inflow velocities, the subscript "i" denotes "ideal."

Ideal Inflow

The ideal wake-induced velocity is obtained from the momentum theory result of Glauert:

$$\lambda_i = \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}} = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}}$$

where $\lambda = \lambda_i + \mu_z$, $\lambda_h^2 = |C_T|/2$ (λ_h is always positive), and $s = \text{sign}(C_T)$. This expression is generalized to $\lambda_i = \lambda_h s F(\mu/\lambda_h, s\mu_z/\lambda_h)$. If μ is zero, the equation for λ_i can be solved analytically. Otherwise, for non-axial flow, an iterative Newton-Raphson solution for λ_i is needed.

An approximate equation is used in the turbulent-wake and vortex-ring states to eliminate the singularity of the momentum theory result at ideal autorotation. The momentum theory result is also extended to the case of a ducted fan.

The wake-induced velocity is reduced when the rotor disk is in the proximity of the ground plane,

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$(\lambda_i)_{\text{IGE}} = f_g(\lambda_i)_{\text{OGE}}$. The factor f_g is a function of the scaled rotor height above the ground, z_g/D. The effects of ground plane tilt and rotor velocity are modeled. Several empirical ground effect models are implemented: from Cheeseman and Bennett (ref. 12), from Law (ref. 13), from Hayden (ref. 14), and a curve fit of the interpolation from Zbrozek (ref. 15).

As a simple approximation to nonuniform induced velocity distribution, a linear variation over the disk is used: $\Delta\lambda = \lambda_x r \cos\psi + \lambda_y r \sin\psi$. There are contributions to $\Delta\lambda$ from forward flight and from hub moments, which influence the relationship between flapping and cyclic. The models implemented for the forward flight gradients are based on references 16 to 19. Differential momentum theory is used to calculate the gradients caused by hub moments.

Rotor Forces

When direct control of the rotor thrust magnitude is used, the rotor collective pitch angle θ_0 must be calculated from the thrust C_T/σ. If the commanded variable is the collective pitch angle, then it is necessary to calculate the rotor thrust, resulting in more computation, particularly since all quantities depending on the thrust (inflow, induced power factor, mean drag coefficient) are also unknown. There may be flight states where the commanded thrust can not be produced by the rotor, even with stall neglected in the section aerodynamics. This condition will manifest as an inability to solve for the collective pitch given the thrust. In this circumstance the trim method can be changed so the required or specified thrust is an achievable value, or commanded collective pitch control can be used.

Cyclic control consists of tip-path plane command, requiring calculation of the rotor cyclic pitch angles from the flapping; or no-feathering plane command, requiring calculation of the tip-path plane tilt from the cyclic control angles. The longitudinal tip-path plane tilt is β_c (positive forward) and the lateral tilt is β_s (positive toward retreating side). The longitudinal cyclic pitch angle is θ_s (positive aft), and the lateral cyclic pitch angle is θ_c (positive toward retreating side). Tip-path plane command is appropriate for main rotors. For rotors with no cyclic pitch, no-feathering plane command must be used.
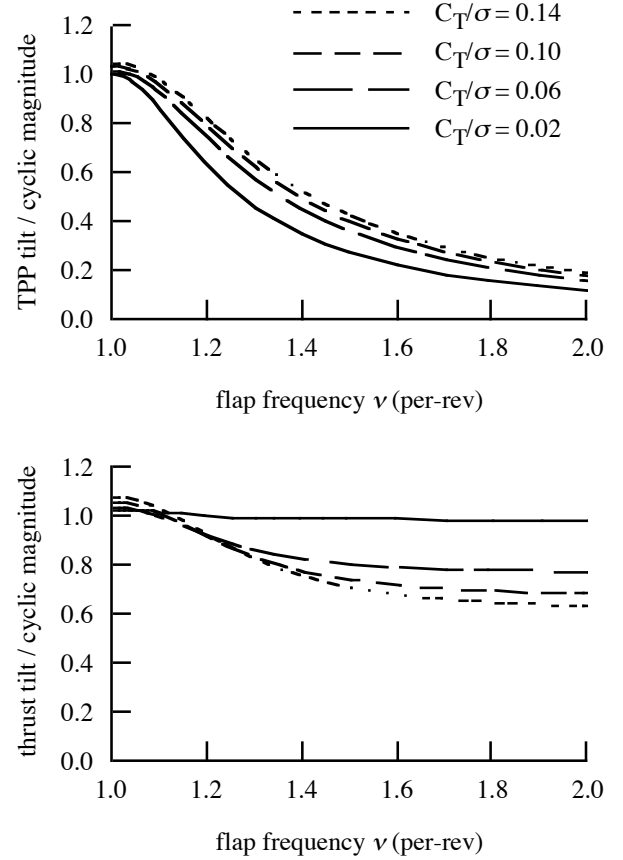

Figure 7. Tip-path plane tilt and thrust vector tilt with cyclic pitch.

The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z, x, y-axis directions respectively). The aerodynamic analysis is conducted for a clockwise rotating rotor, with appropriate sign changes for lateral velocity, flapping, and force. The analysis is conducted in dimensionless form, based on the actual radius and rotational speed of the flight state. The inplane hub forces are produced by tilt of the thrust vector with the tip-path plane, plus forces in the tip-path plane, and profile terms (produced by the blade drag coefficient). The orientation of the tip-path axes relative to the shaft axes is C^{PS}, which is determined by β_c and β_s. Then

$$\begin{pmatrix} C_H \\ C_Y \\ C_T \end{pmatrix} = C^{SP} \begin{pmatrix} 0 \\ 0 \\ C_T/C_{33}^{SP} \end{pmatrix} + \begin{pmatrix} C_{H\,\text{tpp}} \\ rC_{Y\,\text{tpp}} \\ 0 \end{pmatrix} + \begin{pmatrix} C_{Ho} \\ rC_{Yo} \\ 0 \end{pmatrix}$$

The inplane forces relative to the tip-path plane can be neglected or calculated by blade element theory. Note that with thrust and tip-path plane command and $C_{H\,\text{tpp}}$ and $C_{Y\,\text{tpp}}$ neglected, solving for the rotor collective and cyclic pitch angles is not necessary. In general the inplane forces

relative to the tip-path plane are not zero, and may be significant, as for a rotor with large flap stiffness. Figure 7 shows the tip-path plane tilt and thrust vector tilt with cyclic pitch control (no-feathering plane tilt) as a function of flap stiffness (per-rev frequency) for several rotor thrust values. The difference between tip-path plane tilt and thrust vector tilt is caused by tilt of the thrust vector relative to the tip-path plane.

The profile inplane forces can be obtained from simplified equations, or calculated by blade element theory. The simplified method uses:

$$\begin{pmatrix} C_{Ho} \\ C_{Yo} \end{pmatrix} = \frac{\sigma}{8} c_{d\,\mathrm{mean}} F_H \begin{pmatrix} \mu_x/\mu \\ -\mu_y/\mu \end{pmatrix}$$

where the mean drag coefficient $c_{d\,\mathrm{mean}}$ is from the profile power calculation. The function F_H accounts for the increase of the blade section velocity with rotor edgewise and axial speed:

$$F_H = 4\frac{1}{2\pi} \int_0^{2\pi} \int_0^1 ((r+\mu\sin\psi)^2 + (\mu\cos\psi)^2 + \mu_z^2)^{1/2}$$
$$\times (r\sin\psi + \mu)\, dr\, d\psi$$
$$\cong \sqrt{1+V^2}\left(3\mu + \frac{1}{4}\mu^3 \frac{V^2-1}{(1+V^2)^2}\right) + \left(\mu\mu_z^2 + \frac{3}{4}\mu^3\right)\ln\left[\frac{\sqrt{1+V^2}+1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$ (approximation from Harris).

Blade Element Theory

Blade element theory is the basis for the solution for the collective and cyclic pitch angles (or flap angles) and evaluation of the rotor inplane hub forces. The section aerodynamics are described by lift varying linearly with angle-of-attack, $c_\ell = c_{\ell\alpha}\alpha$ (no stall), and a constant mean drag coefficient $c_{d\,\mathrm{mean}}$ (from the profile power calculation). The analysis is conducted in dimensionless form (based on density ρ, rotor rotational speed Ω, and blade radius R of the flight state). The blade section aerodynamic environment is described by the three components of velocity, from which the yaw and inflow angles are obtained, and then the angle-of-attack α. The blade pitch consists of collective, cyclic, twist, and pitch-flap coupling terms. The flap motion is rigid rotation about a hinge with no offset, and only coning and once-per-revolution terms are considered. The inflow includes gradients caused by edgewise flight and hub moments. The effect of the inflow produced by hub moments is to introduce a lift-deficiency function in the flap response.

Integrating the section lift, drag, and radial forces over the blade radial coordinate and azimuth gives the total rotor thrust, drag, and side forces. Integrating terms produced

by the section drag coefficient gives the profile inplane forces, C_{Ho} and C_{Yo}; using blade element theory to evaluate these accounts for the planform and root cutout, while using F_H implies a rectangular blade and no root cutout (plus at most a 1% error approximating the exact integration). The remaining terms in the section forces produce the inplane loads relative to the tip-path plane, $C_{H\,\mathrm{tpp}}$ and $C_{Y\,\mathrm{tpp}}$.

Evaluating these inplane forces requires the collective and cyclic pitch angles, and the flapping motion. The thrust equation must be solved for the rotor collective pitch or the rotor thrust. The relationship between cyclic pitch and flapping is defined by the rotor flap dynamics. The flap motion is rigid rotation about a central hinge, with a flap frequency $\nu > 1$ for articulated or hingeless rotors. The flapping equation of motion, including the effects of precone and the inertial loads of shaft angular motion, is harmonically analyzed. The result is equations for the mean (coning) and 1/rev (tip-path plane tilt) flap motion. The solution for the coning is largely decoupled by introducing the rotor thrust coefficient. A separate flap frequency ν_0 is used for coning, in order to model teetering and gimballed rotors. For an articulated rotor, zero precone should be used.

