NDARC
NASA Design and Analysis of Rotorcraft

Theory

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Contents

Chapter 1

Introduction

The NASA Design and Analysis of Rotorcraft (NDARC) software is an aircraft system analysis tool intended to support both conceptual design efforts and technology impact assessments. The principal tasks are to design (or size) a rotorcraft to meet specified requirements, including vertical takeoff and landing (VTOL) operation, and then analyze the performance of the aircraft for a set of conditions. For broad and lasting utility, it is important that the code have the capability to model general rotorcraft configurations, and estimate the performance and weights of advanced rotor concepts. The architecture of the NDARC code accommodates configuration flexibility, a hierarchy of models, and ultimately multidisciplinary design, analysis, and optimization. Initially the software is implemented with low-fidelity models, typically appropriate for the conceptual design environment.

An NDARC job consists of one or more cases, each case optionally performing design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. For analysis tasks, the aircraft description can come from the sizing task, from a previous case or a previous NDARC job, or be independently generated (typically the description of an existing aircraft).

The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated; and the aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single main-rotor and tail-rotor helicopter; tandem helicopter; coaxial helicopter; and tiltrotors. The architecture of the code accommodates addition of new or higher-fidelity attribute models for a component, as well as addition of new components.

1–1 Background

The definition and development of NDARC requirements benefited substantially from the experiences and computer codes of the preliminary design team of the U.S. Army Aeroflightdynamics Directorate (AFDD) at Ames Research Center.

In the early 1970s, the codes SSP-1 and SSP-2 were developed by the Systems Research Integration Office (SRIO, in St. Louis) of the U.S. Army Air Mobility Research and Development Laboratory. SSP-1 performed preliminary design to meet specified mission requirements, and SSP-2 estimated the performance for known geometry and engine characteristics, both for single main-rotor helicopters (ref. 1). Although similar tools were in use in the rotorcraft community, these computer programs were independently developed, to meet the requirements of government analysis. The Advanced Systems

Research Office (ASRO, at Ames Research Center) of USAAMRDL produced in 1974 two Preliminary Systems Design Engineering (PSDE) studies (refs. 2 and 3) using SSP-1 and SSP-2. These two codes were combined into one code called PSDE by Ronald Shinn.

The MIT Flight Transportation Laboratory created design programs for helicopters (ref. 4) and tiltrotors (ref. 5). Michael Scully, who wrote the helicopter design program and was significantly involved in the development of the tiltrotor design program, joined ASRO in 1975 and ideas from the MIT programs began to be reflected in the continuing development of PSDE. An assessment of design trade-offs for the Advanced Scout Helicopter (ASH) used a highly modified version of PSDE (ref. 6).

A DoD Joint Study Group was formed in April 1975 to perform an Interservice Helicopter Commonality Study (HELCOM) for the Director of Defense Research and Engineering. The final HELCOM study report was published in March 1976 (ref. 7). A result of this study was an assessment by ASRO that PSDE needed substantial development, including better mathematical models and better technical substantiation; more flexible mission analysis; and improved productivity for both design and analysis tasks. Thus began an evolutionary improvement of the code, eventually named RASH (after the developer Ronald A. Shinn, as a consequence of the computer system identification of output by the first four characters of the user name). RASH included improvements in flight performance modeling, output depth, mission analysis, parametric weight estimation, design sensitivity studies, off-design cases, and coding style. The code was still only for single main-rotor helicopters.

In the early 1980s, tool development evolved in two separate directions at the Preliminary Design Team of ASRO. RASH was developed into the HELO (or PDPAC) code, for conventional and compound single main-rotor helicopters. With the addition of conversion models and wing weight estimation methods (refs. 8 and 9), RASH became the TR code, for tiltrotor aircraft. The JVX Joint Technology Assessment of 1982 utilized the HELO and TR codes. A special version called PDABC, including a weight estimation model for lift-offset rotors (ref. 10), was used to analyze the Advancing Blade Concept. The JVX JTA report (ref. 11) documented the methodology implemented in these codes.

Work in support of the LHX program from 1983 on led to a requirement for maneuver analysis of helicopters and tiltrotors, implemented in the MPP code (Maneuver Performance Program) by John Davis. The core aircraft model in MPP was similar to that in TR and HELO, but the trim strategy in particular was new. A design code does not require extensive maneuver analysis capability, but MPP had an impact on the design code development, with the MPP performance and trim methods incorporated into TR87. The sizing analysis of TR88 and the aircraft flight model from MPP were combined into the VAMP code (VSTOL Design and Maneuver Program). VAMP combined the capability to analyze helicopters and tiltrotors in a single tool, although the capability of HELO to analyze compound helicopters was not replicated.

In the early 1990s, the RC code (for RotorCraft) emerged from the evolution of VAMP, with John Preston as the lead developer (refs. 12 and 13). Some maneuver analysis capabilities from MPP were added, and the analysis capability extended to helicopters. The models were confirmed by comparison with results from TR and HELO. RC was operational by 1994, although HELO and RC continued to be used into the mid 1990s. RC97 was a major version, unifying the tiltrotor and helicopter analyses. The RC code introduced new features and capabilities, productivity enhancements, as well as coding standards and software configuration control. Special versions of RC were routinely produced to meet the unique requirements of individual projects (such as ref. 14).

NASA, with support from the U.S. Army, in 2005 conducted the design and in-depth analysis

of rotorcraft configurations that could satisfy the Vehicle Systems Program technology goals (ref. 15). These technology goals and accompanying mission were intended to identify enabling technology for civil application of heavy-lift rotorcraft. The emphasis was on efficient cruise and hover, efficient structures, and low noise. The mission specified was to carry 120 passengers for 1200 nm, at a speed of 350 knots and 30000 ft altitude. The configurations investigated were a Large Civil Tiltrotor (LCTR), a Large Civil Tandem Compound (LCTC), and a Large Advancing Blade Concept (LABC). The results of the NASA Heavy Lift Rotorcraft Systems Investigation subsequently helped define the content and direction of the Subsonic Rotary Wing project in the NASA Fundamental Aeronautics program. The design tool used was the AFDD RC code. This investigation is an example of the role of a rotorcraft sizing code within NASA. The investigation also illustrated the difficulties involved in adapting or modifying RC for configurations other than conventional helicopters and tiltrotors, supporting the requirement for a new tool.

1–2   Requirements

Out of this history, the development of NDARC was begun in early 2007. NDARC is entirely new software, built on a new architecture for the design and analysis of rotorcraft. From the RC theoretical basis, the equations of the parametric weight equations and the Referred Parameter Turboshaft Engine Model were used with only minor changes. Use was also made of the RC component aerodynamic models and rotor performance model. The current users of RC, informed by past and recent applications, contributed significantly to the requirements definition.

The principal tasks are to design (size) rotorcraft to meet specified requirements, and then analyze the performance of the aircraft for a set of flight conditions and missions. Multiple design requirements, from specific flight conditions and various missions, must be used in the sizing task. The aircraft performance analysis must cover the entire spectrum of the aircraft capabilities, and allow general and flexible definition of conditions and missions.

For government applications and to support research, it is important to have the capability to model general rotorcraft configurations, including estimates of the performance and weights of advanced rotor concepts. In such an environment, software extensions and modifications will be routinely required to meet the unique requirements of individual projects, including introduction of special weight and performance models for particular concepts.

Thus the code architecture must accommodate configuration flexibility and alternate models, including a hierarchy of model fidelity. Although initially implemented with low-fidelity models, typical of the conceptual design environment, ultimately the architecture must allow multidisciplinary design, analysis, and optimization. The component performance and engine models must cover all operating conditions. The software design and architecture must facilitate extension and modification of the software.

Complete and thorough documentation of the theory and its software implementation is essential, to support development and maintenance and to enable effective use and modification. Most of the history described above supports this requirement by the difficulties encountered in the absence of good documentation. Documentation of the methodology was often prompted only by the need to substantiate conclusions of major technology assessments, and occasionally by the introduction of new users and developers. For a new software implementation of a new architectures, documentation is required from the beginning of the development.
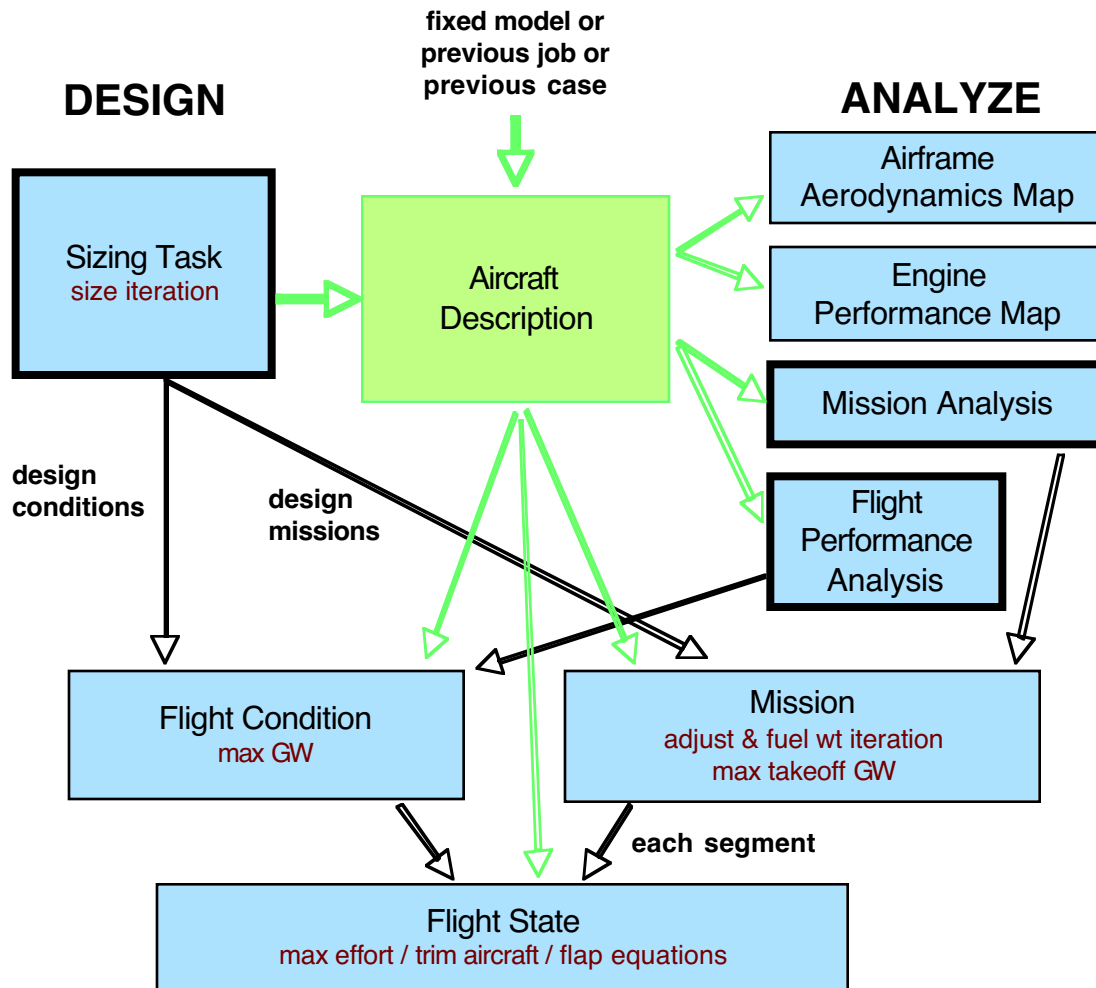
fixed model or
previous job or
previous case

DESIGN

ANALYZE

Sizing Task *size iteration*	Aircraft Description	Airframe Aerodynamics Map

Engine
Performance Map

Mission Analysis

Flight
Performance
Analysis

design
conditions

design
missions

Flight Condition
max GW

Mission
adjust & fuel wt iteration
max takeoff GW

each segment

Flight State
max effort / trim aircraft / flap equations

Figure 1-1. Outline of NDARC tasks.

1–3   Overview

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 1-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with heavy borders. Heavy arrows show control of subordinate tasks.

The aircraft description (figure 1-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light arrows).

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a

specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a number of mission segments, for which time, distance, and fuel burn are evaluated. For the sizing task, certain missions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, and for fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight.

Flight conditions are specified for the sizing task and for the flight performance analysis. For the sizing task, certain flight conditions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, and for antitorque or auxiliary-thrust rotor sizing. The flight condition parameters include gross weight and useful load.

For flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

1–4   Terminology

The following terminology is introduced as part of the development of the NDARC theory and software. Relationships among these terms are reflected in figure 1-1.

a) Job: An NDARC job consists of one or more cases.

b) Case: Each case performs design and/or analysis tasks. The analysis tasks can include off-design mission performance calculation, flight performance calculation for point operating conditions, and generation of airframe aerodynamics or engine performance maps.

c) Design Task: Size rotorcraft to satisfy specified set of design flight conditions and/or design missions. Key aircraft design variables are adjusted until all criteria are met. The resulting aircraft description can be basis for the mission analysis and flight performance analysis tasks.

d) Mission Analysis Task: Calculate aircraft performance for one off-design mission.

e) Flight Performance Analysis Task: Calculate aircraft performance for point operating condition.

f) Mission: Ordered set of mission segments, for which time, distance, and fuel burn are evaluated.

Gross weight and useful load are specified for the beginning of the mission, and adjusted for fuel burn and useful load changes at each segment. Missions are defined for the sizing task and for the mission performance analysis.

g) Flight Condition: Point operating condition, with specified gross weight and useful load. Flight conditions are specified for the sizing task and for the flight performance analysis.

h) Flight State: Aircraft flight condition, part of definition of each flight condition and each mission segment. Flight state solution involves rotor blade motion, aircraft trim, and perhaps a maximum-effort calculation.

i) Component: The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight are calculated.

j) Propulsion Group: A propulsion group is a set of components and engine groups, connected by a drive system. An engine group consists of one or more engines of a specific type. The components define the power required. The engine groups define the power available.

1–5   Analysis Units

The code can use either English or SI units for input, output, and internal calculations. A consistent mass-length-time-temperature system is used, except for weight and power:

	length	mass	time	temperature	weight	power
English:	foot	slug	second	°F	pound	horsepower
SI:	meter	kilogram	second	°C	kilogram	kiloWatt

In addition, the default units for flight conditions and missions are: speed in knots, time in minutes, distance in nautical miles, and rate of climb in feet-per-minute. The user can specify alternate units for these and other quantities.

1–6   Outline of Report

This document provides a complete description of the NDARC theoretical basis and architecture. Chapters 3–5 describe the tasks and solution procedures, while chapters 7–17 present the models for the aircraft and its components. The cost model is described in chapter 6; the engine model in chapter 18; and the weight model in chapter 19. The accompanying NDARC Input Manual describes the use of the code.

1–7   References

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Nomenclature

The nomenclature for geometry and rotations employs the following conventions. A vector x is a column matrix of three elements, measuring the vector relative to a particular basis (or axes, or frame). The basis is indicated as follows:

a) x^A is a vector measured in axes A;

b) $x^{EF/A}$ is a vector from point F to point E, measured in axes A.

A rotation matrix C is a three-by-three matrix that transforms vectors from one basis to another:

c) C^{BA} transforms vectors from basis A to basis B, so $x^B = C^{BA} x^A$.

The matrix C^{BA} defines the orientation of basis B relative to basis A, so it also may be viewed as rotating the axes from A to B. For a vector u, a cross-product matrix $\widetilde{u}$ is defined as follows:

$$
\widetilde{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}
$$

such that $\widetilde{u}v$ is equivalent to the vector cross-product $\mathbf{u} \times \mathbf{v}$. The cross-product matrix enters the relationship between angular velocity and the time derivative of a rotation matrix:

$$
\dot{C}^{AB} = -\widetilde{\omega}^{AB/A} C^{AB} = C^{AB} \widetilde{\omega}^{BA/B}
$$

(the Poisson equations). For rotation by an angle α about the x, y, or z axis $(1, 2, \text{or } 3 \text{ axis})$, the following notation is used:

$$
X_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}
$$

$$
Y_\alpha = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}
$$

$$
Z_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Thus for example, $C^{BA} = X_\phi Y_\theta Z_\psi$ means that the axes B are located relative to the axes A by first rotating by angle ψ about the z-axis, then by angle θ about the y-axis, and finally by angle ϕ about the x-axis.

Acronyms

AFDD	U.S. Army Aeroflightdynamics Directorate
ASM	available seat mile
CAS	calibrated airspeed
CPI	consumer price index
EG	engine group
IGE	in ground effect
IRP	intermediate rated power
IRS	infrared suppressor
ISA	International Standard Atmosphere
ISO	International Organization for Standardization
MCP	maximum continuous power
MRP	maximum rated power
OEI	one engine inoperative
OGE	out of ground effect
PG	propulsion group
RPTEM	referred parameter turboshaft engine model
SDGW	structural design gross weight
SLS	sea level standard
TAS	true airspeed
WMTO	maximum takeoff weight

Weights

W_D	design gross weight
W_E	empty weight
W_{MTO}	maximum takeoff weight
W_{SD}	structural design gross weight
W_G	gross weight, $W_G = W_E + W_{UL} = W_O + W_{\text{pay}} + W_{\text{fuel}}$
W_O	operating weight, $W_O = W_E + W_{FUL}$
W_{UL}	useful load, $W_{UL} = W_{FUL} + W_{\text{pay}} + W_{\text{fuel}}$
W_{pay}	payload
W_{fuel}	fuel weight
W_{FUL}	fixed useful load
W_{burn}	mission fuel burn
W_{vib}	vibration control weight
W_{cont}	contingency weight
χ	technology factor

Fuel Tanks

$W_{\text{fuel}-\text{cap}}$	fuel capacity, maximum usable fuel weight
$V_{\text{fuel}-\text{cap}}$	fuel capacity, volume
N_{auxtank}	number of auxiliary fuel tanks
$W_{\text{aux}-\text{cap}}$	auxiliary fuel tank capacity

Power

P_{reqPG}	power required, propulsion group; $P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$
P_{reqEG}	power required, engine group
P_{avPG}	power available, propulsion group; $\min(\sum f_P P_{avEG}, (\Omega_{\text{prim}}/\Omega_{\text{ref}})P_{DS\text{limit}})$
P_{avEG}	power available, engine group; $(N_{\text{eng}} - N_{\text{inop}})P_{av}$
P_{comp}	component power required
P_{xmsn}	transmission losses
P_{acc}	accessory power
N_{inop}	number of inoperative engines, engine group
$P_{DS\text{limit}}$	drive system torque limit (specified as power limit at reference rotor speed)
$P_{ES\text{limit}}$	engine shaft rating
$P_{RS\text{limit}}$	rotor shaft rating

Engine

P_{eng}	sea level static power available per engine at specified takeoff rating
N_{eng}	number of engines in engine group
P_{av}	power available, installed; $\min(P_a - P_{\text{loss}}, P_{\text{mech}})$
P_a	power available, uninstalled
P_{req}	power required, installed; $P_q - P_{\text{loss}}$
P_q	power required, uninstalled
P_{loss}	installation losses
P_{mech}	mechanical power limit
SP	specific power, $P/\dot{m}$ (conventional units)
sfc	specific fuel consumption, $\dot{w}/P$ (conventional units)
$\dot{m}$	mass flow (conventional units)
$\dot{w}$	fuel flow (conventional units)
F_N	net jet thrust
D_{aux}	momentum drag
N	specification turbine speed
SW	specific weight, P/W

Tip Speed and Rotation

$V_{\text{tip}-\text{ref}}$	reference tip speed, propulsion group primary rotor; each drive state
r	gear ratio; $\Omega_{\text{dep}}/\Omega_{\text{prim}}$ for rotor, $\Omega_{\text{spec}}/\Omega_{\text{prim}}$ for engine
Ω_{prim}	primary rotor rotational speed, $\Omega = V_{\text{tip}-\text{ref}}/R$
Ω_{dep}	dependent rotor rotational speed, $\Omega = V_{\text{tip}-\text{ref}}/R$
Ω_{spec}	specification engine turbine speed
N_{spec}	specification engine turbine speed (rpm)

Mission

T	mission segment time
D	mission segment distance
dR	mission segment range contribution
E	endurance
R	range
$\dot{w}$	fuel flow

Environment

g	gravitational acceleration
h	altitude
c_s	speed of sound
ρ	density
ν	kinematic viscosity
μ	viscosity
T	temperature, °R or °K
τ	temperature, °F or °C
V_w	wind speed

Axis Systems

I	inertial
F	aircraft
A	component aerodynamic
B	component
V	velocity

Geometry

SL, BL, WL	fixed input position (station line, buttline, waterline)
	positive aft, right, up; arbitrary origin
$x/L, y/L, z/L$	scaled input position; positive aft, right, up; origin at reference point
L	reference length (fuselage length, rotor radius, or wing span)
x, y, z	calculated position, aircraft axes; positive forward, right, down;
	origin at reference point for geometry,
	origin at center-of-gravity for motion and loads
z^F	component position vector, in aircraft axes, relative reference point
ℓ	length
S_{wet}	wetted area

Motion

ϕ_F, θ_F, ψ_F	roll, pitch, yaw angles; orientation airframe axes F relative inertial axes
$\dot{\psi}_F$	turn rate
θ_V, ψ_V	climb, sideslip angles; orientation velocity axes V relative inertial axes
v_{AC}^F	aircraft velocity
ω_{AC}^F	aircraft angular velocity
a_{AC}^F	aircraft linear acceleration
n	load factor
V	aircraft velocity magnitude
V_h	horizontal velocity
V_f	forward velocity
V_s	sideward velocity
V_c	climb velocity
V_{cal}	calibrated airspeed

Aerodynamics and Loads

v	component velocity relative air (including interference)
q	dynamic pressure, $\tfrac{1}{2}\rho\lvert v\rvert^2$
α	angle-of-attack, component axes B relative aerodynamic axes A
β	sideslip angle, component axes B relative aerodynamic axes A
ℓ_f	ratio flap chord to airfoil chord, c_f/c
δ_f	flap deflection
F	force
M	moment
D, Y, L	aerodynamic drag, side, lift forces (component aerodynamic axes A)
M_x, M_y, M_z	aerodynamic roll, pitch, yaw moments (component aerodynamic axes A)
c_d, c_ℓ	section drag, lift coefficients
C_D, C_Y, C_L	component drag, side, lift force coefficients
C_ℓ, C_M, C_N	component roll, pitch, yaw moment coefficients
D/q	drag area, SC_D (S = reference area of component)

Aircraft

DL	disk loading, W_D/A_{ref}
A_{ref}	reference rotor area, $\sum f_A A$; typically projected area of lifting rotors
WL	wing loading, W_D/S_{ref}
S_{ref}	reference wing area, $\sum S$; sum area all wings
c_{AC}	aircraft control
T	control matrix
c	component control, $c = STc_{AC} + c_0$
α_{tilt}	tilt control variable
M	aircraft hover figure of merit, $W\sqrt{W/2\rho A_{\mathrm{ref}}}/P$
D_e	aircraft effective drag, P/V
L/D_e	aircraft effective lift-to-drag ratio, WV/P

Rotor

W/A	disk loading, $W = f_W W_D$
C_W/σ	design blade loading, $W/\rho A V_{\text{tip}}^2 \sigma$ ($V_{\text{tip}} =$ hover tip speed)
R	blade radius
A	disk area
σ	solidity (ratio blade area to disk area)
T_{design}	design thrust of antitorque or auxiliary-thrust rotor
r	direction of rotation (1 for counter-clockwise, -1 for clockwise)
r	blade span coordinate
ψ	blade azimuth coordinate
μ	advance ratio
λ	inflow ratio
M_{at}	advancing tip Mach number
ν	blade flap frequency (per-rev)
γ	blade Lock number
C_T/σ	thrust coefficient divided by solidity, $T/\rho A(\Omega R)^2 \sigma$
β_c, β_s	longitudinal, lateral flapping (tip-path plane tilt relative shaft)
$\theta_{0.75}$	blade collective pitch angle (at 75% radius)
θ_c, θ_s	lateral, longitudinal blade pitch angle)
H, Y, T	drag, side, thrust force on hub (shaft axes)
M_x, M_y	roll, pitch moment on hub
Q	shaft torque
P_i, P_o, P_p	induced, profile, parasite power
κ	induced power factor, $P_i = \kappa P_{\text{ideal}}$
$c_{d\text{mean}}$	profile power mean drag coefficient, $C_{Po} = (\sigma/8)c_{d\text{mean}}F_P$
M	rotor hover figure of merit, $Tf_D v/P$
L/D_e	rotor effective lift-to-drag ratio, $VL/(P_i + P_o)$
η	propulsive efficiency, TV/P

Wing

W/S	wing loading, $W = f_W W_D$
S	area
b	span
c	chord, S/b
$A\!R$	aspect ratio, b^2/S

Tasks

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps.

3–1 Size Aircraft for Design Conditions and Missions

3-1.1 Sizing Method

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight (W_D) or weight empty (W_E), rotor radius (R), and engine power available (P_{eng}). The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

> a) Engine power: Determine P_{eng}, for fixed R. The engine power is the maximum of the power required for all designated sizing flight conditions and sizing missions (typically including vertical flight, forward flight, and one-engine inoperative). Hence the engine power is changed by the ratio $\max(P_{reqPG}/P_{avPG})$ (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). This approach is the one most commonly used for the sizing task.

> b) Rotor radius: Determine R for input P_{eng}. The maximum power required for all designated sizing flight conditions and sizing missions is calculated, and then the rotor radius determined such that sthe power required equals the input power available. The change in radius is estimated as $R = R_{\text{old}}\sqrt{P_{reqPG}/P_{avPG}}$ (excluding flight states for which zero power margin is calculated, such as maximum gross weight or maximum effort). For multi-rotor aircraft, the radius can be fixed rather than sized for some rotors.

Alternatively, P_{eng} and R can be input rather than sized. Aircraft parameters can be determined by a subset of the design conditions and missions.

> a) Design gross weight W_D: maximum gross weight from designated conditions

and missions (for which gross weight is not fixed).

b) Maximum takeoff gross weight W_{MTO}: maximum gross weight from designated conditions (for which gross weight is not fixed).

c) Drive system torque limit $P_{DS\text{limit}}$: maximum torque from designated conditions and missions (for each propulsion group; specified as power limit at reference rotor speed).

d) Fuel tank capacity $W_{\text{fuel}-\text{cap}}$: maximum fuel weight from designated missions (without auxiliary tanks).

e) Antitorque or auxiliary thrust rotor design thrust T_{design}: maximum rotor thrust from designated conditions and missions.