The thrust and flapping equations of motion must be solved for the unknown angles. For tip-path plane command, the flapping os known, so the equations are solved for θ_c, and θ_s; for no-feathering plane command, the cyclic pitch is known, so the equations are solved for β_c, and β_s. For thrust command the equations are solved for $\theta_{0.75}$; for collective pitch command, the equations are solved for C_T/σ. However, if $\partial T/\partial\theta_{0.75}$ is small, the solution may not produce a reasonable collective. A Newton–Raphson solution method is used.

Power

The rotor power consists of induced, profile, and parasite terms: $P = P_i + P_o + P_p$. The rotor parasite power (including climb/descent power for the aircraft) is obtained from the wind axis drag force: $P_p = -XV = (v^F)^T F^F$.

The induced power is calculated from the ideal power: $P_i = \kappa P_{\mathrm{ideal}} = \kappa f_D T v_{\mathrm{ideal}}$. The empirical factor κ accounts for the effects of nonuniform inflow, non-ideal span loading, tip losses, swirl, blockage, and other phenomenon that increase the induced power losses ($\kappa > 1$). For a ducted fan, $f_D = f_W/2$ is introduced, where f_W is the ratio of the far wake induced velocity to the induced velocity at the disk. The induced power at zero thrust is

zero in this model (or accounted for as a profile power increment). If κ is deduced from an independent calculation of induced power, nonzero P_i at low thrust will be reflected in large κ values.

The profile power is calculated from a mean blade drag coefficient: $P_o = \rho A (\Omega R)^3 C_{Po}$, $C_{Po} = (\sigma/8)c_{d\,\text{mean}}F_P$. The function $F_P(\mu, \mu_z)$ accounts for the increase of the blade section velocity with rotor edgewise and axial speed:

$$F_P = 4\frac{1}{2\pi}\int_0^{2\pi}\int_0^1 ((r+\mu\sin\psi)^2 + (\mu\cos\psi)^2 + \mu_z^2)^{3/2}dr\,d\psi$$

$$\cong \sqrt{1+V^2}\left(1 + \tfrac{5}{2}V^2 + \tfrac{3}{8}\mu^2\frac{4+7V^2+4V^4}{(1+V^2)^2} - \frac{9}{16}\frac{\mu^4}{1+V^2}\right)$$

$$+\left(\tfrac{3}{2}\mu_z^4 + \tfrac{3}{2}\mu_z^2\mu^2 + \tfrac{9}{16}\mu^4\right)\ln\left[\frac{\sqrt{1+V^2}+1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$ (approximation from Harris). This expression is exact when $\mu = 0$, and $F_P \approx 4V^3$ for large V.

Two performance methods are implemented: the energy method and the table method. The induced power factor and mean blade drag coefficient are obtained from equations with the energy method, or from tables with the table method. With the table method, the induced power factor κ is obtained from an input array (linearly interpolated) that can be a function of edgewise advance ratio μ or axial velocity ratio μ_z, and of the blade loading C_T/σ; and the mean drag coefficient $c_{d\,\text{mean}}$ or $c_{d\,\text{mean}}F_P$ is obtained from an input array (linearly interpolated) that can be a function of μ and C_T/σ. Optionally κ and $c_{d\,\text{mean}}$ can be specified for each flight state, superseding the performance method values.

Energy Performance Method: Induced Power

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. Reference values of κ are specified for hover, axial cruise (propeller), and edgewise cruise (helicopter): κ_{hover}, κ_{prop}, κ_{edge}. Two models are implemented: constant model and standard model. The constant model uses $\kappa = \kappa_{\text{hover}}$ if $\mu = \mu_z = 0$; or $\kappa = \kappa_{\text{prop}}$ if $|\mu| < 0.1\,|\mu_z|$; or $\kappa = \kappa_{\text{edge}}$ otherwise.

The standard model calculates an axial flow factor κ_{axial} from the values κ_{hover}, κ_{climb}, and κ_{prop}. Let $\Delta = C_T/\sigma - (C_T/\sigma)_{\text{ind}}$. For hover and low speed axial climb, including a variation with thrust, the inflow factor is

$$\kappa_h = \kappa_{\text{hover}} + k_{h1}\Delta_h + k_{h2}|\Delta_h|^{X_{h2}}$$

$$+ (\kappa_{\text{climb}} - \kappa_{\text{hover}})\tfrac{2}{\pi}\tan^{-1}\left[((|\mu_z|/\lambda_h)/M_{\text{axial}})^{X_{\text{axial}}}\right]$$

where $|\mu_z|/\lambda_h = M_{\text{axial}}$ is the midpoint of the transition between hover and climb. Figure 8 illustrates κ in hover (with a minimum value). A polynomial describes the variation with axial velocity, scaled so $\kappa = \kappa_h$ at $\mu_z = 0$ and $\kappa = \kappa_p$ at $\mu_{z\,\text{prop}}$. Including variations with thrust and shaft angle:

$$\kappa_p = \kappa_{\text{prop}} + k_{p1}\Delta_p + k_{p2}|\Delta_p|^{X_{p2}} + k_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}}$$

$$\kappa_{\text{axial}} = \kappa_h + k_{a1}\mu_z + S_a(k_{a2}\mu_z^2 + k_{a3}\mu_z^{X_a})$$

where S_a accomplishes the scaling. A polynomial describes the variation with edgewise advance ratio, scaled so $\kappa = \kappa_{\text{axial}}$ at $\mu = 0$ and $\kappa = f_{\text{off}}\kappa_{\text{edge}}$ at $\mu = \mu_{\text{edge}}$. Thus the induced power factor is

$$\kappa = \kappa_{\text{axial}} + k_{e1}\mu + S_e(k_{e2}\mu^2 + k_{e3}\mu^{X_e})$$

where S_e accomplishes the scaling. The function $f_{\text{off}} = 1 - k_{o1}(1 - e^{-k_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$. Figure 9 illustrates κ in edgewise flight. Minimum and maximum values of the induced power factor are also specified.
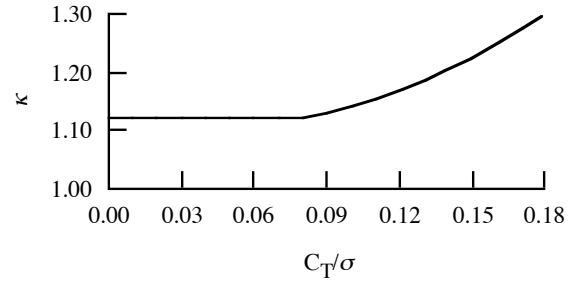
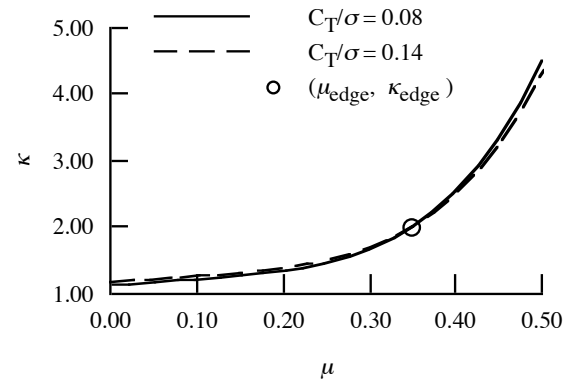Figure 8. Induced power factor in hover (with minimum $\kappa = 1.12$).

Figure 9. Induced power factor in edgewise flight.

Energy Performance Method: Profile Power

The profile power is calculated from a mean blade drag coefficient: $C_{Po} = (\sigma/8)c_{d\,\text{mean}}F_P$. Since the blade mean lift coefficient is $c_\ell \cong 6C_T/\sigma$, the drag coefficient is

estimated as a function of blade loading C_T/σ (using thrust-weighted solidity). With separate estimates of the basic, stall, and compressibility drag, the mean drag coefficient is

$$c_{d\,\text{mean}} = \chi S(c_{d\,\text{basic}} + c_{d\,\text{stall}} + c_{d\,\text{comp}})$$

where χ is a technology factor. The factor $S = (Re_{\text{ref}}/Re)^{0.2}$ accounts for Reynolds number effects on the drag coefficient; Re is based on the thrust-weighted chord, $0.75V_{\text{tip}}$, and the kinematic velocity of the flight state; and Re_{ref} corresponds to the input drag coefficient information. The following models are implemented for the basic drag.

a) Array model: The basic drag $c_{d\,\text{basic}}$ is input as a function of C_T/σ (linearly interpolated array).

b) Equation model: The basic drag $c_{d\,\text{basic}}$ is a quadratic function of C_T/σ, plus an additional term allowing faster growth at high (sub-stall) angles of attack. Let $\Delta = |C_T/\sigma - (C_T/\sigma)_{D\,\text{min}}|$, where $(C_T/\sigma)_{D\,\text{min}}$ corresponds to the minimum drag; and $\Delta_{\text{sep}} = |C_T/\sigma| - (C_T/\sigma)_{\text{sep}}$. Values of the basic drag equation are specified for helicopter (hover and edgewise) and propeller (axial climb and cruise) operation:

$$c_{dh} = d_{0\,\text{hel}} + d_{1\,\text{hel}}\Delta + d_{2\,\text{hel}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}}$$

$$c_{dp} = d_{0\,\text{prop}} + d_{1\,\text{prop}}\Delta + d_{2\,\text{prop}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}} + d_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}}$$

The separation term is present only if $\Delta_{\text{sep}} > 0$. The helicopter and propeller values are interpolated as a function of μ_z:

$$c_{d\,\text{basic}} = c_{dh} + (c_{dp} - c_{dh})\tfrac{2}{\pi}\tan^{-1}\left(|\mu_z|/\lambda_h\right)$$

so $|\mu_z|/\lambda_h = 1$ is the midpoint of the transition.