Alternatively, these parameters can be fixed at input values. The design gross weight (W_D) can be fixed. The weight empty can be fixed (achieved by changing the contingency weight).

A successive substitution method is used for the sizing iteration, with an input tolerance ϵ. Relaxation is applied to P_{eng} or R, W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel}-\text{cap}}$, and T_{design}. Convergence is tested in terms of these parameters, and the aircraft weight empty W_E. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, for each propulsion group. The inner loop is an iteration on parameters: W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel}-\text{cap}}$, and T_{design}. Either loop can be absent, depending on the definition of the size task.

For each flight condition and each mission, the gross weight and useful load are specified. The gross weight can be input, maximized, or fallout. For flight conditions, the payload or fuel weight can be specified, and the other calculated; or both payload and fuel weight specified, with gross weight fallout. For missions, the payload or fuel weight can be specified, the other fallout, and then time or distance of mission segments adjusted; or fuel weight calculated from mission, and payload fallout; or both payload and fuel weight specified (or payload specified and fuel weight calculated from mission), with gross weight fallout. For each flight condition and mission segment, the following checks are performed.

a) The power required does not exceed the power available: $P_{reqPG} \leq (1+\epsilon)P_{avPG}$ (for each propulsion group).

b) The torque required does not exceed the drive system limit: for each propulsion group $P_{reqPG}/\Omega \leq (1+\epsilon)P_{DS\text{limit}}/\Omega_{\text{prim}}$. Rotor shaft torque and engine shaft torque are also checked.

c) The fuel weight does not exceed the fuel capacity: $W_{\text{fuel}} \leq (1+\epsilon)(W_{\text{fuel}-\text{cap}} + \sum N_{\text{auxtank}}W_{\text{aux}-\text{cap}})$ (including auxiliary tanks).

These checks are performed using an input tolerance ϵ.

Sizing flight conditions typically include takeoff (hover or specified vertical rate-of-climb), one-engine inoperative, cruise or dash, perhaps transmission, and perhaps mission midpoint hover. Sizing missions typically include a design mission and a mission to determine fuel tank capacity.

3-1.2   Component Sizing

3-1.2.1   Engine Power

The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines N_{eng} is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power P_{eng} of at least one engine group is found (including the first engine group). The total power required is $P_{PG} = r \sum N_{\text{eng}} P_{\text{eng}}$, where $r = \max(P_{reqPG}/P_{avPG})$. The sized power is $P_{\text{sized}} = P_{PG} - \sum_{\text{fixed}} N_{\text{eng}} P_{\text{eng}}$, where the sum is over the engine groups for which the power is fixed. Then the sized engine power is $P_{\text{eng}} = f_n P_{\text{sized}}/N_{\text{eng}}$ for the n-th engine group (with f_n an input ratio and $f_1 = \sum_{n \neq 1,\text{sized}} f_n$ for the first group).

3-1.2.2   Main Rotor

The main rotor size is defined by the radius R or disk loading W/A, thrust-weighted solidity σ, hover tip speed V_{tip}, and blade loading $C_W/\sigma = W/\rho A V_{\text{tip}}^2 \sigma$. With more than one main rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density ρ for C_W/σ is obtained from a specified takeoff condition.

If the rotor radius is fixed for the sizing task, three of (R or W/A), C_W/σ, V_{tip}, σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor.

If the sizing task determines the rotor radius (R and W/A), then two of C_W/σ, V_{tip}, σ are input, and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others. The radii of all sized rotors are changed by the same factor.

3-1.2.3   Antitorque or Auxiliary Thrust Rotor

For antitorque and auxiliary thrust rotors, three of (R or W/A), C_W/σ, V_{tip}, σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. Optionally the radius can be scaled with the main rotor radius. The disk loading and blade loading are based on $f_T T_{\text{design}}$, where f_T is an input factor and T_{design} is the maximum thrust from designated design conditions and missions.

3-1.2.4   Wing

The wing size is defined by the wing area S or wing loading W/S, span (perhaps calculated from other geometry), chord, and aspect ratio. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$.

Two of the following parameters are input: area (or wing loading), span, chord, and aspect ratio; the other parameters are derived. Optionally the span can be calculated from the rotor radius, fuselage width, and clearance (typically used for tiltrotors). Optionally the span can be calculated from a specified ratio to the span of another wing.

3-1.2.5   Fuel Tank

The fuel tank capacity $W_{\text{fuel-cap}}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel-miss}}$ (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{\text{fuel-cap}} = \max(f_{\text{fuel-cap}} W_{\text{fuel-miss}}, W_{\text{fuel-miss}} + W_{\text{reserve}})$$

where $f_{\text{fuel-cap}} \geq 1$ is an input factor. Alternative, the fuel tank capacity $W_{\text{fuel-cap}}$ can be input.

3-1.2.6   Weights

The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input, or specified as an increment d plus a fraction f of a weight W:

$$W_{SD} = d_{SDGW} + f_{SDGW}W = \begin{cases} d_{SDGW} + f_{SDGW}W_D \\ d_{SDGW} + f_{SDGW}(W_D - W_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}) \end{cases}$$

$$W_{MTO} = d_{WMTO} + f_{WMTO}W = \begin{cases} d_{WMTO} + f_{WMTO}W_D \\ d_{WMTO} + f_{WMTO}(W_D - W_{\text{fuel}} + W_{\text{fuel-cap}}) \end{cases}$$

This convention allows the weights to be input directly ($f = 0$), or scaled with W_D. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity). Alternatively, W_{SD} can be calculated as the gross weight at a designated sizing flight condition. For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition.

3-1.2.7   Drive System Rating

The drive system rating is defined as a power limit, $P_{DS\text{limit}}$. The rating is properly a torque limit, $Q_{DS\text{limit}} = P_{DS\text{limit}}/\Omega_{\text{ref}}$, but is expressed as a power limit for clarity. The drive system rating can be specified as follows:

a) Input $P_{DS\text{limit}}$.
b) From the engine takeoff power rating, $P_{DS\text{limit}} = f_{\text{limit}} \sum N_{\text{eng}}P_{\text{eng}}$ (summed over all engine groups).
c) From the power available at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}}P_{av}$ (largest of all conditions and segments).
d) From the power required at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}}P_{req}$ (largest of all conditions and segments).

with f_{limit} an input factor. The drive system rating is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft ($P_{RS\text{limit}}$) and of each engine group ($P_{ES\text{limit}}$). The engine shaft rating is calculated as for the drive system rating, without the sum over engine groups. The rotor shaft rating is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system rating $P_{DS\text{limit}}$ is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft rating $P_{ES\text{limit}}$ is the corresponding engine turbine speed. The rotation speed for the rotor shaft rating $P_{RS\text{limit}}$ is the corresponding speed of that rotor.

3–2   Mission Analysis

For the mission analysis, the fuel weight or payload weight is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits. Missions can be fixed or adjustable.

3–3   Flight Performance Analysis

For each performance flight condition, the power required is calculated or maximum gross weight

is calculated. Power required, torque (drive system, engine shaft, and rotor shaft), and fuel weight are then verified to be within limits.

3–4 Maps

3-4.1 Engine Performance

The engine performance can be calculated for a specified range of power, altitude, and speed.

3-4.2 Airframe Aerodynamics

The airframe aerodynamic loads can be calculated for a specified range of angle-of-attack, sideslip angle, and control angles. The aerodynamic analysis evaluates the component lift, drag, and moments given the velocity. The aircraft velocity is here $v_{AC}^F = C^{FA}(v\,0\,0)^T$; interference velocity from the rotors is not considered. From the angle-of-attack α and sideslip angle β, the transformation from wind axes to airframe axes is $C^{FA} = Y_\alpha Z_{-\beta}$ (optionally $C^{FA} = Z_{-\beta} Y_\alpha$ can be used, for better behavior in sideward flight). The loads are summed in the airframe axes (with and without tail loads), and then the wind axis loads are:

$$F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} = C^{AF} F^F \qquad M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = C^{AF} M^F$$

The center of action for the total loads is the fuselage location z_{fuse}. The ratio of the loads to dynamic pressure is required, so a nominal velocity $v = 100$ (ft/sec or m/sec) and sea level standard density are used.

Operation

4–1   Flight Condition

Flight conditions are specified for the sizing task and for the flight performance analysis. For each condition, a flight state is also defined. For the sizing task, certain flight conditions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, or for rotor thrust sizing. The flight condition parameters include gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method.

a) Design gross weight, W_D (calculated or input).

b) Structural design gross weight, W_{SD}, or maximum takeoff weight, W_{MTO} (which may depend on W_D).

c) Function of W_D: $W = d + fW_D$ (with d an input weight and f an input factor).

d) Function of W_{SD} ($W = d + fW_{SD}$); or function of W_{MTO} ($W = d + fW_{MTO}$).

e) Input W.

f) Gross weight from specified mission segment or flight condition.

g) Gross weight maximized, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$ (in general, $\min((fP_{avPG} + d) - P_{reqPG}) = 0$, minimum over all propulsion groups) with d an input power and f an input factor; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$.

h) Gross weight maximized, such that transmission torque equals limit: zero torque margin, $\min(P_{\text{limit}} - P_{req}) = 0$ (mininum over all propulsion groups, engine groups, and rotors).

i) Gross weight maximized, such that power required equals specified power or transmission torque equals limit (most restrictive).

j) Gross weight fallout from input payload and fuel weights: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

Only the last four options are available for W_D design conditions in the sizing task. The gross weight can be obtained from a mission segment only for the sizing task. Optionally the altitude can be obtained from the specified mission segment or flight condition. The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance ϵ and a perturbation Δ are specified.

The useful load can be specified as follows, consistent with the sizing method and the gross weight specification.

a) Input payload weight W_{pay}, fuel weight fallout: $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$.

b) Input fuel weight W_{fuel}, payload weight fallout: $W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.

c) Input payload and fuel weights, gross weight fallout (must match gross weight option): $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

The input fuel weight is $W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}, W_{\text{fuel-cap}}) + \sum N_{\text{auxtank}}W_{\text{aux-cap}}$. For fallout fuel weight, N_{auxtank} is changed (optionally only increased). If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight changed instead. The fixed useful load can have increments, including crew weight increment; equipment weight increment; and installed folding, wing, wing extension, and other kits. These increments are reflected in the fallout weight.

<div align="center">

4–2 Mission

</div>

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments. A flight state is defined for each mission segment. For the sizing task, certain missions are designated to be used for engine sizing, for design gross weight calculations, for transmission sizing, or for fuel tank sizing. The mission parameters include mission takeoff gross weight and useful load. The gross weight can be specified as follows, consistent with the sizing method.

a) Design gross weight, W_D (calculated or input).
b) Structural design gross weight, W_{SD}, or maximum takeoff weight, W_{MTO} (which may depend on W_D).
c) Function of W_D: $W = d + fW_D$ (with d an input weight and f an input factor).
d) Function of W_{SD}, $W = d + fW_{SD}$; or function of W_{MTO}, $W = d + fW_{MTO}$.
e) Input W.
f) Gross weight maximized at specified mission segments, such that power required equals specified power: $P_{reqPG} = fP_{avPG} + d$ (in general, $\min((fP_{avPG} + d) - P_{reqPG}) = 0$, minimum over all propulsion groups) with d an input power and f an input factor; default $d = 0$ and $f = 1$ gives zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$.
g) Gross weight maximized at specified mission segments, such that transmission torque equals limit: zero torque margin, $\min(P_{\text{limit}} - P_{req}) = 0$ (mininum over all propulsion groups, engine groups, and rotors).
h) Gross weight maximized at specified mission segments, such that power required equals specified power or transmission torque equals limit (most restrictive).
i) Gross weight fallout from input initial payload and fuel weights: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.
j) Gross weight fallout from input initial payload weight and calculated mission fuel weight: $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

If maximum gross weight is specified for more than one mission segment, then the minimum takeoff gross weight increment is used; so the power or torque margin is zero for the critical segment and positive for other designated segments. Only the last five options are available for W_D design conditions in the sizing task. The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance ϵ and a perturbation Δ are specified.

The useful load can be specified as follows, consistent with the sizing method and the gross weight

specification.

> a) Input initial payload weight W_{pay}, fuel weight fallout: $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$.
> b) Input fuel weight W_{fuel}, initial payload weight fallout: $W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.
> c) Calculated mission fuel weight, initial payload weight fallout: $W_{\text{pay}} = W_G - W_O - W_{\text{fuel}}$.
> d) Input payload and fuel weights, takeoff gross weight fallout (must match gross weight option): $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.
> e) Input payload weight and calculated mission fuel weight, takeoff gross weight fallout (must match gross weight option): $W_G = W_O + W_{\text{pay}} + W_{\text{fuel}}$.

The input fuel weight is $W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel−cap}}, W_{\text{fuel−cap}}) + \sum N_{\text{auxtank}}W_{\text{aux−cap}}$. If the fuel weight is not calculated from the mission, then the mission is changed. The fixed useful load can have increments, including installed folding kits; other increments are specified for individual mission segments.

The takeoff gross weight is evaluated at the start of the mission, perhaps maximized for zero power margin at a specified mission segment (either takeoff conditions or midpoint). Then the aircraft is flown for all segments. For calculated mission fuel weight, the fuel weight at takeoff is set equal to the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on mission fuel weight. Range credit segments (defined below) can also require an iteration. A successive substitution method is used if an iteration is required, with a tolerance ϵ specified. The iteration to maximize takeoff gross weight could be an outer loop around the mission iteration, but instead it is executed as part of the mission iteration. At the specified mission segment, the gross weight is maximized for zero power margin, and the resulting gross weight increment added to the takeoff gross weight for the next mission iteration. Thus takeoff gross weight is also a variable of the mission iteration.

Each mission consists of a specified number of mission segments. The following segment types can be specified.

> a) Taxi or warm-up (fuel burned but no distance added to range).
> b) Distance: fly segment for specified distance (calculate time).
> c) Time: fly segment for specified time (calculate distance).
> d) Hold: fly segment for specified time (loiter, so fuel burned but no distance added to range).
> e) Climb: climb or descend from present altitude to next segment altitude (calculate time and distance).
> f) Spiral: climb or descend from present altitude to next segment altitude; fuel burned but no distance added to range.

For each mission segment a payload weight can be specified; or a payload weight change can be specified, as an increment from the initial payload or as a fraction of the initial payload.

The number of auxiliary fuel tanks can change with each mission segment: N_{auxtank} is changed based on the fuel weight (optionally only increased relative to the input number at takeoff, optionally fixed during mission). For input fuel weight, N_{auxtank} is specified at takeoff. For fallout fuel weight, the takeoff fuel weight is changed for the auxiliary fuel tank weight given N_{auxtank} (fixed $W_G - W_{\text{pay}} = W_O + W_{\text{fuel}}$).

If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\text{fuel}} = W_G - W_O - W_{\text{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the takeoff payload weight changed instead. For fuel tank design missions, N_{auxtank} and fuel tank capacity is determined from W_{fuel}. Optionally the aircraft can refuel (either on the ground or in the air) at the start of a mission segment, by either filling all tanks to capacity or adding a specified fuel weight. Optionally fuel can be dropped at the start of a mission segment. The fixed useful load can have changes, including crew weight increment; equipment weight increment; and installed wing extension and other kits.

For calculation of the time or distance in a mission segment, a headwind or tailwind can be specified. The wind velocity is a linear function of altitude h: $V_w = \pm(\max(0, d_{\text{wind}} + f_{\text{wind}}h))$, with the plus sign for a headwind and the minus sign for a tailwind. For example, California-to-Hawaii 85$^{\text{th}}$ percentile winter quartile headwind profile is $V_w = 9.59 + 0.00149h$ (with altitude h in ft).

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at speed for best endurance. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{burn}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{burn}} + f_{\text{res}}W_{\text{fuel}-\text{cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used. Time and distance in reserve segments are not included in endurance and range.

To facilitate specification of range, range calculated for a group of segments (typically climb and descent segments) can be credited to a designated distance segment. For mission analysis, missions can be fixed or adjustable. In an adjustable mission, the fuel is input, so the time or distance in specified segments is adjusted based on the calculated fuel burned. If more than one segment is adjusted, all must be distance or all must be time or hold. Each segment can have only one special designation: reserve, adjustable, or range credit.

A segment with a large distance, time, or altitude change can be split into several segments, for more accurate calculation of the performance and fuel burned. The number of segments n can be input, or calculated from an input increment Δ: $n = [x/\Delta] + 1$, where the brackets indicate integer truncation, and x is the total distance, time, or altitude change. Then the change for each split segment is $\Delta = x/n$.

Table 4-1 summarizes the time T, distance D, and range dR calculations for each segment. The segment fuel burned is $dW_{\text{burn}} = T\dot{w}$, where $\dot{w}$ is the fuel flow. The horizontal velocity is V_h, and the vertical velocity (climb or descent) is V_c. The altitude at the start of the segment is h, and at the end of the segment (start of next segment) h_{end}. The wind speed is V_w, and the ground speed is $V_h - V_w$. The air distance is calculated from the time and speed (D/V_h), without the wind speed.

In an adjusted mission, the distances or times are changed at the end of the mission such that the sum of the fuel burned increments will equal the difference between takeoff fuel weight (plus any added fuel) and the calculated mission fuel: $\sum dW_{\text{burn}} = \sum \dot{w}\,dT = \sum \dot{w}\,dD/(V_h - V_w) = \Delta W_{\text{fuel}}$. The increments are apportioned among the adjusted segments by the factor f, determined from the ratio of the input distances or times: $dD = f\Delta D$ or $dT = f\Delta T$. Hence $\Delta D = \Delta W_{\text{fuel}}/\left(\sum f\dot{w}/(V_h - V_w)\right)$ or $\Delta T = \Delta W_{\text{fuel}}/\left(\sum f\dot{w}\right)$. For a segment that is a source of range credit, the range increment is set to zero and the distance D is added to D_{other} of the destination segment. For the destination segment, the range contribution remains fixed at the input value, but the time and hence fuel burned are calculated from $(\text{dist} - D_{\text{other}})$. It is necessary to separately accumulate D_{other} from earlier segments and D_{other} from later segments; D_{other} from later segments are estimated initially from the last iteration. At the end of

the mission, the times and fuel burned are recalculated for all range credit destination segments.

Table 4-1. Mission segment calculations.

segment kind	time T	distance D	range dR
taxi	time	0	D
distance	$D/(V_h - V_w)$	dist	D
time	time	$T(V_h - V_w)$	D
hold	time	0	D
climb	$(h - h_{\text{end}})/V_c$	$T(V_h - V_w)$	D
spiral	$(h - h_{\text{end}})/V_c$	0	D
range credit			
source	T	D	0
destination	$D/(V_h - V_w)$	$\text{dist} - D_{\text{other}}$	dist
adjusted			
distance	$T + dD/(V_h - V_w)$	$D + dD = D + f\Delta D$	D_{new}
time	$T + dT = T + f\Delta T$	$D + dT(V_h - V_w)$	D_{new}
hold	$T + dT = T + f\Delta T$	0	D_{new}

The segment time, distance, and fuel burned are evaluated by integrating over the segment duration. This integration can be performed by using the horizontal velocity, climb velocity, and fuel flow obtained for the flight state with the gross weight and altitude at the start of the segment; or at the middle of the segment; or the average of the segment start and segment end values (trapezoidal integration). The gross weight at the segment middle equals the gross weight at the segment start, less half the segment fuel burned (obtained from the previous mission iteration). The gross weight at the segment end equals the gross weight at the segment start, less the segment fuel burned. With trapezoidal integration, for the output the flight state is finally evaluated at the segment middle.

The mission endurance (block time), range, and fuel burned are $E = \sum T$, $R = \sum dR$, $W_{\text{burn}} = \sum dW_{\text{burn}}$ (sum over all non-reserve segments). The reserve fuel from mission segments is $W_{\text{res}} = \sum dW_{\text{burn}}$ (sum over all reserve segments). Optionally the reserve fuel is the maximum of that from mission segments and the fraction $f_{\text{res}}W_{\text{burn}}$, or the fraction $f_{\text{res}}W_{\text{fuel-cap}}$. The calculated mission fuel is then $W_{\text{fuel}} = W_{\text{burn}} + W_{\text{res}}$.

A fuel efficiency measure for the mission is the product of the payload and range, divided by the fuel weight: $e = W_{\text{pay}}R/W_{\text{burn}}$ (ton-nm/lb or ton-nm/kg). A productivity measure for the mission is $p = W_{\text{pay}}V/W_O$ (ton-kt/lb or ton-kt/kg), where W_O is the operating weight and V the block speed; or $p = W_{\text{pay}}V/W_{\text{burn}}$ (ton-kt/lb or ton-kt/kg). The Breguet range equation $R = RF \ln(W_0/W_1)$ is obtained by integrating $dR = -RF(dW/W)$ for constant range factor

$$RF = \frac{L/D_e}{\text{sfc}} = \frac{WV/P}{\text{sfc}}$$

The endurance $E = EF\, 2\left(\sqrt{W_0/W_1} - 1\right)$ is obtained by integrating $dE = -EF\sqrt{W_0}(dW/W^{3/2})$ for constant endurance factor

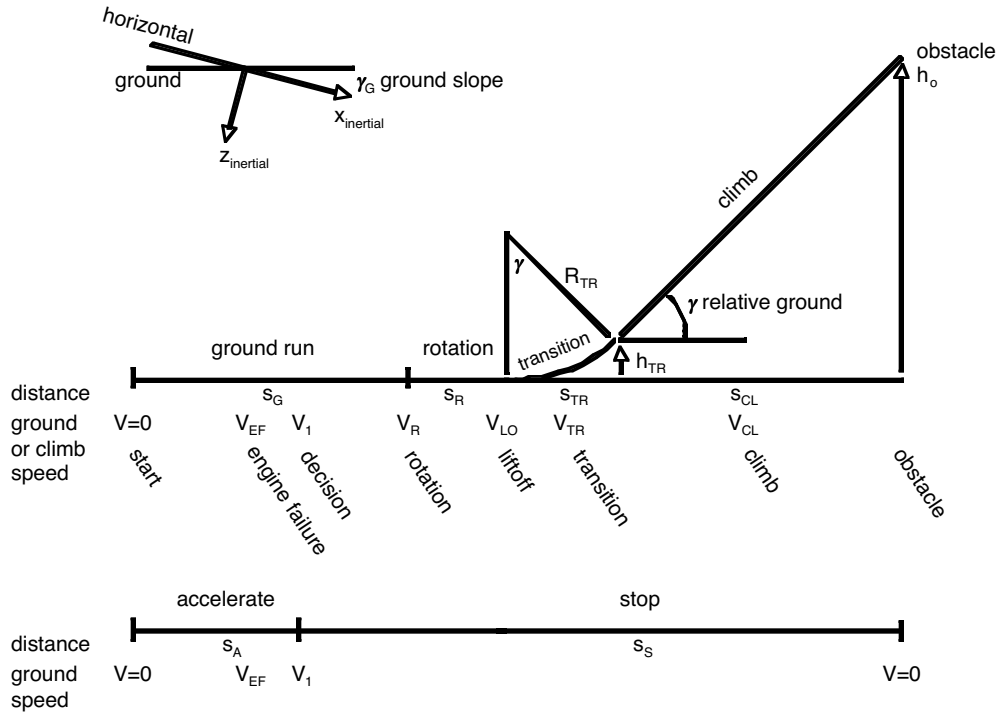$$EF = \frac{L/D_e}{\text{sfc}}\frac{\sqrt{W/W_0}}{V} = \frac{W/P\sqrt{W/W_0}}{\text{sfc}}$$

Figure 4-1. Takeoff distance and accelerate-stop distance elements.

Constant RF implies operation at constant $L/D_e = WV/P$. Constant EF implies operation at constant $(L/D_e)\sqrt{W}/V = W^{3/2}/P$ (or constant $C_L^{3/2}/C_D$ for an airplane). It follows that overall range and endurance factors can be calculated from the mission performance:

$$RF = \frac{R}{\ln W_0/W_1}$$

$$EF = \frac{E}{2\left(\sqrt{W_0/W_1} - 1\right)}$$

where $W_0 = W_{to}$ is the takeoff weight, and $W_1 = W_{to} - W_{\mathrm{burn}}$.

4–3   Takeoff Distance

The takeoff distance can be calculated, either as ground run plus climb to clear an obstacle or accelerate-stop distance in case of engine failure. The obstacle height h_o is typically 35 ft for commercial transport aircraft, or 50 ft for military aircraft and general aviation. This calculation allows determination of the balanced field length: engine failure at critical speed, such that the distance to clear the obstacle equals the distance to stop. Landing and VTOL takeoff calculations are not implemented, as these are best solved as an optimal control problem.

The takeoff distance consists of a ground run, from zero ground speed to liftoff speed V_{LO}, perhaps including engine failure at speed V_{EF}; then rotation, transition, and climb; or decelerate to stop. Figure 4-1 describes the elements of the takeoff distance and the accelerate-stop distance, with the associated

speeds. The ground is at angle γ_G relative to the horizontal (inertial axes), with γ_G positive for takeoff up hill. The takeoff profile is defined in terms of ground speed or climb speed, input as calibrated airspeed (CAS). The aircraft speed relative to the air is obtained from the ground speed, wind, and ground slope. The aircraft acceleration as a function of ground speed is integrated to obtain the ground distance, as well as the time, height, and fuel burned. Usually the speed increases from the start to liftoff (or engine failure), but the calculated acceleration depends on the flight state specification. The analysis checks for consistency of the input velocity and the calculated acceleration (on the ground) and for consistency of the input height and input or calculated climb angle (during climb).

The takeoff profile consists of a set of mission segments. The first segment is at the start of the takeoff, $V = 0$. Subsequent segments correspond to the ends of the integration intervals. The last segment has the aircraft at the required obstacle height, or stopped on the ground. The mission can consist of just one takeoff; more than one takeoff; or both takeoff and non-takeoff segments. Takeoff segments contribute to the mission fuel burned, but do not contribute to the mission time, distance, or range. The takeoff distance calculation is performed for a set of adjacent segments, the first segment specified as the takeoff start, and the last segment identified as before a non-takeoff segment or before another takeoff start. The takeoff distance is calculated if a liftoff segment (with V_{LO}) is specified; otherwise the accelerate-stop distance is calculated. Table 4-2 summarizes the mission segments for takeoff calculations. There can be only one liftoff, engine failure, rotation, and transition segment (or none). The engine failure segment must occur before the liftoff segment. Rotation and transition segments must occur after liftoff. All ground run segments must be before liftoff, and all climb segments must be after liftoff. Takeoff segments (except start, rotation, and transition) can be split, in terms of height for climb and in terms of velocity for other segments. Splitting the takeoff or engine failure segment produces additional ground run segments. Separately defining multiple ground run, climb, or brake segments allows configuration variation during the takeoff.