The stall drag increment represents the rise of profile power caused by the occurrence of significant stall on the rotor disk. Let $\Delta_s = |C_T/\sigma| - (f_s/f_{\text{off}})(C_T/\sigma)_s$, where f_s is an input factor. The function $f_{\text{off}} = 1 - d_{o1}(1 - e^{-d_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$. Then

$$c_{d\,\text{stall}} = d_{s1}\Delta_s^{X_{s1}} + d_{s2}\Delta_s^{X_{s2}}$$

(zero if $\Delta_s \leq 0$). The blade loading at which the stall affects the entire rotor power, $(C_T/\sigma)_s$, is an input function of the velocity ratio $V = (\mu^2 + \mu_z^2)^{1/2}$.

Figure 10 shows typical stall functions $(C_T/\sigma)_s$ for two rotors with different airfoils. For reference, typical rotor steady and transient load limits are also shown.
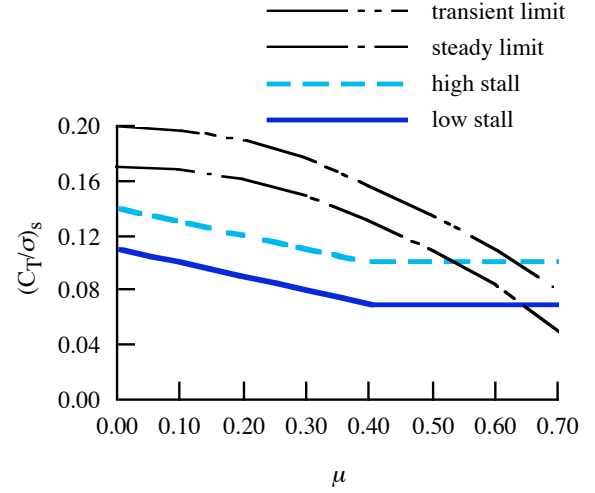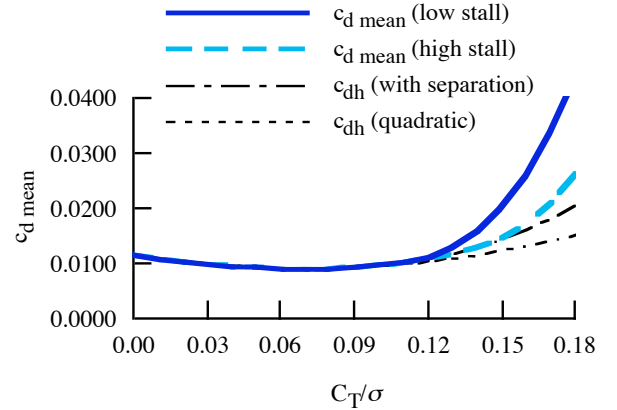

Figure 10. Profile power stall function.

Figure 11. Mean drag coefficient in hover.

The compressibility drag increment depends on the advancing tip Mach number M_{at}. Drag divergence and similarity models are implemented. For the drag divergence model, let $\Delta M = M_{at} - M_{dd}$, where M_{dd} is the drag divergence Mach number of the tip section. Then the compressibility increment in the mean drag coefficient is

$$c_{d\,\text{comp}} = d_{m1}\Delta M + d_{m2}\Delta M^{X_m}$$

(ref. 20). M_{dd} is a function of the advancing tip lift coefficient (available from blade element theory) and the tip airfoil thickness-to-chord ratio (ref. 21). The similarity model is based on transonic small-disturbance theory scaling (refs. 22–23).

Figure 11 illustrates the mean drag coefficient in hover, showing c_{dh} without and with the separation term, and the total for the high-stall and-low stall cases. Figure 12 illustrates the mean drag coefficient in forward flight,

showing the compressibility term $c_{d\,\text{comp}}$, and the growth in profile power with C_T/σ and μ as the stall drag increment increases.
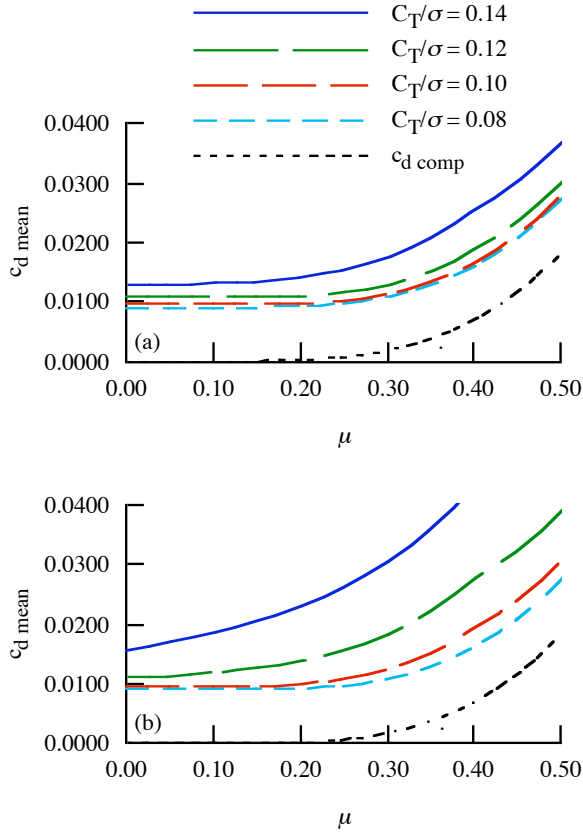

Figure 12. Mean drag coefficient in forward flight; (a) high stall; (b) low stall.

Twin Rotors

For twin rotors, the induced power is determined by the induced velocity of the rotor system, not the individual rotors. The induced power is still obtained using $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$ for each rotor, but the ideal induced velocity is calculated for an equivalent thrust C_{Te} based on the thrust and geometry of both rotors (see refs. 4 and 6). The profile power calculation is not changed for twin rotors.

Performance Indices

Several performance indices are calculated for each rotor. The induced power factor is $\kappa = P_i/P_{\text{ideal}}$. The rotor mean drag coefficient is $c_d = (8C_{Po}/\sigma)/F_P$. The rotor effective lift-to-drag ratio is a measure of the induced and profile power: $L/D_e = VL/(P_i + P_o)$. The hover figure of merit is

$M = Tf_D v/P$. The propeller propulsive efficiency is $\eta = TV/P$.

Interference

The rotor can produce aerodynamic interference velocities at the other components (fuselage, wings, tails). The induced velocity at the rotor disk is κv_i, acting opposite the thrust (z-axis of tip-path plane axes). So $v_{\text{ind}}^P = -k^P \kappa v_i$, and $v_{\text{ind}}^F = C^{FP} v_{\text{ind}}^P$. The total velocity of the rotor disk relative to the air consists of the aircraft velocity and the induced velocity from this rotor: $v_{\text{total}}^F = v^F - v_{\text{ind}}^F$. The direction of the wake axis is thus $e_w^P = -C^{PF} v_{\text{total}}^F / |v_{\text{total}}^F|$. The angle of the wake axis from the thrust axis is $\chi = \cos^{-1} |(k^P)^T e_w^P|$.

The interference velocity v_{int}^F at each component is proportional to the induced velocity v_{ind}^F (and is in the same direction), with factors accounting for the stage of wake development and the position of the component relative to the rotor wake:

$$v_{\text{int}}^F = K_{\text{int}} f_W f_z f_r f_t v_{\text{ind}}^F$$

The factors $f_W f_z$ account for axial development of the wake velocity, the factor f_r accounts for immersion in the wake, and K_{int} is an input empirical factor. An additional factor f_t for twin rotors is included.

Optionally the development along the wake axis can be a step function ($f_W f_z = 0$, 1, or f_W for above the rotor, on the rotor disk, or below the rotor disk, respectively); or the wake develops with the nominal or input rate of change. Optionally the wake immersion can use the contracted radius or the uncontracted radius; be a step function ($f_r = 1$ and 0 inside and outside the wake boundary); be always immersed ($f_r = 1$ always); or use an input transition distance.

Optionally the interference factor K_{int} can be reduced from an input value at low speed to zero at high speed, with linear variation over a specified speed range. To account for the extent of the wing or tail area immersed in the rotor wake, the interference velocity can be calculated at several points along the span and averaged.

Drag

The rotor component includes drag forces acting on the hub and spinner and on the pylon. The aerodynamic velocity relative to the air is calculated in component axes, v^B. The angle-of-attack α and dynamic pressure q are calculated from v^B. The reference areas for the drag coefficients are the rotor disk area (for hub drag), pylon wetted area, and spinner wetted area.

The hub drag can be fixed, specified as a drag area D/q; or scaled, specified as a drag coefficient C_D based on the rotor disk area; or estimated based on the gross weight, using either a squared-cubed relationship or a square-root relationship. Based on historical data, the drag coefficient $C_D = 0.004$ for typical hubs, $C_D = 0.0024$ for current low drag hubs, and $C_D = 0.0015$ for faired hubs. For the squared-cubed relationship: $(D/q)_{hub} = k(W_{MTO}/1000)^{2/3}$ where W_{MTO} is the maximum take-off gross weight. Based on historical data, $k = 1.4$ for typical hubs, $k = 0.8$ for current low drag hubs, and $k = 0.5$ for faired hubs (k in English units). For the square-root relationship: $(D/q)_{hub} = k(W_{MTO}/N_{rotor})^{1/2}$ where W_{MTO}/N_{rotor} is the maximum take-off gross weight per lifting rotor. Based on historical data (ref. 24), $k = 0.074$ for single rotor helicopters, $k = 0.049$ for tandem rotor helicopters (probably a blade number effect), $k = 0.038$ for hingeless rotors, and $k = 0.027$ for faired hubs (k in English units).

The hub vertical drag can be fixed, specified as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on the rotor disk area.

The pylon forward flight drag and vertical drag are specified as drag area or drag coefficient, based on the pylon wetted area. The spinner drag is specified as drag area or drag coefficient, based on the spinner wetted area.

The drag coefficient for the hub or pylon at angle-of-attack α is $C_D = C_{D0} + (C_{DV} - C_{D0}) |\sin \alpha|^{X_d}$; optionally the variation can be quadratic ($X_d = 2$). Then the total component drag force is

$$D = qA C_{D\,hub} + q S_{pylon} C_{D\,pylon} + q S_{spin} C_{D\,spin}$$

FORCE

The force component is an object that can generate a force acting on the aircraft, possibly used for lift, or propulsion, or control. The amplitude of the force can be a fixed value, or connected to an aircraft control for trim. The direction of the force can be fixed, or connected to aircraft control.

The force generation requires a fuel flow that is calculated from an input thrust specific fuel consumption: $\dot{w} = A(\text{sfc})$, where A is the force amplitude, and the units of sfc are pound/hour/pound or kilogram/hour/Newton.

The force component weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group. The force component weight is calculated from a specific weight and the design maximum force F_{max}, plus a fixed increment: $W = SF_{max} + \Delta W$.

WING

The aircraft can have one or more wings, or no wings. The wing is described by planform area S, span b, mean chord $c = S/b$, and aspect ratio $AR = b^2/S$. These parameters are for the entire wing. The geometry is specified in terms of two of the following parameters: S or wing loading W/S, b (perhaps calculated from other geometry), c, AR. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$. Optionally the span can be calculated from a specified ratio to the span of another wing.

For the tiltrotor configuration, the wing span can be calculated from the fuselage and rotor geometry (rotor radius, rotor-fuselage clearance, and fuselage width). The wing span can be calculated from the rotor hub position (regardless of how the rotor position is determined). Optionally the wing span can be calculated from an appropriate specification of all wing panel widths.

η_{Ep}	outboard panel edge, $\eta = y/(b/2)$
η_{Op}	wing station
λ_{Op}	chord ratio, $\lambda = c/c_{ref}$
c_{Op}	chord
Λ_p	sweep (+ aft)
δ_p	dihedral (+ up)
x_{Ip}	aero center offset (inboard, + aft)
z_{Ip}	aero center offset (inboard, + up)
b_p	span (each side)
c_p	mean chord
S_p	area = $2b_p c_p$
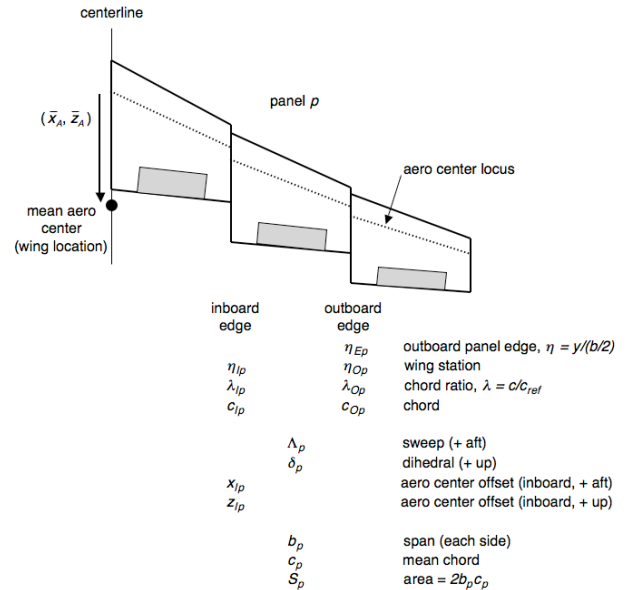
Figure 13. Wing geometry (symmetric, only right half-wing shown).

Panels

The wing planform is defined in terms of one or more wing panels (figure 13). Symmetry of the wing is assumed. Each panel is a trapezoid, with a straight aerodynamic center and linear taper. The aerodynamic center locus (in wing axes) is defined by sweep, dihedral, and offsets at the inboard edge relative to the aerodynamic center of the previous panel. The wing position z^F is the mean aerodynamic center. The offset of the mean

aerodynamic center from the root chord aerodynamic center is calculated (so the wing planform can be drawn; typically the aerodynamic center is drawn as the quarter-chord).

A panel is characterized by span b_p (each side), mean chord c_p, and area $S_p = 2b_p c_p$ (both sides). The taper is defined by inboard and outboard chord ratios. The span for each panel (if there are more than two panels) can be a fixed input, a fixed ratio of the wing span, or free. The panel outboard edge (except for the wing tip) can be at a fixed input position, at a fixed station, calculated from the fuselage and rotor geometry, calculated from the hub position, or adjusted. The specification of panel spans and panel edges must be consistent, and sufficient to determine the wing geometry. To complete the definition of the geometry, one of the following quantities is specified for each panel: panel area S_p; ratio of panel area to wing area, S_p / S; panel mean chord c_p; ratio of panel mean chord to wing mean chord, c_p / c; inboard and outboard chord ratios; or free. The total wing area equals the sum of all panel areas.

Controls

The wing control variables are flap, flaperon, aileron , and incidence. The flaperon deflection can be specified as a fraction of flap deflection, or as an increment relative to the flap deflection, or the flaperon can be independent of the flap. The flaperon and aileron are the same surface, generating symmetric and antisymmetric loads respectively, hence with different connections to pilot controls. With more than one wing panel, each panel can have control variables. The outboard panel control or incidence and be specified independently, or in terms of the root panel control or incidence (either fraction or increment). Each control is described by the ratio of the control surface chord to the wing panel chord; and by the ratio of the control surface span to wing panel span.

Aerodynamics

The wing is at position z^F, where the aerodynamic forces act. The component axes are the aircraft body axes. The aerodynamic velocity of the wing relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack and dynamic pressure are calculated from v^B. The reference area for the wing aerodynamic coefficients is the planform area, S.

The wing vertical drag can be fixed, specified as a drag area $(D/q)_V$; or scaled, specified as a drag coefficient C_{DV} based on the wing area; or calculated from an airfoil section drag coefficient (for -90 deg angle-of-attack) and the wing area immersed in the rotor wake (including changes in wing area due to flap and flaperon deflection).

The wing lift, pitch moment, and roll moment are defined in scaled form (coefficients). From the control surface deflection and geometry, the lift coefficient, maximum lift angle, moment coefficient, and drag coefficient increments are evaluated. These increments are the sum of contributions from flap and flaperon deflection, hence weighted by the control surface area. The drag coefficient increment includes the contribution from aileron deflection.

The wing lift is defined in terms of lift curve slope and maximum lift coefficient. The three-dimensional lift curve slope is input directly or calculated from the two-dimensional lift curve slope and the wing aspect ratio.

The only wing roll moment considered is that produced by aileron control. Typically the flaperon and aileron are the same surface, but they are treated separately in this model. The aileron effectiveness is specified in terms of the lateral position of the aileron aerodynamic center, measured from the wing centerline (defined as a fraction of the wing semi-span).

The drag area or drag coefficient is defined for forward flight and vertical flight. For small angle-of-attack, the drag increases proportional to $(1 + K |\alpha|^X)$, using input factor K and exponent X, plus an additional term representing separation drag. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e \, AR}$$

Conventionally, the Oswald efficiency e can represent the wing parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). The wing-body interference is specified as a drag area, or a drag coefficient based on the wing area. Then

$$D = qSC_D = qS(C_{Dp} + C_{Di} + C_{Dwb})$$

is the drag force.

The wing panels can have separate controls and different incidence angles. Thus the lift, drag, and moment coefficients are evaluated separately for each panel, based on the panel area S_p and mean chord c_p. Then the total wing coefficients are the sum of the panel coefficients, weighted by the area ratios (and chord ratio for pitching moment). The three-dimensional lift curve slope is

calculated for the entire wing and used for each panel. The induced drag is calculated for the entire wing, from the total lift.

Interference

With more than one wing, the interference velocity at other wings is proportional to the induced velocity of the wing producing the interference: $v_{int}^F = K_{int} v_{ind}^F$. The induced velocity is obtained from the induced drag:

$$\alpha_{ind} = v_{ind} / | v^B | = C_{Di} / C_L = C_L / (\pi e \, AR)$$

and $v_{ind}^F = C^{FB} k^B | v^B | \alpha_{ind}$. For tandem wings, typically $K_{int} = 2$ for the interference of the front wing on the aft wing, and $K_{int} = 0$ for the interference of the aft wing on the front wing. For biplane wings, the mutual interference is typically $K_{int} = 0.7$ (upper on lower, and lower on upper). The induced drag is then

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e \, AR} + C_L \sum K_{int} \alpha_{int}$$

where the sum is over all other wings.

The wing interference at the tail produces an angle-of-attack change $\varepsilon = E(C_L / C_{L\alpha})$, where $E = d\varepsilon / d\alpha$ is an input factor determined by the aircraft geometry. The change in orientation of the wing velocity v^B produces the interference velocity v_{int}^F at the tail.

Wing Extensions

The wing can have extensions, defined as wing portions of span b_X at each wing tip. For the tiltrotor configuration in particular, the wing weight depends on the distribution of wing area outboard (the extension) and inboard of the rotor and nacelle location. Wing extensions are defined as a set of wing panels at the tip. The extension span and area are the sum of the panel quantities.

Optionally the wing extensions can be considered a kit, hence the extensions can be absent for designated flight conditions or missions. As a kit, the wing extension weight is considered fixed useful load. With wing extensions removed, the aerodynamic analysis considers only the remaining wing panels.

EMPENNAGE

The aircraft can have one or more tail surfaces, or no tail surface. Each tail is designated as horizontal or vertical. The tail is described by planform area S, span b, chord $c = S / b$, and aspect ratio $AR = b^2 / S$. The tail volume V can be referenced to rotor radius and disk area; to wing area and chord for horizontal tails; or to wing area and span for vertical tails. The geometry is specified in terms of S or V, and b or AR or c.

The tail is at position z^F, where the aerodynamic forces act. The scaled input for tail position can be referenced to the fuselage length, or the rotor radius.

The horizontal tail can have a cant angle ϕ (positive tilt to left). The control variables are elevator and incidence.

The convention for nominal orientation of the vertical tail is positive lift to the left, so aircraft sideslip (positive to right) generates positive tail angle-of-attack and positive tail lift. The vertical tail can have a cant angle ϕ (positive tilt to right). The control variables are rudder and incidence.

The aerodynamic velocity of the tail relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack and dynamic pressure are calculated from v^B. The reference area for the tail aerodynamic coefficients is the planform area, S.