Table 4-2. Mission segments for takeoff calculation.

takeoff distance		accelerate-stop distance	
start	$V = 0$	start	$V = 0$
ground run	V	ground run	V
engine failure	V_{EF}	engine failure	V_{EF}
ground run	V	brake	V
liftoff	V_{LO}	brake	$V = 0$
rotation	V_R		
transition	V_{TR}		
climb, to h	V_{CL}		
climb, to h_o	V_{CL}		

Each takeoff segment requires that the flight state specify the appropriate configuration, trim option, and maximum effort. In particular, the number of inoperative engines for a segment is part of the flight state specification, regardless of whether or not an engine failure segment is defined. The engine failure segment (if present) serves to implement a delay in decision after failure: for a time t_1 after engine failure, the engine rating, power fraction, and friction of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The number of inoperative engines specified must be consistent with the presence of the engine failure segment. The takeoff is assumed

to occur at fixed altitude (so the maximum-effort variable can not be altitude). The flight state velocity specification is superseded by the ground or climb speed input for the takeoff segment. The flight state specification of height above ground level is superseded by the height input for the takeoff segment.

The ground distance, time, height, and fuel burned are calculated for each takeoff segment. The takeoff distance or accelerate-stop distance is the sum of the ground distance of all segments. Takeoff segments do not contribute to mission time, distance, or range.

4-3.1   Ground Run

The takeoff starts at zero ground speed and accelerates to liftoff ground speed V_{LO} (input as CAS). Possibly an engine failure speed $V_{EF} < V_{LO}$ is specified. Start, liftoff, and engine failure segments designate events, but otherwise are analyzed as ground run segments. The decision speed V_1 is t_1 seconds after engine failure (typically $t_1 = 1$ to 2 sec). Up to t_1 after engine failure, conditions of the engine failure segment are used (so the engine failure segment corresponds to conditions before failure). The aircraft acceleration is obtained from the thrust minus drag ($T - D$ in airplane notation), plus a friction force proportional to the weight on wheels ($W - L$ in airplane notation):

$$Ma = T - D - \mu(W - L) = \sum F_x - \mu \sum F_z$$

from the force components in ground axes (rotated by the ground slope angle γ_G from inertial axes). Table 4-3 gives typical values of the friction coefficient μ. The velocity of the aircraft relative to the air is obtained from the ground velocity V, wind velocity V_w (assumed parallel to the ground here), and the ground slope: $V_h = (V + V_w)\cos\gamma_G$ and $V_c = (V + V_w)\sin\gamma_G$. The takeoff configuration is specified, including atmosphere, in-ground-effect, gear down, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically fixed attitude with longitudinal force trimmed using collective, for a given longitudinal acceleration. Perhaps the net aircraft yaw moment is trimmed with pedal. The maximum-effort condition is specified: maximum longitudinal acceleration (ground axes) for zero power margin. The aircraft acceleration as a function of ground speed is integrated to obtain the segment time, ground distance, height, and fuel burned:

$$t_G = \int dt = \int \frac{dt}{dv}\, dv = \int \frac{dv}{a} = \sum_{\text{seg}} \frac{1}{2}\left(\frac{1}{a_2} + \frac{1}{a_1}\right)(v_2 - v_1) = \sum_{\text{seg}} \Delta t$$

$$s_G = \int v\, dt = \int v\frac{dt}{dv}\, dv = \int \frac{v\, dv}{a} = \int \frac{d(v^2)}{2a} = \sum_{\text{seg}} \frac{1}{2}\left(\frac{1}{2a_2} + \frac{1}{2a_1}\right)(v_2^2 - v_1^2) = \sum_{\text{seg}} \left(\frac{v_2 + v_1}{2}\right)\Delta t$$

$$h_G = 0$$

$$w_G = \int \dot{w}_f\, dt = \sum_{\text{seg}} \left(\frac{\dot{w}_{f2} + \dot{w}_{f1}}{2}\right)\Delta t$$

Trapezoidal integration is used; each segment corresponds to the end of an integration integral.

Table 4-3. Typical friction coefficient μ.

surface	rolling	braking
dry and hard	0.03–0.05	0.30–0.50
grass	0.08	0.20
ice	0.02	0.06–0.10

4-3.2   Brake

After engine failure, the aircraft can decelerate to a stop. The operating engines are at idle. Reverse thrust is not permitted for the accelerate-stop distance calculation. The braking configuration is specified. Typically no trim option is executed; rather the aircraft has fixed attitude with controls for zero rotor thrust (such as zero collective and pedal). The aircraft acceleration as a function of ground speed is integrated, as for ground run.

4-3.3   Rotation

Rotation occurs at speed V_R; usually $V_R = V_{LO}$ is used. The duration t_R is specified, then $s_R = V_R t_R$, $h_R = 0$, $w_R = \dot{w}_f t_R$ are the ground distance, height, and fuel burned. Typically $t_R = 1$ to 3 sec.

4-3.4   Transition

Transition from liftoff to climb is modeled as a constant load factor pull-up to the specified climb angle γ, at speed V_{TR}. Usually $V_{TR} = V_{LO}$ is used, and typically $n_{TR} \cong 1.2$. From the load factor $n_{TR} = 1 + V_{TR}^2/gR_{TR}$, the flight path radius is $R_{TR} = V_{TR}^2/(g(n_{TR}-1))$ and the pitch rate is $\dot{\theta} = V_{TR}/R_{TR}$. Then

$$t_{TR} = \gamma/\dot{\theta} = \gamma \frac{R_{TR}}{V_{TR}} = \gamma \frac{V_{TR}}{g(n_{TR} - 1)}$$

$$s_{TR} = R_{TR} \sin \gamma$$

$$h_{TR} = R_{TR}(1 - \cos \gamma)$$

$$w_{TR} = \dot{w}_f t_{TR}$$

are the time, ground distance, height, and fuel burned.

4-3.5   Climb

Climb occurs at angle γ relative to the ground and air speed V_{CL}, from the transition height h_{TR} to the obstacle height h_o (perhaps in several climb segments). The climb configuration is specified, including atmosphere, in-ground-effect, gear down or retracted, power rating, nacelle tilt, flap setting, and number of inoperative engines. An appropriate trim option is specified, typically aircraft force and moment trimmed with attitude and controls. The climb angle and air speed can be fixed or a maximum-effort condition can be specified. The maximum-effort options are fixed air speed and maximum rate of climb for zero power margin; or airspeed for best climb rate or best climb angle with maximum rate of climb for zero power margin. Not implemented is a maximum-effort calculation of maximum flight path acceleration for zero power margin, for specified climb angle; this calculation would require integration of the acceleration as a function of flight speed. For the climb segment, the input V_{CL} is the magnitude of the aircraft velocity relative to the air, and the climb angle relative to the horizontal is $\theta_V = \gamma + \gamma_G$. Hence from the maximum-effort calculation, the climb angle relative to the ground is $\gamma = \theta_V - \gamma_G$ and the ground speed is $V_{\text{ground}} = V_{CL} \cos \gamma - V_w$ (from the wind speed V_w). Then

$$t_{CL} = s_{CL}/V_{\text{ground}}$$

$$s_{CL} = (h - h_{\text{last}})/\tan \gamma$$

$$h_{CL} = h$$

$$w_{CL} = \dot{w}_f t_{CL}$$

are the time, ground distance, height, and fuel burned.

4–4   Flight State

A flight state is defined for each flight condition (sizing task design conditions and flight performance analysis), and for each mission segment. The following parameters are required.

a) Speed: flight speed and vertical rate of climb, with the following options.

> 1) Specify horizontal speed (or forward speed or velocity magnitude), rate of climb (or climb angle), and sideslip angle.
> 2) Hover or vertical flight (input vertical rate of climb; climb angle 0 or ± 90 deg).
> 3) Left or right sideward flight (input velocity and rate of climb; sideslip angle ± 90 deg).
> 4) Rearward flight (input velocity and rate of climb; sideslip angle 180 deg).

b) Aircraft motion.

> 1) Pitch and roll angles (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).
> 2) Turn, pull-up, or linear acceleration.

c) Altitude: For mission segment, optionally input, or from last mission segment; climb segment end altitude from next segment.

d) Atmosphere:

> 1) Standard day, polar day, tropical day, or hot day at specified altitude.
> 2) Standard day, polar day, or tropical day plus temperature increment.
> 3) Standard day, polar day, or tropical day and specified temperature.
> 4) Input density and temperature.
> 5) Input density, speed of sound, and viscosity.

e) Height of landing gear above ground level. Landing gear state (extended or retracted).
f) Aircraft control state: input; or conversion schedule.
g) Aircraft control values (Aircraft values or flight state input; initial values for trim variables, fixed otherwise).
h) Aircraft center-of-gravity position (increment or input value).

For each propulsion group, the following parameters are required:

i) Drive system state.
j) Rotor tip speed for primary rotor:

> 1) Input.
> 2) Reference.
> 3) Conversion schedule or function speed.

4) Default for hover, cruise, maneuver, OEI, or transmission sizing condition.

5) From input $C_T/\sigma = t_0 - \mu t_1$, or μ, or M_{at} (where μ is the rotor advance ratio, and M_{at} is the rotor advancing tip Mach number).

And for each engine group of each propulsion group:

k) Number of inoperative engines.

l) Infrared suppressor state: off (hot exhaust) or on (suppressed exhaust).

m) Engine rating, and fraction of rated engine power available.

Aircraft and rotor performance parameters for each flight state:

o) Aircraft drag: forward flight drag increment, accounting for payload aerodynamics.

p) Rotor performance: induced power factor κ and profile power mean c_d.

The aircraft trim state and trim targets are also specified.

The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. For the maximum effort, a quantity and variable are specified. The available maximum-effort quantities include:

a) Best endurance: maximum $1/\dot{w}$.

b) Best range: 99% maximum $V/\dot{w}$ (high side); or low side; or 100%.

c) Best climb or descent rate: maximum V_z or $1/P$.

d) Best climb or descent angle: maximum V_z/V_h or V/P.

e) Ceiling: maximum altitude.

f) Power limit: zero power margin, $\min(P_{avPG} - P_{reqPG}) = 0$ (minimum over all propulsion groups).

g) Torque limit: zero torque margin, $\min(Q_{\text{limit}} - Q_{req}) = 0$ (minimum over all propulsion groups, engine groups, and rotors; Q_{limit} expressed as power at reference rotation speed).

h) Power limit or torque limit: most restrictive.

i) Wing stall: zero wing lift margin, $C_{L\max} - C_L = 0$ (for designated wing).

j) Rotor stall: zero rotor thrust margin, $(C_T/\sigma)_{\max} - C_T/\sigma = 0$ (for designated rotor, steady or transient limit).

Here $\dot{w}$ is the aircraft fuel flow, and P the aircraft power. The available maximum-effort variables include:

a) Horizontal velocity V_h or vertical rate of climb V_z (times an input factor).

b) Aircraft altitude.

c) Aircraft angular rate, $\dot{\theta}$ (pull-up) or $\dot{\psi}$ (turn).

d) Aircraft linear acceleration (airframe, inertial, or ground axes).

If the variable is velocity, first the velocity is found for the specified maximum effort; then the performance is evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized

(slope zero), then the variable is found such that $V/\dot{w}$ equals 99% of that maximum. Two maximum-effort quantity/variable pairs can be specified, and solved in nested iterations. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance ϵ and a perturbation Δ are specified.

Given the gross weight and useful load (from the flight condition or mission specification), the performance is calculated for this flight state. The calculated state information includes weight, speed and velocity orientation (climb and sideslip), aircraft Euler angles, rotor tip speeds, and aircraft controls.

A number of performance metrics are calculated for each flight state. The specific range is the ratio of the speed to the fuel flow: $V/\dot{w}$ (nm/lb or nm/kg). From the Breguet range equation, it follows that the range for fuel equal 1% of the gross weight is

$$R_{1\%GW} = \frac{L/D_e}{\mathrm{sfc}} \ln\left(\frac{1}{.99}\right)$$

where $L/D_e = WV/P$. A fuel efficiency measure is the product of the payload and specific range: $e = W_{\mathrm{pay}}(V/\dot{w})$ (ton-nm/lb or ton-nm/kg). A productivity measure is $p = W_{\mathrm{pay}}V/W_O$ (ton-kt/lb or ton-kt/kg), where W_O is the operating weight.

The aircraft weight statement defines the fixed useful load and operating weight for the design configuration. For each flight state, the fixed useful load may be different from the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

4–5 Environment and Atmosphere

The aerodynamic environment is defined by the speed of sound c_s, density ρ, and kinematic viscosity $\nu = \mu/\rho$ of the air (or other fluid). These quantities can be obtained from the standard day (International Standard Atmosphere), or input directly. Polar day, tropical day, and hot day atmospheres can also be used. The following options are implemented:

a) Input the altitude h_{geom} and a temperature increment ΔT. Calculate the temperature from altitude and pressure from temperature for the standard day (or polar day, tropical day, hot day), add ΔT, and then calculate the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.

b) Input the pressure altitude h_{geom} and the temperature τ (°F or °C). Calculate the pressure (from temperature vs. altitude) for the standard day (or polar day, tropical day, hot day), and then the density from the equation of state for a perfect gas. Calculate the speed of sound and viscosity from the temperature.

c) Input the density ρ and the temperature τ (°F or °C). Calculate the speed of sound and viscosity from the temperature.

d) Input the density ρ, sound speed c_s, and viscosity μ. Calculate the temperature from the sound speed.