The tail lift is defined in scaled form (coefficients). From the elevator or rudder deflection and geometry, the lift coefficient, maximum lift angle, and drag coefficient increments are evaluated. The tail lift is defined in terms of lift curve slope and maximum lift coefficient. The three-dimensional lift curve slope is input directly or calculated from the two-dimensional lift curve slope and the wing aspect ratio.

The drag area or drag coefficient is defined for forward flight and vertical flight. For small angle-of-attack, the drag increases proportional to $(1 + K | \alpha |^X)$, using input factor K and exponent X. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e \, AR}$$

Conventionally, the Oswald efficiency e can represent the tail parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). Then

$$D = qSC_D = qS(C_{Dp} + C_{Di})$$

is the drag force.

FUEL TANK

The fuel tank capacity $W_{fuel\text{-}cap}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{fuel\text{-}miss}$ (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}}W_{\text{fuel-miss}}, W_{\text{fuel-miss}} + W_{\text{reserve}})$$

where $f_{\text{fuel-cap}} \geq 1$ is an input factor. Alternatively, the fuel tank capacity $W_{\text{fuel-cap}}$ can be input. The corresponding volumetric fuel tank capacity is $V_{\text{fuel-cap}} = W_{\text{fuel-cap}}/\rho_{\text{fuel}}$ (gallons or liters), where ρ_{fuel} is the fuel density (input as weight per volume).

For missions that are not used to size the fuel tank, the fuel weight may be fallout, or the fuel weight may be specified (with or without auxiliary tanks). The fuel weight for a flight condition or the start of a mission is specified as an increment, plus a fraction of the fuel tank capacity, plus auxiliary tanks:

$$W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}, W_{\text{fuel-cap}})$$
$$+ \Sigma N_{\text{auxtank}}W_{\text{aux-cap}}$$

where $W_{\text{aux-cap}}$ is the capacity of each auxiliary fuel tank.

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at V_{be}. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{fuel-miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{miss-seg}} + f_{\text{res}}W_{\text{fuel-cap}}$, If more than one criterion for reserve fuel is specified, the maximum reserve is used.

Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $W_{\text{aux-cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, N_{auxtank} for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.

The weight and drag of N_{auxtank} tanks are included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel weight at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is an input fraction of the tank capacity. The drag area for each auxiliary tank is specified, $(D/q)_{\text{auxtank}}$.

<div align="center">

PROPULSION

</div>

The propulsion group is a set of components and engine groups connected by a drive system. The engine model describes a particular engine, which is used in one or more

engine groups. The components (rotors) define the power required. The engine groups define the power available. Figure 14 illustrates the power flow.
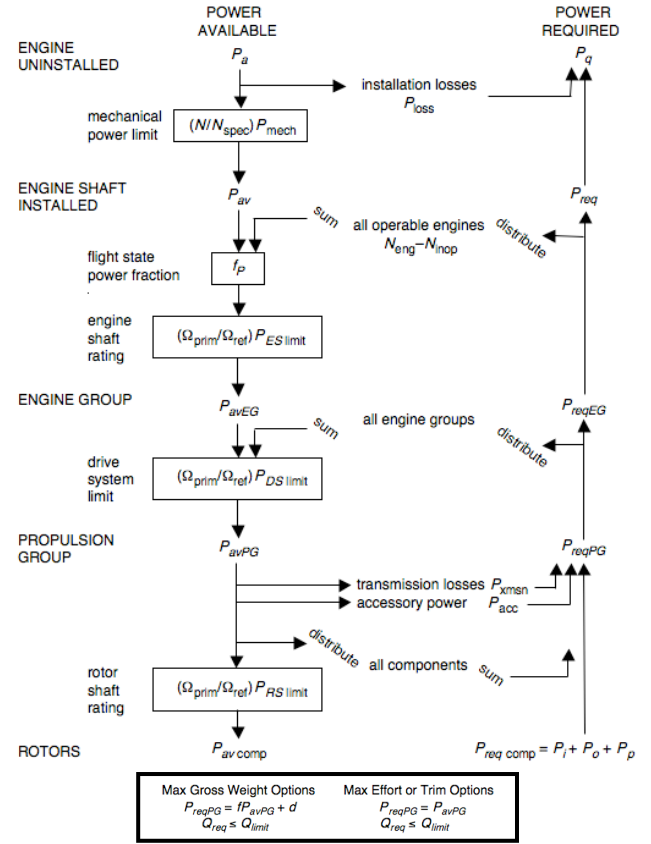

Figure 14. Power flow.

Drive System

The drive system defines gear ratios for all the components that it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\text{tip-ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different

engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$.

For dependent rotors, either the gear ratio is specified (for each drive system state); or a tip speed is specified, and the gear ratio calculated ($r = \Omega_{dep}/\Omega_{prim}$, $\Omega = V_{tip\text{-}ref}/R$).

For the engine group, either the gear ratio is specified (for each drive system state); or the gear ratio calculated from the specification engine turbine speed $\Omega_{spec} = (2\pi/60)N_{spec}$ and the reference tip speed of the primary rotor ($r = \Omega_{spec}/\Omega_{prim}$, $\Omega_{prim} = V_{tip\text{-}ref}/R$). The latter option means the specification engine turbine speed N_{spec} corresponds to $V_{tip\text{-}ref}$ for all drive system states.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius the rotational speed of the primary rotor is obtained ($\Omega_{prim} = V_{tip\text{-}ref}/R$); from the gear ratios, the rotational speed of dependent rotors ($\Omega_{dep} = r\Omega_{prim}$) and the engine groups ($N = (60/2\pi)r_{eng}\Omega_{prim}$) are obtained; and from the rotor radius, the tip speed of the dependent rotor ($V_{tip} = \Omega_{dep}R$) is obtained. The flight state specification of the tip speed can be an input value; the reference tip speed; a function of flight speed or a conversion schedule; or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor f_{gear} on the gear ratio when the speeds of the dependent rotors and engines are evaluated. The factor f_{gear} is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional control conversion schedule is defined in terms of conversion speeds: hover and helicopter mode for speeds below V_{Chover}, cruise mode for speeds above $V_{Ccruise}$, and conversion mode between. The tip speed is $V_{tip\text{-}hover}$ in helicopter and conversion mode, and $V_{tip\text{-}cruise}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function

of flight speed. Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from an input C_T/σ or μ or M_{at}.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), or the reference tip speed of a dependent rotor, or a rotor radius, or the specification engine turbine speed N_{spec}. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

Power Required

The component power required P_{comp} is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power: $P_{reqPG} = P_{comp} + P_{xmsn} + P_{acc}$. The transmission losses are calculated as an input fraction ℓ_{xmsn} of the component power, plus windage loss. Accessory losses are calculated as an input constant, a fraction of power required (such as ECU losses), IRS fan loss (if IR suppression system is on), and terms that scale with air density and rotor speed.

The power required for the propulsion group must be distributed to the engine groups. With only one engine group, $P_{reqEG} = P_{reqPG}$. An engine group power can be fixed at a fraction of the engine power available: $P_{reqEG} = f_P(N_{eng} - N_{inop})P_{eng}$ at the specified rating (each operating engine at $f_P P_{eng}$). The power required for the remaining (perhaps all) engine groups is distributed proportional to the engine rated power. The fuel flow of the propulsion group is obtained from the sum over the engine groups: $\dot{w}_{reqPG} = \Sigma\, \dot{w}_{reqEG}$.

Drive System Rating

The drive system rating is defined as a power limit, $P_{DS\,limit}$. The rating is properly a torque limit, $Q_{DS\,limit} = P_{DS\,limit}/\Omega$, but is expressed as a power limit for clarity. The drive system rating is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft and of each engine group. The power limit is associated with a reference rotational speed, and when applied it is scaled with the rotational speed of the flight state. The rotation speed for the drive system rating $P_{DS\,limit}$ is the hover

speed of the primary rotor of the propulsion group (for the first drive state).

ENGINE GROUP

The engine group consists of one or more engines of a specific type. For each engine type an engine model is defined. The RPTEM engine model is implemented.

The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines N_{eng} is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power P_{eng} of at least one engine group is found (including the first engine group).

Power Available

Given the flight condition and engine rating, the power available P_a is calculated (from the specific power SP_a and mass flow $\dot{m}_a$). The flight condition information includes the altitude, temperature, flight speed, and primary rotor speed; a power fraction f_P; and the states of the engine, drive system, and IR suppressor. The engine turbine speed is $N = (60/2\pi) r_{\text{eng}}/\Omega_{\text{prim}}$, where Ω_{prim} is the current rotor speed and r_{eng} is the gear ratio (depending on the drive system state). If the reference primary rotor speed Ω_{prim} corresponds to the specification turbine speed N_{spec}, then $r_{\text{eng}} = \Omega_{\text{spec}}/\Omega_{\text{prim}}$; alternatively, the engine gear ratio can be a fixed input.

In the engine model, installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$), and then the mechanical limit applied: $P_{av} = \min(P_{av}, rP_{\text{mech}R})$, $r = N/N_{\text{spec}}$. The mechanical limit is properly a torque limit, but is expressed as a power limit for clarity.

The engine model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines): $P_{avEG} = (N_{\text{eng}} - N_{\text{inop}})P_{av}$. The propulsion group power available is obtained from the sum over the engine groups: $P_{avPG} = \Sigma f_P P_{avEG}$, including a specified power fraction f_P.

The drive system rating at the flight condition is $(\Omega_{\text{prim}}/\Omega_{\text{ref}})P_{DS\,\text{limit}}$. Optionally this limit is applied to the propulsion group power: $P_{avPG} = \min(P_{avPG}, P_{DS\,\text{limit}})$. Similarly the engine shaft limit at the flight condition is optionally applied to the engine group power.

Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q, flight condition, and engine rating. The flight condition includes the altitude, temperature, flight speed, and primary rotor speed; and engine, drive system, and IR suppressor states. The engine turbine speed is $N_{\text{eng}} = r_{\text{eng}}\Omega_{\text{prim}}$.