Here h_{geom} is the geometric altitude above mean sea level.

The gravitational acceleration g can have the standard value or an input value.

The International Standard Atmosphere (ISA) is a model for the variation with altitude of pressure, temperature, density, and viscosity, published as International Standard ISO 2533 by the International Organization for Standardization (ISO) (ref. 1). The ISA is intended for use in calculations and design of flying vehicles, to present the test results of flying vehicles and their components under identical conditions, and to allow unification in the field of development and calibration of instruments. The ISA is defined up to 80 km geopotential altitude and is identical to the ICAO Standard Atmosphere up to 32 km.

Dry air is modeled in the ISA as a perfect gas with a mean molecular weight, and hence a gas constant R, defined by adopted values for sea level pressure, temperature, and density (p_0, T_0, ρ_0). The speed of sound at sea level c_{s0} is defined by an adopted value for the ratio of specific heats γ. The variation of temperature with geopotential altitude is defined by adopted values for vertical temperature gradients (lapse rates, L_b) and altitudes (h_b). The variation of pressure with geopotential altitude is further defined by an adopted value for the standard acceleration of free fall (g). The variation of dynamic viscosity μ with temperature is defined by adopted values for Sutherland's empirical coefficients β and S.

The required parameters are given in table 4-4, including the acceleration produced by gravity, g. The temperature T is in $°K$, while τ is $°C$ (perhaps input as $°F$); $T = T_{\mathrm{zero}} + \tau$. The gas constant is $R = p_0/\rho_0 T_0$, and μ_0 is actually obtained from S and β. The ISA is defined in SI units. Although table 4-4 gives values in both SI and English units, all the calculations for the aerodynamic environment are performed in SI units. As required, the results are converted to English units using the exact conversion factors for length and force.

The ISA consists of a series of altitude ranges with constant lapse rate L_b (linear temperature change with altitude). Thus at altitude h_g, the standard day temperature is

$$T_{\mathrm{std}} = T_b + L_b(h_g - h_b)$$

for $h_g > h_b$. The altitude ranges and lapse rates are given in table 4-5. Note that h_0 is sea level, and h_1 is the boundary between the troposphere and the stratosphere. This altitude h_g is the geopotential height, calculated assuming constant acceleration due to gravity. The geometric height h is calculated using an inverse square law for gravity. Hence $h_g = rh/(r + h)$, where r is the nominal radius of the Earth. The standard day pressure is obtained from hydrostatic equilibrium ($dp = -\rho g\, dh_g$) and the equation of state for a perfect gas ($p = \rho RT$, so $dp/p = -(g/RT)dh_g$). In isothermal regions ($L_b = 0$) the standard day pressure is

$$\frac{p_{\mathrm{std}}}{p_b} = e^{-(g/RT)(h_g - h_b)}$$

and in gradient regions ($L_b \neq 0$)

$$\frac{p_{\mathrm{std}}}{p_b} = \left(\frac{T}{T_b}\right)^{-g/RL_b}$$

where p_b is the pressure at h_b, obtained from these equations by working up from sea level. Let T_0, p_0, ρ_0, c_{s0}, μ_0 be the temperature, pressure, density, sound speed, and viscosity at sea level standard conditions.

Then the density, sound speed, and viscosity are obtained from

$$\rho = \rho_0 \left(\frac{p}{p_0} \right) \left(\frac{T}{T_0} \right)^{-1}$$

$$c_s = c_{s0} \left(\frac{T}{T_0} \right)^{1/2}$$

$$\mu = \mu_0 \left(\frac{(T/T_0)^{3/2}}{\alpha(T/T_0) + 1 - \alpha} \right)$$

where $\mu_0 = \beta T_0^{3/2}/(T_0 + S)$ and $\alpha = T_0/(T_0 + S)$. For the cases using input temperature, $T = T_{\text{zero}} + \tau$. The density altitude and pressure altitude are calculated for reference. From the density and the standard day (troposphere only), the density altitude is:

$$h_d = \frac{T_0}{|L_0|} \left(1 - \left(\frac{\rho}{\rho_0} \right)^{1/(g/R|L_0|-1)} \right)$$

From the pressure $p = \rho R T$ and the standard day,

$$h_p = \frac{T_0}{|L_0|} \left(1 - \left(\frac{p}{p_0} \right)^{1/(g/R|L_0|)} \right) = \frac{T_0}{|L_0|} \left(1 - \left(\frac{\rho}{\rho_0} \frac{T}{T_0} \right)^{1/(g/R|L_0|)} \right)$$

is the pressure altitude.

The polar and tropical days are atmospheric models that describe realistic profiles of extremes of temperature and density, needed to calculate performance in near-worst-case conditions (ref. 6). These atmospheres are hydrodynamically balanced and can be used in calculations involving engine performance and aerodynamic characteristics, including calculations of true vertical velocity (ref. 4). Tables of air properties for the polar and tropical days are given in MIL-STD-3013A, based on MIL-STD-210A. The break points in the lapse rates are evident in MIL-STD-210A, in terms of degrees C as a function of altitude in ft (geopotential altitude in MIL-C-5011B). The pressure ratio at sea level is $\delta = 30.268/29.92 = 1.0116$ for the polar day, and $\delta = 1$ for the tropical day. The altitude ranges and lapse rates are given in table 4-6 for the polar day, and in table 4-7 for the tropical day. Figure 4-2 compares the temperature profiles. The air properties for the polar and tropical days are calculated from h_b, L_b, and T_b using the equations of hydrostatic equilibrium, as for the standard day. The primary data for the polar and tropical days are the altitude in ft and temperature in °C; h_b in km and L_b are derived using the exact conversion factor for length.

The conditions of the standard day, polar day, and tropical day are applicable to free air conditions. Temperatures close to the surface of the earth, even at high elevations, can be considerably higher than those for free air. The hot day is a model of ground-level atmospheric conditions, to be used for takeoff and other ground operations at elevations up to 15000 ft (ref. 6). The hot day properties are statistically sampled and are not hydrodynamically balanced, hence should be used for approximately constant altitude conditions (ref. 4).

The origins of the hot day are in ref. 2, which has $103°F$ for sea level, and a lapse rate $-3.7°F$ per 1000 ft (geometric) to 40000 ft. The pressure in the table is from the equations for equilibrium, which gives the pressure altitude; these data are not used further. According to MIL-STD-210A (paragraph 3.1.1), the pressure (hence pressure altitude) as a function of altitude was obtained from statistics. Up to 15000 ft, the ratio of geometric altitude to pressure altitude is 1.050. The -3.7 geometric lapse rate
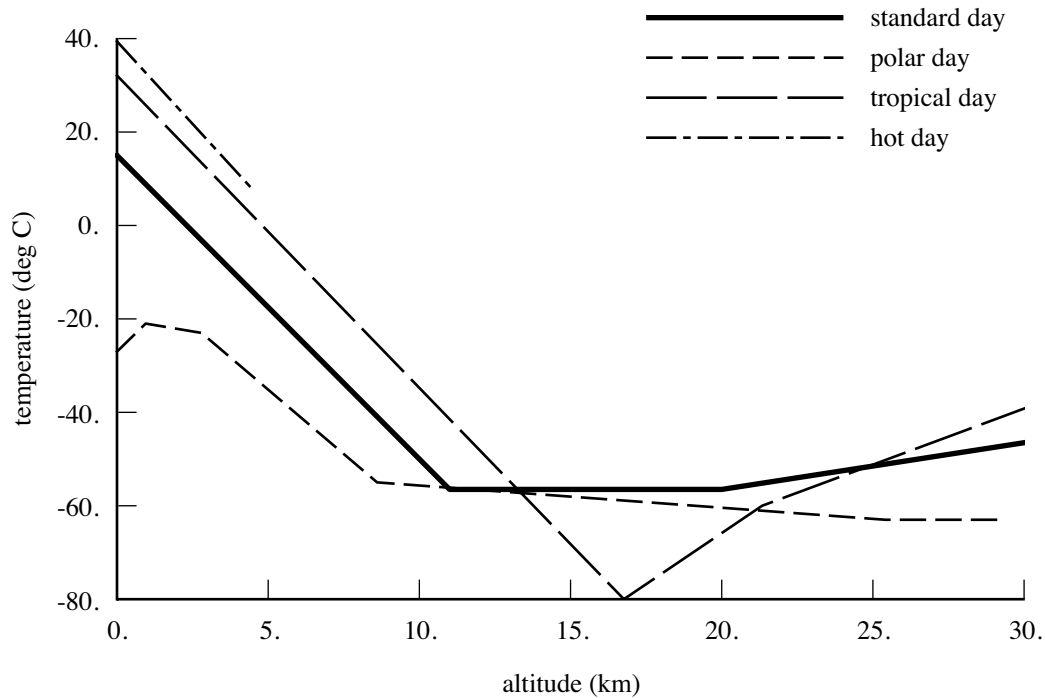
Figure 4-2. Temperature as a function of altitude.

plus the mapping of geometric to pressure altitude gave °F vs. pressure altitude, rounded to 1 decimal place. Then °C was calculated from °F, rounded to 1 decimal place. The pressure ratio δ followed from the pressure altitude. The geometric altitude (to 35000 ft) in MIL-STD-210A was calculated from the temperature and the -3.7 lapse rate, rounded to 100s. MIL-C-5011B has same data as MIL-STD-210A, but does not give geometric altitude, and truncates the table at 15000 ft. MIL-STD-3013A took °C (already rounded to 1 decimal place) vs. pressure altitude (ft) from MIL-STD-210A as the temperature profile up to 15000 ft, and calculated °F (2 decimal places, so no more loss of information) from °C. The geopotential altitude was calculated from °C and -3.7°F lapse rate, rounded to 100s; but these altitude information is not meaningful, since the are not in equilibrium; only the pressure altitude is used. Thus the hot day model has a sea level temperature of 39.4°C (102.92°F). Curve fitting the data to 15000 ft gives a lapse rate of -7.065°C per 1000 m = -2.1534°C per 1000 ft (table 4-8).

Given the pressure altitude h, the hot day temperature and pressure ratio are

$$T_{\text{hot}} = T_b + L_b(h - h_b)$$
$$\frac{p_{\text{hot}}}{p_0} = \left(1 + \frac{L_0}{T_0}h\right)^{-g/RL_b}$$

using the standard day $L_0 = -6.5$. A constant lapse rate fits the tabular data for the hot day atmosphere to only about 0.1°C. Thus the tabular data can be used directly instead (table 4-8, from MIL-C-5011B), with linear interpolation to the specified pressure altitude. MIL-STD-3013A has the same data, but only for altitudes a multiple of 1000 ft.

4–6   References

1) International Organization for Standardization. "Standard Atmosphere." ISO 2533-1975(E), May 1975.

2) Theiss, E.C. "Proposed Standard Cold and Hot Atmospheres for Aeronautical Design." Wright Air Development Center, United States Air Force, Technical Memorandum Report WCSE 141, June 1952.

3) Department of Defense Military Specification. "Climatic Extremes for Military Equipment." MIL-STD-210A, August 1957.

4) Department of Defense Military Specification. "Charts: Standard Aircraft Characteristics and Performance, Piloted Aircraft (Fixed Wing). Appendix IC, Atmospheric Tables." MIL-C-005011B(USAF), June 1977.

5) Department of Defense Military Specification. "Climatic Information to Determine Design and Test Requirements for Military Systems and Equipment." MIL-STD-210C, January 1987.

6) Department of Defense Military Specification. "Glossary of Definitions, Ground Rules, and Mission Profiles to Define Air Vehicle Performance Capability." MIL-STD-3013A, September 2008.

Table 4-4. Constants adopted for calculation of the ISA.

parameter	SI units	English units
units h	m	ft
units τ	°C	°F
m per ft		0.3048
kg per lbm		0.45359237
T_0	288.15 °K	518.67 °R
T_{zero}	273.15 °K	459.67 °R
p_0	101325.0 N/m^2	2116.22 lb/ft^2
ρ_0	1.225 kg/m^3	0.002377 slug/ft^3
c_{s0}	340.294 m/sec	1116.45 ft/sec
μ_0	1.7894E-5 kg/m-sec	3.7372E-7 slug/ft-sec
S	110.4 °K	
β	1.458E-6	
γ	1.4	
g	9.80665 m/sec^2	32.17405 ft/sec^2
r	6356766 m	20855531 ft

Table 4-5. Temperatures and vertical temperature gradients: standard day.

level		base altitude h_b	lapse rate L_b	temperature T_b		
		km	°K/km	°K	°C	°F
	troposphere	-2	-6.5	301.15	28	82.4
0	troposphere	0	-6.5	288.15	15	59
1	tropopause	11	0	216.65	-56.5	-69.7
2	stratosphere	20	+1.0	216.65	-56.5	-69.7
3	stratosphere	32	+2.8	228.65	-44.5	-48.1
4	stratopause	47	0	270.65	-2.5	27.5
5	mesosphere	51	-2.8	270.65	-2.5	27.5
6	mesosphere	71	-2.0	214.65	-58.5	-73.3
7	mesopause	80	0	196.65	-76.5	-105.7

Table 4-6. Temperatures and vertical temperature gradients: polar day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km *	°K	°C	°F *	$\delta = p/p_0$
0	0	6.326	246.15	-27	-16.6	30.268/29.92
3111.871	0.948	-1.083	252.15	-21	-5.8	
9172.604	2.796	-5.511	250.15	-23	-9.4	
28224.543	8.603	-0.476	218.15	-55	-67.0	
83363.393	25.409	0	210.15	-63	-81.4	

* derived

Table 4-7. Temperatures and vertical temperature gradients: tropical day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km *	°K	°C	°F *	$\delta = p/p_0$
0	0	-6.687	305.25	32.1	89.78	1
55000	16.764	4.374	193.15	-80	-112.00	
70000	21.336	2.401	213.15	-60	-76.00	
100745	30.707		235.65	-37.5	-35.50	

* derived

Table 4-8. Temperatures and vertical temperature gradients: hot day.

base altitude h_b		lapse rate L_b	temperature T_b			pressure ratio
ft	km *	°K/km	°K	°C	°F *	$\delta = p/p_0$
0	0	-7.065	312.55	39.4	102.92	1
15000	4.572		280.25 *	7.10 *	44.78	

* derived

Table 4-9. Temperature table for hot day (from MIL-C-5011B).

pressure altitude		temperature		
ft	km *	°K	°C	°F *
0	0	312.55	39.4	102.92
500	0.152	311.55	38.4	101.12
1000	0.305	310.45	37.3	99.14
1500	0.457	309.45	36.3	97.34
2000	0.610	308.35	35.2	95.36
2500	0.762	307.25	34.1	93.38
3000	0.914	306.25	33.1	91.58
3500	1.067	305.15	32.0	89.60
4000	1.219	304.05	30.9	87.62
4500	1.372	302.95	29.8	85.64
5000	1.524	301.85	28.7	83.66
5500	1.676	300.75	27.6	81.68
6000	1.829	299.65	26.5	79.70
6500	1.981	298.55	25.4	77.72
7000	2.134	297.45	24.3	75.74
7500	2.286	296.35	23.2	73.76
8000	2.438	295.25	22.1	71.78
8500	2.591	294.15	21.0	69.80
9000	2.743	293.05	19.9	67.82
9500	2.896	291.95	18.8	65.84
10000	3.048	290.85	17.7	63.86
10500	3.200	289.85	16.7	62.06
11000	3.353	288.85	15.7	60.26
11500	3.505	287.75	14.6	58.28
12000	3.658	286.75	13.6	56.48
12500	3.810	285.65	12.5	54.50
13000	3.962	284.55	11.4	52.52
13500	4.115	283.55	10.4	50.72
14000	4.267	282.45	9.3	48.74
14500	4.420	281.35	8.2	46.76
15000	4.572	280.35	7.2	44.96

* derived

Solution Procedures

The NDARC code performs design and analysis tasks. The design task involves sizing the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 5-1 illustrates the tasks. The principal tasks (sizing, mission analysis, and flight performance analysis) are shown in the figure as boxes with dark borders. Dark arrows show control of subordinate tasks.

The aircraft description (figure 5-1) consists of all the information, input and derived, that defines the aircraft. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by light arrows).

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments, for which time, distance, and fuel burn are evaluated. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on fuel weight.

Flight conditions are specified for the sizing task and for the flight performance analysis.

For flight conditions and mission takeoff, the gross weight can be maximized such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified in terms of a quantity such as best endurance or best range, and a variable such as speed, rate of climb, or altitude. The aircraft must be trimmed, by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states. Evaluating the rotor hub forces may require solution of the blade flap equations of motion.

The sizing task is described in more detail in chapter 3. The flight condition, mission, and flight state calculations are described in chapter 4. The solution of the blade flap equations of motion is described in chapter 11. The present chapter provides details of the solution procedures implemented for each iteration of the analysis.

The nested iteration loops involved in the solution process are indicated by the subtitles in the boxes of figure 5-1, and illustrated in more detail in figure 5-2. The flight state solution involves up
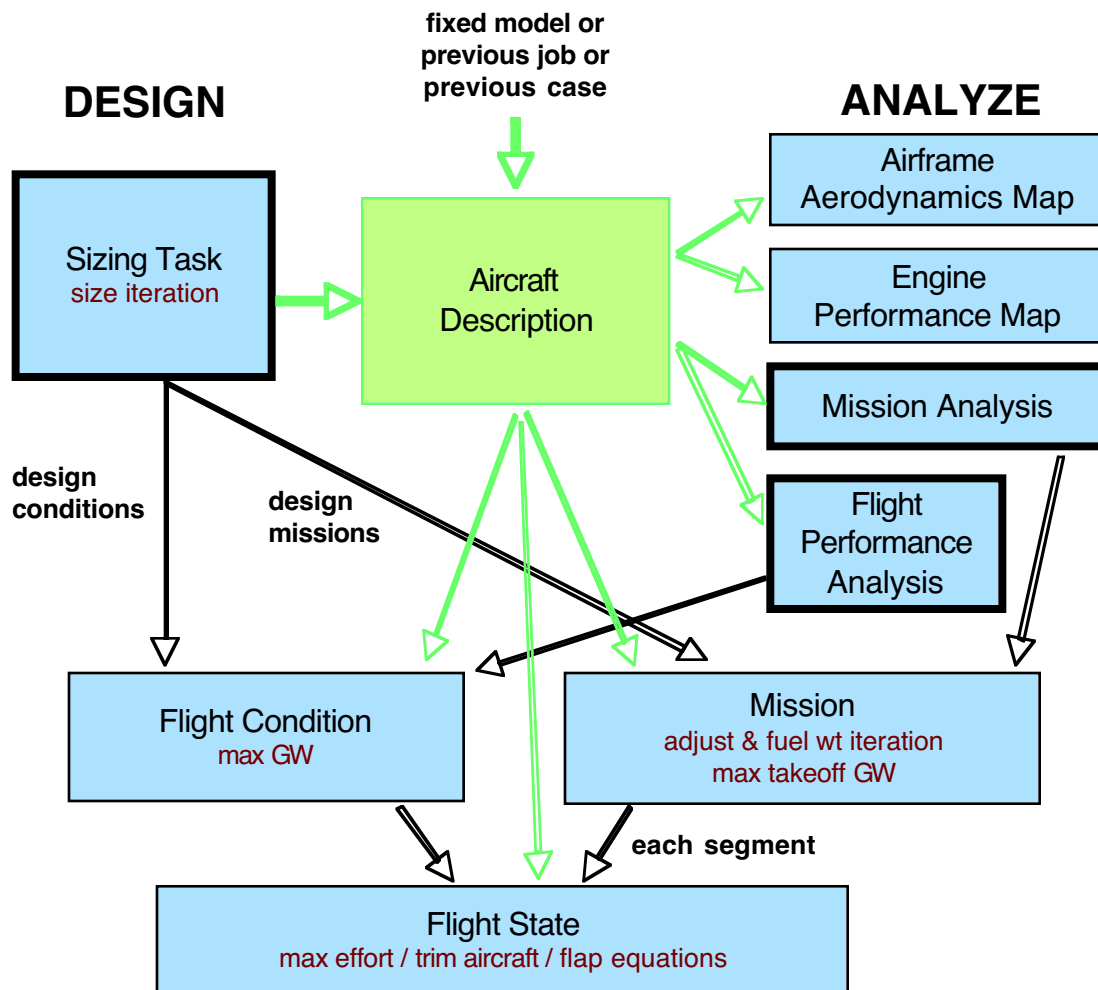
Figure 5-1. Outline of NDARC tasks.

to three loops. The innermost loop is the solution of the blade flap equations of motion, needed for an accurate evaluation of the rotor hub forces. The next loop is the trim solution, which is required for most flight states. The flight state optionally has one or two maximum-effort iterations. The flight state solution is executed for each flight condition and for each mission segment. A flight condition solution or any mission segment solution can optionally maximize the aircraft gross weight. The mission usually requires an iterative solution, for fuel weight or for adjustable segment time or distance. Thus each flight condition solution involves up to four nested iterations: maximum gross weight (outer), maximum effort, trim, and blade motion (inner). Each mission solution involves up to five nested iterations: mission (outer), and then for each segment maximum gross weight, maximum effort, trim, and blade motion (inner). Finally, the design task introduces a sizing iteration, which is the outermost loop of the process.

Sizing Task

Size Iteration
method: successive substitution

> Flight Conditions
>
> Missions

Mission Analysis

> Missions

Flight Performance Analysis

> Flight Conditions

Flight Condition

Maximum GW
method: secant or false position

> Flight State

Mission

Mission Iteration fuel weight, adjust time/distance
method: successive substitution

Segments

Maximum GW
method: secant or false position

> Flight State

Flight State

Maximum Effort
method: golden section search for maximum endurance, range, or climb; otherwise secant or false position

Trim
method: Newton-Raphson

Component Performance Evaluation

Blade Flapping
method: Newton-Raphson

Figure 5-2. Design and analysis tasks, with nested loops and solution methods.

5–1   Iterative Solution Tasks

5-1.1   Tolerance and Perturbation

For each solution procedure, a tolerance ϵ and a perturbation Δ may be required. Single values are specified for the task, and then scaled for each element tested or perturbed.

The scaling is based on a reference weight W (design gross weight, or derived from aircraft $C_T/\sigma = 0.07$), a reference length L (fuselage length, rotor radius, or wing span), and a reference power P (aircraft installed power, or derived from $P = W\sqrt{W/2\rho A}$). Then the force reference is $F = W$, the moment reference is $M = WL/10$, and the angle reference is $A = 1$ deg. The velocity reference is $V = 400$ knots. The angular velocity reference is $\Omega = V/L$ (in deg/sec). The coefficient reference is $C = 0.6$ for wings and $C = 0.1$ for rotors. Altitude scale is $H = 10000$ ft. Acceleration scale is $G = g$ (acceleration due to gravity). The range scale is $X = 100$ nm. These scaling variables are referred to in the subsections that follow, and in tables 5-1, 5-3, and 5-4.

5-1.2   Size Aircraft

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

A successive substitution method is used for the sizing iteration, with an input tolerance ϵ. Relaxation is applied to P_{eng} or R, W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$, and T_{design}. Two successive substitution loops are used. The outer loop is an iteration on performance: engine power or rotor radius, for each propulsion group. The inner loop is an iteration on parameters: W_D, W_{MTO}, $P_{DS\text{limit}}$, $W_{\text{fuel-cap}}$, and T_{design}. Either loop can be absent, depending on the definition of the sizing task. Convergence is tested in terms of these parameters and the aircraft weight empty W_E. The tolerance is $0.1P\epsilon$ for engine power and drive system limit; $0.01W\epsilon$ for gross weight, maximum takeoff weight, fuel weight, and design rotor thrust; and $0.1L\epsilon$ for rotor radius.

5-1.3   Mission

Missions consist of a specified number of segments, for which time, distance, and fuel burn are evaluated. For calculated mission fuel weight, the fuel weight at takeoff is adjusted to equal the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on fuel weight. Range credit segments can also require an iteration.

A successive substitution method is used if an iteration is required, with a tolerance ϵ specified. The principal iteration variable is takeoff fuel weight, for which the tolerance is $0.01W\epsilon$. For calculated mission fuel weight, the relaxation is applied to the mission fuel value used to update the takeoff fuel weight. For specified takeoff fuel weight, the relationship is applied to the fuel weight increment used to adjust the mission segments. The tolerance for the distance flown in range credit segments is $X\epsilon$. The

relaxation is applied to the distance flown in the destination segments for range credit.

5-1.4   Maximum Gross Weight

Flight conditions are specified for the sizing task and for the flight performance analysis. Mission takeoff conditions are specified for the sizing task and for the mission analysis. Optionally for flight conditions and mission takeoff, the gross weight can be maximized, such that the power required equals the power available, $\min(P_{avPG} - P_{reqPG}) = 0$ (zero power margin, minimum over all propulsion groups); or such that the power required equals an input power, $\min((d + f P_{avPG}) - P_{reqPG}) = 0$ (minimum over all propulsion groups, with d an input power and f an input factor; this convention allows the power to be input directly, $f = 0$, or scaled with power available).

The secant method or the method of false position is used to solve for the maximum gross weight. A tolerance ϵ and a perturbation Δ are specified. The variable is gross weight, with initial increment of $W\Delta$, and tolerance of $0.01W\epsilon$. Note that the convergence test is applied to the magnitude of the gross weight increment.

5-1.5   Maximum Effort

The aircraft performance can be analyzed for the specified state or a maximum-effort performance can be identified. The secant method or the method of false position is used to solve for the maximum effort. The task of finding maximum endurance, range, or climb is usually solved using the golden-section or curve-fit method. A tolerance ϵ and a perturbation Δ are specified.

A quantity and variable are specified for the maximum-effort calculation. Tables 5-1 and 5-2 summarize the available choices, with the tolerance and initial increment used for the variables. Note that the convergence test is applied to the magnitude of the variable increment. Optionally two quantity/variable pairs can be specified, solved in nested iterations. The two variables must be unique. The two variables can maximize the same quantity (endurance, range, or climb). If the variable is velocity, first the velocity is found for the specified maximum effort; the performance is then evaluated at that velocity times an input factor. For endurance, range, or climb, the slope of the quantity to be maximized must be zero; hence in all cases the target is zero. The slope of the quantity is evaluated by first-order backward difference. For the range, first the variable is found such that $V/\dot{w}$ is maximized (slope zero), and then the variable is found such that $V/\dot{w}$ equals 99% of that maximum; for the latter the variable perturbation is increased by a factor of 4 to ensure that the solution is found on the correct side of the maximum.