The engine model deals with a single engine. The power required of a single engine is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines): $P_{req} = P_{reqEG}/(N_{\text{eng}} - N_{\text{inop}})$. In the engine model, installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{\text{loss}}$).

The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

$$\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{req}$$
$$\dot{w}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}})\dot{w}_{req}K_{ffd}$$
$$F_{N\,EG} = (N_{\text{eng}} - N_{\text{inop}})F_N$$
$$D_{\text{aux}EG} = (N_{\text{eng}} - N_{\text{inop}})D_{\text{aux}}$$

The fuel flow has also been multiplied by a factor K_{ffd} accounting for deterioration of the engine efficiency.

Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}: $P_{av} = P_a - P_{\text{loss}}$ and $P_q = P_{req} + P_{\text{loss}}$. The inlet ram recovery efficiency η_d is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. Installation effects on the jet thrust are included in the engine model.

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

Control and Loads

The engine orientation is specified by selecting a nominal direction in body axes (usually thrust forward), then applying a yaw angle, then an incidence or tilt angle. The yaw and incidence angles can be connected to the aircraft controls. Hence the incidence and yaw angles can be fixed orientation or can be control variables.

The lateral position of the engine group can be set equal to that of a designated rotor (useful for tiltrotors when the rotor hub lateral position is calculated from the clearance or wing geometry).

The engine group produces a jet thrust F_N, acting in the direction of the engine; a momentum drag D_{aux}, acting in the wind direction; and a nacelle drag D_{nac}, acting in the wind direction. The engine group is at location z^F. The force acting on the aircraft in body axes is thus:

$$F^F = e_f F_N + e_d D_{aux} + e_d D_{nac}$$

where e_f is the engine thrust direction, and e_d is the drag direction.

Nacelle Drag

The engine group includes a nacelle, which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per engine is input, or calculated either from the engine system (engine, exhaust, and accessories) weight or from the engine system plus drive system weight: $S_{wet} = k(w/N_{eng})^{2/3}$, where $w = W_{ES}$ or $w = W_{ES} + W_{gbrs}/N_{EG}$. The reference area is then $S_{nac} = N_{eng}S_{wet}$. The nacelle area is included in the aircraft wetted area if the drag coefficient is nonzero.

REFERRED PARAMETER TURBOSHAFT ENGINE MODEL (RPTEM)

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. This means that engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, a model for the total engine performance is used. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of engine performance data for existing or projected engines, over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck (a computer program). The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The use of referred parameters to curve-fit engine performance data was suggested by David Woodley from Boeing during the JVX program (1983). The RPTEM was developed and documented by Michael P. Scully and Henry Lee of ASRAO, U.S. Army Aeroflightdynamics Directorate (AFDD), with a subsequent implementation written by Sam Ferguson (1995).

Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure $p = p_{std}$. Here the temperatures are in deg R or deg K. The engine characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$. The flight Mach number M is obtained from the aircraft speed.

The inlet ram air temperature ratio and pressure ratio are obtained from M and the inlet ram recovery efficiency η_d: $\theta_M = (1 + 0.2M^2)$ and $\delta_M = (1 + \eta_d 0.2M^2)^{3.5}$, using the ratio of specific heats $\gamma = 1.4$.

Engine Ratings

The power available from a turboshaft engine depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine. Typical engine ratings are MCP (maximum continuous power, no time limit), IRP (intermediate rated power, 30 min), MRP (maximum rated power, 10 min), and CRP (contingency rated power). Engine power is generally specified in terms of SLS static MCP. Takeoff typically uses MRP. CRP is restricted to use in one-engine inoperative (OEI) emergencies.

Performance Characteristics

The engine performance is described by the power available P_a, at each engine rating and the specification engine turbine speed N_{spec}; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce power required P_q at engine turbine speed N; and the gross jet thrust F_g at a given power required P_q. Then the specific power is $SP = P/\dot{m}$, and the specific fuel consumption is $\text{sfc} = \dot{w}/P$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). Referred or corrected engine parameters are used in the model: power $P/(\delta\sqrt{\theta})$, mass flow $\dot{m}/(\delta/\sqrt{\theta})$, specific power $(P/\dot{m})/\theta$, fuel flow $\dot{w}/(\delta\sqrt{\theta})$, thrust F/δ, and turbine speed $N/\sqrt{\theta}$.

For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: power P_{0R}, specific power SP_{0R}, and mechanical power limit $P_{\text{mech}\,R}$. The mass flow is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$. The gross jet thrust F_{g0C} is given at MCP. These characteristics are at the specification turbine speed N_{spec}.

The installed power required P_{req} and power available $P_{av} > P_{req}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust F_N, from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such IR suppressor or cooling air) are treated as momentum drag D_{aux}.

The relationship between net and gross jet thrust is $F_n = F_g - \dot{m}_{req}V = \dot{m}_{req}(V_j - V)$, where V_j is the engine jet exhaust velocity.

The uninstalled power required is P_q, the power available P_a, the gross jet thrust F_g, and net jet thrust F_n. The engine model calculates P_a as a function of flight condition and engine rating; or calculates engine mass flow, fuel flow, and jet thrust at P_q.

Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}: $P_{av} = P_a - P_{\text{loss}}$ and $P_{req} = P_q - P_{\text{loss}}$. The inlet ram recovery efficiency η_d (through θ_M) is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. So

$$P_{av} = P_a(1 - \ell_{in} - \ell_{ex})$$
$$P_q = P_{req}/(1 - \ell_{in} - \ell_{ex})$$

The engine model gives uninstalled power and the gross thrust F_g, for a nominal exhaust nozzle area. The gross jet thrust F_G and exhaust power loss (ℓ_{ex}) are both functions of the exhaust nozzle area. Smaller exhaust nozzle areas increase exhaust losses and increase gross thrust. Thus the ratio of installed to uninstalled thrust $F_G/F_g = K_{fgr}$ is approximated by a third-order polynomial in the exhaust power loss ℓ_{ex}. So the net installed jet thrust is $F_N = K_{fgr}F_g - \dot{m}_{req}V$.

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency.

Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

Power Turbine Speed

The shaft power available is a function of the gas power available P_G and the power turbine efficiency η_t: $P_a = \eta_t P_G$. Generally the power turbine speed N has a significant effect on η_t, but almost no effect on P_G. The model used for the efficiency variation is

$$\eta_t \cong 1 - |(N/N_{\text{opt}}) - 1|^{X_N}$$

where N_{opt} is the speed for peak efficiency. Two approximations for the optimum turbine speed are used, the first a linear expression in $p = P/(P_{0R}\delta\sqrt{\theta})$, the second a cubic function of $p = P/(P_{0C}\delta\sqrt{\theta})$.

Power Available

Given the flight condition and engine rating, the power available P_a is calculated as follows. The specific power

and referred mass flow (at N_{spec}, relative to SP_0 and $\dot{m}_0$ for this rating) are approximated by functions of the ambient temperature ratio θ and inlet ram air ratios:

$$SP_a(N_{\text{spec}}) = SP_0\, \theta\, K_{spa}\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{spa}}$$

$$\dot{m}_a(N_{\text{spec}}) = \dot{m}_0(\delta/\sqrt{\theta})e^{K_{mfa}}\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{mfa}}$$

where the static lapse rate (K_{spa}, K_{mfa}) and ram air exponents (X_{spa}, X_{mfa}) are piecewise linear functions of θ. The power available is then

$$P_a(N_{\text{spec}}) = P_0(\delta\sqrt{\theta})K_{spa}e^{K_{mfa}}\left[\delta_M\,\sqrt{\theta_M}\right]^{X_{spa}+X_{mfa}}$$

This expression for $\dot{m}_a$ is only used to calculate P_a; elsewhere the $\dot{m}_q$ expression below is used to obtain the mass flow at a power P_q. Finally

$$P_a(N) = P_a(N_{\text{spec}})\frac{1 - |(N/N_{\text{opt}}) - 1|^{X_N}}{1 - |(N_{\text{spec}}/N_{\text{opt}}) - 1|^{X_N}}$$

is the power available at turbine speed N. Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$). Then the mechanical limit is applied: $P_{av} = \min(P_{av}, rP_{\text{mech}R})$, $r = N/N_{\text{spec}}$.

Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q (which might equal the power available), flight condition, and engine rating. Installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{\text{loss}}$). The referred quantities (relative to SLS static MCP quantities) are approximated by cubic functions of $q = P_q(N_{\text{spec}})/(P_{0C}\delta\sqrt{\theta})$

$$\dot{w}_{req} = \dot{w}_{0C}(\delta\sqrt{\theta})(K_{ffq0} + K_{ffq1}q$$
$$+ K_{ffq2}q^2 + K_{ffq3}q^3)\left[\theta_M\right]^{-X_{ffq}}$$

$$\dot{m}_{req} = \dot{m}_{0C}(\delta/\sqrt{\theta})(K_{mfq0} + K_{mfq1}q$$
$$+ K_{mfq2}q^2 + K_{mfq3}q^3)\left[\theta_M\right]^{X_{mfq}}$$

$$F_g = F_{g0C}\,\delta(K_{fgq0} + K_{fgq1}q$$
$$+ K_{fgq2}q^2 + K_{fgq3}q^3)\left[\theta_M\right]^{X_{fgq}}$$

at N_{spec}, with $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The mass flow and fuel flow are primarily functions of the gas power P_G, and are assumed to be independent of η_t, hence independent of turbine speed. However, these equations are functions of $P_q(N_{\text{spec}})$, obtained from $P_q(N)$ using

$$P_q(N_{\text{spec}}) = P_q(N)\frac{1 - |(N_{\text{spec}}/N_{\text{opt}}) - 1|^{X_N}}{1 - |(N/N_{\text{opt}}) - 1|^{X_N}}$$

Then the installed net jet thrust F_N and momentum drag D_{aux} are calculated.

Scaling

The parameters of the engine model can be defined for a specific engine, but as part of the aircraft sizing task the parameters must be scaled, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and N_{spec}).

The engine model includes reference values of the engine performance parameters: P_{0R}, SP_{0R}, $P_{\text{mech}R}$, sfc_{0C}, SF_{0C}, N_{spec}, $N_{\text{opt}0C}$. Mass flow and fuel flow are obtained from $\dot{m}_{0R} = P_{0R}/SP_{0R}$ and $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The reference power at each engine rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$; and similarly for specific power and mechanical limits. These ratios are kept fixed when the engine is scaled.

The engine size is specified as takeoff power $P_{to} = P_{\text{eng}}$: power at rating R, for SLS static conditions and specification turbine speed N_{spec}. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows.

The engine technology parameters SP_{0C} and sfc_{0C} are assumed to vary linearly with mass flow $\dot{m}_{0C}$ up to a limit $\dot{m}_{\text{lim}}$, and constant thereafter at SP_{lim} and sfc_{lim}. Usually the effect of size is that specific power increases and specific fuel consumption decreases with mass flow. Using $\dot{m}_{0C} = P_{0C}/SP_{0C}$, the specific power equation can be solved for the mass flow given the power. From this mass flow, SP_{0C} and sfc_{0C} are calculated, hence the fuel flow $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The specific thrust available at MCP is assumed to be constant, and the specification power turbine speed decreases with the square-root of the mass flow.

Engine Speed

The above model may not adequately account for variation of engine performance with engine speed, so it is also possible to define the parameters corresponding to a set of engine speed ratios N/N_{spec}. Then the engine performance and power available quantities are linearly interpolated to obtain the values at the required engine speed N. If this option is used, then the correction based on $P(N)/P(N_{\text{spec}}) = \eta_t(N)/\eta_t(N_{\text{spec}})$ is not applied.

Weight

The engine weight can be a fixed input value, calculated as a function of power, or scaled with engine mass flow. As a function of power, the weight of one engine is:

$$W_{\text{one eng}} = K_{0\text{eng}} + K_{1\text{eng}}P + K_{2\text{eng}}P^{X_{\text{eng}}}$$

where P is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power is given by using only the $K_{1\text{eng}}$ term. Alternatively, the specific weight $SW = P/W$ can be scaled with the mass flow $\dot{m}_{0C}$: linear up to the limit $\dot{m}_{\text{lim}}$, and constant thereafter at SW_{lim}. Usually the effect of size is that the specific weight decreases with mass flow.

WEIGHTS

The design gross weight W_D is a principal parameter defining the aircraft, usually determined by the sizing task for the design conditions and missions. The aircraft weight statement defines the empty weight, fixed useful load, and operating weight for the design configuration. The aircraft weight statement is the sum of the weight statements for all the aircraft components, the component weight determined by input or by parametric calculations with technology factors. The definition of the weight terms is as follows.

operating weight: $W_O = W_E + W_{FUL}$

useful load: $W_{UL} = W_{FUL} + W_{\text{pay}} + W_{\text{fuel}}$

gross weight: $W_G = W_E + W_{UL} = W_O + W_{\text{pay}} + W_{\text{fuel}}$

where W_E is the weight empty; W_{FUL} the fixed useful load; W_{pay} the payload weight; and W_{fuel} the usable fuel weight. The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. If the weight empty is input, then the contingency weight is adjusted so W_E equals the required value. If the design gross weight is input, then the payload or fuel weight must be fallout.

The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input or specified as $d + fW$, for input increment d and fraction f. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity). For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition.

The design ultimate load factor $n_{z\,\text{ult}}$ at the structural design gross weight W_{SD} is specified, in particular for use in the component weight estimates. The structural design gross weight W_{SD} and design ultimate load factor $n_{z\,\text{ult}}$ are used for the fuselage, rotor, and wing weight estimation. The maximum takeoff weight W_{MTO} is used for the cost and drag (scaled aircraft and hub drag), and for the weights (system, fuselage, landing gear, and engine group).

The gross weight W_G is specified for each flight condition and mission, perhaps in terms of the design gross weight W_D. For a each flight state, the fixed useful load may be different than the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or furnishings weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

Weight Groups

Aircraft weight information is stored in a data structure that follows SAWE RP8A Group Weight Statement format (ref. 25), as outlined in table 3. The asterisks designate extensions of RP8A. There are 2 or 3 additional levels in the data structure for some weight groups, based on the weight breakdown for parametric estimation.

For each weight group, fixed (input) weights can be specified or weight increments dW added to the results of the parametric weight model. The parametric weight model includes technology factors χ. Thus typically a component or element weight is obtained from $W = \chi W_{\text{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

The vibration control weight can be input, or specified as a fraction of weight empty: $W_{\text{vib}} = f_{\text{vib}}W_E$. The contingency weight can be input, or specified as a fraction of weight empty: $W_{\text{cont}} = f_{\text{cont}}W_E$. However, if the weight empty is input, then the contingency weight is adjusted so W_E equals the required value.

Folding weights can be calculated in several groups, including wing, rotor, and fuselage. These are the total weights for folding and the impact of folding on the group. A fraction f_{foldkit} of these weights can be in a kit, hence optionally removable. Thus of the total folding weight, the fraction f_{foldkit} is a kit weight in the fixed useful load of

the weight statement, while the remainder is kept in the component group weight.

Table 3. Weight statement (* indicates extension of RP8A).

WEIGHT EMPTY
STRUCTURE
wing group
rotor group
empennage group
fuselage group
alighting gear group
engine section or nacelle group
air induction group
PROPULSION GROUP
engine system
propeller/fan installation
fuel system
drive system
SYSTEMS AND EQUIPMENT
flight controls group
auxiliary power group
instruments group
hydraulic group
pneumatic group
electrical group
avionics group (mission equipment)
armament group
furnishings & equipment group
environmental control group
anti-icing group
load & handling group
VIBRATION (*)
CONTINGENCY
FIXED USEFUL LOAD
crew
fluids (oil, unusable fuel) (*)
auxiliary fuel tanks
other fixed useful load (*)
kits (*)
PAYLOAD
USABLE FUEL
standard tanks (*)
auxiliary tanks (*)

The fixed useful load W_{FUL} consists of crew, trapped fluids (oil and unusable fuel), auxiliary fuel tanks, furnishings increment, kits (folding, wing extension, other), and other fixed useful load.

Systems and equipment includes the following fixed (input) weights: auxiliary power group, instruments group, pneumatic group, electrical group, avionics group (mission equipment), armament group (armor and armament provisions), furnishings and equipment group, environmental control group, and load and handling group. Systems and equipment includes the following scaled weights: flight controls group, hydraulic group, and anti-icing group.

Flight controls group includes the following fixed (input) weights: cockpit controls, and automatic flight control system. Flight controls group includes the following scaled weights: fixed wing systems, rotary wing systems, and conversion or thrust vectoring systems. Rotary wing flight control weights can be calculated for the entire aircraft (using rotor parameters such as chord and tip speed for a designated rotor), which is consistent with parametric weight equations developed for conventional two-rotor configurations. Alternatively, rotary wing flight control weights can be calculated separately for each rotor and then summed. The fixed wing flight controls and the conversion controls can be absent.

The fuselage group consists of the basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure.

The alighting gear group consists of basic structure, retraction, and crashworthiness structure.

The rotor configuration determines where the weights occur in the weight statement. The rotor group consists of blade assembly, hub and hinge, fairing/spinner, and blade fold structure. The tail rotor (in empennage group) or the propeller/fan installation (in propulsion group) consists of the blade assembly, the hub and hinge, and the rotor/fan duct and rotor support. There are separate weight models for main rotors, tail rotors, and auxiliary-thrust systems (propellers).

The force component weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group.

The wing group consists of basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, spoilers). There are

separate models for a tiltrotor or tiltwing configuration and for other configurations (including compound helicopter). The AFDD wing weight models are based on parameters for the basic wing plus the wing tip extensions (not the total wing and extensions). The tiltrotor wing model requires the weight on the wing tips (both sides), consisting of rotor group, engine system, drive system (except drive shaft), engine section or nacelle group, air induction group, rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions.

The tail weight (empennage group) model depends on the configuration: helicopters and compounds, or tiltrotors and tiltwings. Separate weight models are available for horizontal and vertical tails.

The fuel system consists of the tanks (including support) and the plumbing. The auxiliary fuel tanks weight is part of the fixed useful load.

The drive system consists of gear boxes and rotor shafts, drive shafts, rotor brakes, clutches, and gas drive.

The engine group component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the engine group, consisting of N_{eng} engines.

AFDD Weight Models

For scaled weights of all components, the rotorcraft weight models developed by the U.S. Army Aeroflightdynamics Directorate (AFDD) are implemented. The weights are estimated from parametric equations based on the weights of existing turbine-powered helicopters and tiltrotors (and some fixed wing aircraft component weights). For some weight groups two models are available, designated AFDDnn. Table 4 summarizes the statistics of the parametric weight estimation equations.

Wing weight equations for a tiltrotor aircraft are based on methodology developed by Chappell and Peyran (refs. 26 and 27). The wing is primarily sized to meet torsional stiffness requirements. For lift offset rotors, the blade and hub weights can be calculated based on the methodology of ref. 28. These equations were developed for the coaxial rotor configuration, and the blade weight estimate is based on the stiffness required for tip clearance of the two rotors.

Table 4. Statistics of parametric weight equations.

group		number of aircraft	average error %
wing		25	3.4
rotor blade	AFDD82	37	7.7
rotor hub	AFDD82	37	10.2
rotor blade	AFDD00	51	7.9
rotor hub	AFDD00	51	12.2
horizontal tail		13	22.4
vertical tail		12	23.3
tail rotor		19	16.7
fuselage	AFDD82	30	8.7
fuselage	AFDD84	35	6.5
alighting gear		28	8.4
engine support		12	11.0
engine cowling		12	17.9
air induction		12	11.0
accessory		16	11.5
fuel tank		15	4.6
gear box + rotor shaft	AFDD83	30	7.7
gear box + rotor shaft	AFDD00	52	8.6
drive shaft		28	16.0
rotor brake		23	25.1
rotary wing flight controls	non-boosted	20	10.4
rotary wing flight controls	boost mechanisms	21	6.5
rotary wing flight controls	boosted	20	9.7

Figure 15 shows the error of the calculated weight for the sum of all parametric weight, accounting on average for 42% of the empty weight. This sum is composed of the structural group (based on the AFDD00 equation for rotor blade and hub weights, and the AFDD84 equation for body weight), the propulsion group (based on the AFDD00 equation for drive system weight), and the flight controls group. Based on 42 aircraft, the average error of the sum of all parametric weight is 5.3%. The corresponding average error is 6.1% for the structural group (8.6% for the rotor group alone), 10.9% for the propulsion group, and 8.7% for the flight controls group.
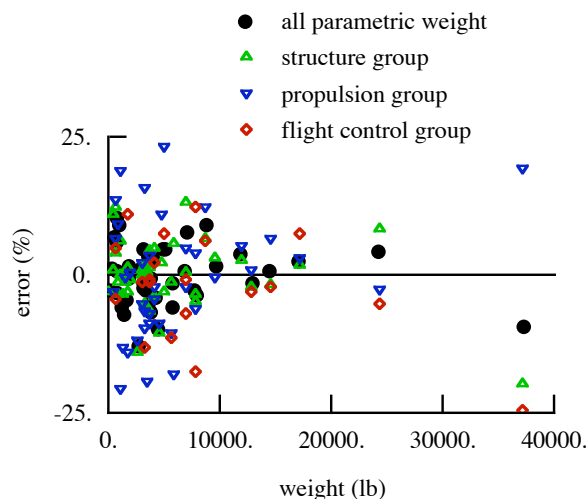
Figure 15. Accuracy of sum of all parametric weight.

NDARC SOFTWARE

The NDARC program is entirely new software, built on an architecture that enables routine extensions and modifications. The software has been implemented with low-fidelity models, typical of the conceptual design environment. Incorporation of higher-fidelity models will be possible, as the architecture of the code accommodates configuration flexibility and a hierarchy of models.

The program is written in Fortran 95, using a special-purpose software tool to manage the data structures, construct the input manual, and automatically generate some input and output subroutines. The program has been compiled on several platforms and operating systems.

On typical computers, NDARC execution times range from seconds for a job with just a few analysis tasks, to minutes for a job that sizes an aircraft based on multiple flight conditions and missions.

Input is in namelist-based text format. The program output includes text files formatted for printing and for spreadsheets, and special files to support functions such as preparing layout drawings. Java graphical user interfaces are being developed by the user community to facilitate dealing with the input and output.

The program is supported by complete and thorough documentation, including the theory manual (Ref. 6), input manual, and data structures manual. A NASA-hosted Wiki has been established to support user communication.

Distribution of the NDARC program is controlled by the Software Release Authority at NASA Ames Research Center. Source code and documentation are available to users, subject to a Software Usage Agreement.

CONCLUDING REMARKS

The theoretical basis and architecture of the conceptual design tool NDARC (NASA Design and Analysis of Rotorcraft) has been described. The principal tasks of NDARC are to design a rotorcraft to satisfy specified design conditions and missions, and then analyze the performance of the aircraft for a set of off-design missions and point operating conditions. NDARC provides a capability to model general rotorcraft configurations, and estimate the performance and attributes of advanced rotor concepts.

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NOMENCLATURE

Acronyms

AFDD	U.S. Army Aeroflightdynamics Directorate
EG	engine group
IGE	in ground effect
IRP	intermediate rated power
IRS	infrared suppressor
ISA	International Standard Atmosphere
MCP	maximum continuous power
MRP	maximum rated power
OEI	one engine inoperative
OGE	out of ground effect
PG	propulsion group
RPTEM	referred parameter turboshaft engine model
SDGW	structural design gross weight
SLS	sea level standard
WMTO	maximum takeoff weight

Weights

W_D	design gross weight
W_E	empty weight
W_{MTO}	maximum takeoff weight
W_{SD}	structural design gross weight
W_G	gross weight, $W_E + W_{UL} = W_O + W_{pay} + W_{fuel}$
W_O	operating weight, $W_E + W_{FUL}$
W_{UL}	useful load, $W_{FUL} + W_{pay} + W_{fuel}$
W_{pay}	payload
W_{fuel}	fuel weight
W_{FUL}	fixed useful load
W_{burn}	mission fuel burn
W_{vib}	vibration control weight
W_{cont}	contingency weight
χ	technology factor

Fuel Tanks

$W_{fuel-cap}$	fuel capacity, maximum usable fuel weight
$N_{auxtank}$	number of auxiliary fuel tanks
$W_{aux-cap}$	auxiliary fuel tank capacity

Power

P_{reqPG} power required, propulsion group;
$$P_{comp} + P_{xmsn} + P_{acc}$$

P_{reqEG} power required, engine group

P_{avPG} power available, propulsion group;
$$\min(\Sigma\, f_P P_{avEG}, (\Omega_{prim}/\Omega_{ref})P_{DS\,limit})$$

P_{avEG} power available, engine group;
$$(N_{eng} - N_{inop})P_{av}$$

P_{comp} component power required

P_{xmsn} transmission losses

P_{acc} accessory power

N_{inop} number of inoperative engines, engine group

$P_{DS\,limit}$ drive system torque limit (specified as power limit at reference rotor speed)

$P_{ES\,limit}$ engine shaft rating

$P_{RS\,limit}$ rotor shaft rating

Engine

P_{eng} sea level static power available per engine at specified takeoff rating

N_{eng} number of engines in engine group

P_{av} power available, installed;
$$\min(P_a - P_{loss}, P_{mechR})$$

P_a power available, uninstalled

P_{req} power required, installed; $P_q - P_{loss}$

P_q power required, uninstalled

P_{loss} installation losses

P_{mechR} mechanical power limit

SP specific power, $P/\dot{m}$ (conventional units)

sfc   specific fuel consumption, $\dot{w}/P$ (conventional units)

$\dot{m}$ mass flow (conventional units)

$\dot{w}$ fuel flow (conventional units)

F_N net jet thrust

D_{aux} momentum drag

N specification turbine speed

SW specific weight, P/W

Tip Speed and Rotation

$V_{tip\text{-}ref}$ reference tip speed, propulsion group primary rotor; each drive state

r gear ratio; $\Omega_{dep}/\Omega_{prim}$ for rotor, $\Omega_{spec}/\Omega_{prim}$ for engine

Ω_{prim} primary rotor rotational speed, $V_{tip\text{-}ref}/R$

Ω_{dep} dependent rotor rotational speed

Ω_{spec} specification engine turbine speed

N_{spec} specification engine turbine speed (rpm)

Mission

T mission segment time

D mission segment distance

E endurance

R range

$\dot{w}$ fuel flow

Environment

g gravitational acceleration

h altitude

c_s speed of sound

ρ density

T temperature, deg R or deg K

Axis Systems

I   inertial

F   aircraft

B   component

Geometry

SL,BL,WL   fixed input position

$x/L, y/L, z/L$ scaled input position

L reference length

z^F component position vector, in aircraft axes

Motion

ψ_F, θ_F, ϕ_F roll, pitch, yaw angles; orientation airframe axes F relative inertial axes

$\dot{\psi}_F$ turn rate

ψ_V, θ_V climb, sideslip angles; orientation velocity axes relative inertial axes

v_{AC}^F aircraft velocity

ω_{AC}^F aircraft angular velocity

a_{AC}^F aircraft linear acceleration

n load factor

V aircraft velocity magnitude

Aerodynamics and Loads

v velocity relative air (including interference)

q dynamic pressure, $\frac{1}{2}\rho v^2$

α angle-of-attack

β sideslip angle

F force

M moment

D, Y, L drag, side, lift forces (aerodynamic axes)

M_x, M_y, M_z roll, pitch, yaw moments (aerodynamic axes)

c_d, c_ℓ section drag, lift coefficients

C_D, C_Y, C_L drag, side, lift force coefficients

C_ℓ, C_M, C_N roll, pitch, yaw moment coefficients

D/q drag area, SC_D (S = reference area)

Aircraft

DL disk loading, W_D/A_{ref}

A_{ref} reference rotor area, $\Sigma\, f_A A$; typically projected area of lifting rotors

WL	wing loading, W_D / S_{ref}	ν	blade flap frequency (per-rev)
S_{ref}	reference wing area, ΣS; sum area all wings	γ	blade Lock number
c_{AC}	aircraft control	C_T/σ	thrust coefficient divided by solidity,
T	control matrix		$T / \rho A (\Omega R)^2 \sigma$
c	component control, $c = STc_{AC} + c_0$	β_c, β_s	longitudinal, lateral flapping (tip-path plane
α_{tilt}	tilt control variable		tilt relative shaft)
		$\theta_{0.75}$	blade collective pitch angle (at 75% radius)
Rotor		θ_c, θ_s	lateral, longitudinal blade pitch angle
W/A	disk loading, $W = f_W W_D$	H, Y, T	drag, side, thrust force on hub (shaft axes)
C_W/σ	design blade loading, $W / \rho A V_{\text{tip}}^2 \sigma$ ($V_{\text{tip}} =$	M_x, M_y	roll, pitch moment on hub
	hover tip speed)	Q	shaft torque
R	blade radius	P_i, P_o, P_p	induced, profile, parasite power
A	disk area	κ	induced power factor, $P_i = \kappa P_{\text{ideal}}$
σ	solidity (ratio blade area to disk area)	$c_{d\,\text{mean}}$	profile power mean drag coefficient,
T_{design}	design thrust of antitorque or auxiliary-thrust		$C_{Po} = (\sigma / 8) c_{d\,\text{mean}} F_P$
	rotor		
r	direction of rotation ($r = 1$ for counter-	**Wing**	
	clockwise, $r = -1$ for clockwise)	W / S	wing loading, $W = f_W W_D$
r	blade span coordinate	S	area
ψ	blade azimuth coordinate	b	span
μ	advance ratio	c	chord, S/b
λ	inflow ratio	AR	aspect ratio, b^2 / S
M_{at}	advancing tip Mach number		