5-1.6   Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance ϵ and a perturbation Δ are specified.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. Tables 5-3 and 5-4 summarize the available choices, with the tolerances and perturbations used.

Table 5-1. maximum-effort solution.

maximum-effort variable		initial increment	tolerance
horizontal velocity	V_h	$0.1V\Delta$	$0.1V\epsilon$
vertical rate of climb	V_z	$0.1V\Delta$	$0.1V\epsilon$
aircraft altitude		$H\Delta$	$H\epsilon$
aircraft angular rate	$\dot{\theta}$ (pullup), $\dot{\psi}$ (turn)	$\Omega\Delta$	$\Omega\epsilon$
aircraft linear acceleration	a_x, a_y, a_z	$G\Delta$	$G\epsilon$

Table 5-2. maximum-effort solution.

maximum-effort quantity		
best endurance	maximum $1/\dot{w}$	
best range	99% maximum $V/\dot{w}$	high or low side, or 100%
best climb or descent rate	maximum V_z or $1/P$	
best climb or descent angle	maximum V_z/V or V/P	
ceiling	maximum altitude	
power limit	power margin, $\min(P_{avPG} - P_{reqPG}) = 0$	over all propulsion groups
torque limit	torque margin, $\min(Q_{\text{limit}} - Q_{req}) = 0$	over all limits
power and torque limit	power margin or torque margin	most restrictive
wing stall	lift margin, $C_{L\text{max}} - C_L = 0$	for designated wing
rotor stall	thrust margin, $(C_T/\sigma)_{\text{max}} - C_T/\sigma = 0$	for designated rotor

Table 5-3. Trim solution.

trim quantity		target	tolerance
aircraft total force	x, y, z components	0	$F\epsilon$
aircraft total moment	x, y, z components	0	$M\epsilon$
aircraft load factor	x, y, z components	Flight State	ϵ
propulsion group power		Flight State	$P\epsilon$
power margin	$P_{avPG} - P_{reqPG}$	Flight State	$P\epsilon$
torque margin	$P_{DS\text{limit}} - P_{reqPG}$	Flight State	$P\epsilon$
rotor force	lift, vertical, propulsive	Flight State, component schedule	$F\epsilon$
rotor thrust	C_T/σ	Flight State, component schedule	$C\epsilon$
rotor thrust margin	$(C_T/\sigma)_{\text{max}} - C_T/\sigma$	Flight State	$C\epsilon$
rotor flapping	β_c, β_s	Flight State	$A\epsilon$
rotor hub moment	x (roll), y (pitch)	Flight State	$M\epsilon$
rotor torque		Flight State	$M\epsilon$
wing force	lift	Flight State, component schedule	$F\epsilon$
wing lift coefficient	C_L	Flight State, component schedule	$C\epsilon$
wing lift margin	$C_{L\text{max}} - C_L$	Flight State	$C\epsilon$
tail force	lift	Flight State	$F\epsilon$

Table 5-4. Trim solution.

trim variable		perturbation
aircraft orientation	θ (pitch), ϕ (roll)	$100A\Delta$
aircraft velocity	V_h (horizontal velocity)	$V\Delta$
aircraft velocity	V_z (vertical velocity)	$V\Delta$
aircraft velocity	β (sideslip)	$100A\Delta$
aircraft angular rate	$\dot{\theta}$ (pullup), $\dot{\psi}$ (turn)	$\Omega\Delta$
aircraft control	angle	$100A\Delta$

5-1.7 Rotor Flap Equations

Evaluating the rotor hub forces may require solution of the flap equations $E(v) = 0$. For tip-path plane command, the thrust and flapping are known, so $v = (\theta_{0.75}\,\theta_c\,\theta_s)^T$. For no-feathering plane command, the thrust and cyclic pitch are known, so $v = (\theta_{0.75}\,\beta_c\,\beta_s)^T$. A Newton–Raphson solution method is used: from $E(v_{n+1}) \cong E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C\,E(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor f. The derivative matrix for axial flow can be used. Alternatively, the derivative matrix dE/dv can be obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where ϵ is an input tolerance.

5–2 Theory

The analysis uses several methods to solve nonlinear algebraic equations. Such equations may be written in two forms: (a) fixed point $x = G(x)$, and (b) zero point $f(x) = 0$; where x, G, and f are vectors. The analysis provides operations that implement the function G or f. Solution procedures appropriate for the zero point form can be applied to equations in fixed point form, by defining $f(x) = x - G(x)$. In this context, f can be considered the iteration error.

Efficient and convergent methods are required to find the solution $x = \alpha$ of these equations. Note that $f'(\alpha) = 0$ or $G'(\alpha) = 1$ means that α is a higher-order root. For nonlinear problems, the method will be iterative: $x_{n+1} = F(x_n)$. The operation F depends on the solution method. The solution error is:

$$\epsilon_{n+1} = \alpha - x_{n+1} = F(\alpha) - F(x_n) = (\alpha - x_n)F'(\xi_n) \cong \epsilon_n F'(\alpha)$$

Thus the iteration will converge if F is not too sensitive to errors in x: $|F'(\alpha)| < 1$ for scalar x. For x a vector, the criterion is that all the eigenvalues of the derivative matrix $\partial F/\partial x$ have magnitude less than one. The equations in this section are generally written for scalar x; the extension to vector x is straightforward. Convergence is linear for F' nonzero, quadratic for $F' = 0$. Iterative methods have a relaxation factor (and other parameters) to improve convergence, and a tolerance to measure convergence.

The following subsections describe the solution methods used for the various iterations, as shown in figure 5-2.

```
successive substitution iteration
    save:  x_old = x
    evaluate  x
    relax:  x = λx + (1 − λ)x_old
    test convergence:  error = ‖x − x_old‖ ≤ λtolerance × weight
```

Figure 5-3. Outline of successive substitution method.

5-2.1   Successive Substitution Method

The successive substitution method (with relaxation) is an example of a fixed point solution. A direct iteration is simply $x_{n+1} = G(x_n)$, but $|G'| > 1$ for many practical problems. A relaxed iteration uses $F = (1 − \lambda)x + \lambda G$:

$$x_{n+1} = (1 − \lambda)x_n + \lambda G(x_n) = x_n − \lambda f(x_n)$$

with relaxation factor λ. The convergence criterion is then

$$|F'(\alpha)| = |1 − \lambda + \lambda G'| < 1$$

so a value of λ can be found to ensure convergence for any finite G'. Specifically, the iteration converges if the magnitude of λ is less than the magnitude of $2/(1 − G') = 2/f'$ (and λ has the same sign as $1 − G' = f'$). Quadratic convergence ($F' = 0$) is obtained with $\lambda = 1/(1 − G') = 1/f'$. Over-relaxation ($\lambda > 1$) can be used if $|G'| < 1$. Since the correct solution $x = \alpha$ is not known, convergence must be tested by comparing the values of two successive iterations:

$$\text{error} = \|x_{n+1} − x_n\| \leq \text{tolerance}$$

where the error is some norm of the difference between iterations (typically absolute value for scalar x). Note that the effect of the relaxation factor is to reduce the difference between iterations:

$$x_{n+1} − x_n = \lambda\big(G(x_n) − x_n\big)$$

Hence the convergence test is applied to $(x_{n+1} − x_n)/\lambda$, in order to maintain the definition of tolerance independent of relaxation. The process for the successive substitution method is shown in figure 5-3.

5-2.2   Newton–Raphson Method

The Newton–Raphson method (with relaxation and identification) is an example of a zero point solution. The Taylor series expansion of $f(x) = 0$ leads to the iteration operator $F = x − f/f'$:

$$x_{n+1} = x_n − [f'(x_n)]^{-1} f(x_n)$$

which gives quadratic convergence. The behavior of this iteration depends on the accuracy of the derivative f'. Here it is assumed that the analysis can evaluate directly f, but not f'. It is necessary to evaluate f' by numerical perturbation of f, and for efficiency the derivatives may not be evaluated for each x_n. These approximations compromise the convergence of the method, so a relaxation factor λ is introduced to compensate. Hence a modified Newton–Raphson iteration is used, $F = x − Cf$:

$$x_{n+1} = x_n − Cf(x_n) = x_n − \lambda D^{-1}f(x_n)$$

```
initialize
    evaluate h
    test convergence: error = |h_j − h_targetj| ≤ tolerance × weight_j
    initialize derivative matrix D to input matrix
    calculate gain matrix: C = λD⁻¹
iteration
    identify derivative matrix
        optional perturbation identification
            perturb each element of x: δx_i = Δ × weight_i
            evaluate h
            calculate D
            calculate gain matrix: C = λD⁻¹
    increment solution: δx = −C(h − h_target)
    evaluate h
    test convergence: error = |h_j − h_targetj| ≤ tolerance × weight_j
```

Figure 5-4. Outline of Newton–Raphson method.

where the derivative matrix D is an estimate of f'. The convergence criterion is then

$$|F'(\alpha)| = |1 - Cf'| = |1 - \lambda D^{-1} f'| < 1$$

since $f(\alpha) = 0$. The iteration converges if the magnitude of λ is less than the magnitude of $2D/f'$ (and λ has the same sign as D/f'). Quadratic convergence is obtained with $\lambda = D/f'$ (which would require λ to change during the iteration however). The Newton–Raphson method ideally uses the local derivative in the gain factor, $C = 1/f'$, so has quadratic convergence:

$$F'(\alpha) = \frac{ff''}{f'^2} = 0$$

since $f(\alpha) = 0$ (if $f' \neq 0$ and f'' is finite; if $f' = 0$, then there is a multiple root, $F' = \frac{1}{2}$, and the convergence is only linear). A relaxation factor is still useful, since the convergence is only quadratic sufficiently close to the solution. A Newton–Raphson method has good convergence when x is sufficiently close to the solution, but frequently has difficulty converging elsewhere. Hence the initial estimate x_0 that starts the iteration is an important parameter affecting convergence. Convergence of the solution for x may be tested in terms of the required value (zero) for f:

$$\text{error} = \|f\| \leq \text{tolerance}$$

where the error is some norm of f (typically absolute value for scalar f).

The derivative matrix D is obtained by an identification process. The perturbation identification can be performed at the beginning of the iteration, and optionally every M_{PID} iterations thereafter. The derivative matrix is calculated from a one-step finite-difference expression (first order). Each element x_i of the vector x is perturbed, one at a time, giving the i-th column of D:

$$D = \begin{bmatrix} \cdots & \dfrac{\partial f}{\partial x_i} & \cdots \end{bmatrix} = \begin{bmatrix} \cdots & \dfrac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i} & \cdots \end{bmatrix}$$

```
initialize
    evaluate f₀ at x₀, f₁ at x₁ = x₀ + Δx, f₂ at x₂ = x₁ + Δx
iteration
    calculate derivative f′
        secant: from f₀ and f₁
        false position: from f₀, and f₁ or f₂ (opposite sign from f₀)
    calculate gain: C = λ/f′
    increment solution: δx = −Cf
    shift: f₂ = f₁, f₁ = f₀
    evaluate f
    test convergence
```

<center>Figure 5-5. Outline of secant method or method of false position.</center>

Alternatively, a two-step finite-difference expression (second order) can be used:

$$D = \left[\cdots \quad \frac{\partial f}{\partial x_i} \quad \cdots \right] = \left[\cdots \quad \frac{f(x_i + \delta x_i) - f(x_i - \delta x_i)}{2 \delta x_i} \quad \cdots \right]$$

With this procedure, the accuracy of D (hence convergence) can be affected by both the magnitude and sign of the perturbation (only the magnitude for a two-step difference).

The process for the Newton–Raphson method is shown in figure 5-4. A problem specified as $h(x) = h_{\text{target}}$ becomes a zero point problem with $f = h - h_{\text{target}}$. A successive substitution problem, $x = G(x)$, becomes a zero point problem with $f = x - G$. At the beginning of the solution, x has an initial value. The perturbation identification can optionally never be performed (so an input matrix is required), be performed at the beginning of the iteration, or be performed at the beginning and every M_{PID} iterations thereafter.

5-2.3   Secant Method

The secant method (with relaxation) is developed from the Newton–Raphson method. The modified Newton–Raphson iteration is:

$$x_{n+1} = x_n - Cf(x_n) = x_n - \lambda D^{-1} f(x_n)$$

where the derivative matrix D is an estimate of f'. In the secant method, the derivative of f is evaluated numerically at each step:

$$f'(x_n) \cong \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

It can be shown that then the error reduces during the iteration according to:

$$|\epsilon_{n+1}| \cong |f''/2f'| \, |\epsilon_n| \, |\epsilon_{n-1}| \cong |f''/2f'|^{.62} |\epsilon_n|^{1.62}$$

which is slower than the quadratic convergence of the Newton–Raphson method (ϵ_n^2), but still better than linear convergence. In practical problems, whether the iteration converges at all is often more important than the rate of convergence. Limiting the maximum amplitude of the derivative estimate may also be appropriate. Note that with $f = x - G(x)$, the derivative f' is dimensionless, so a universal limit (say

```
initialize
    evaluate f₀ at x₀, f₁ at x₁ = x₀ + Δx, f₂ at x₂ = x₁ + Δx
bracket maximum: while not f₁ ≥ f₀, f₂
    if f₂ > f₀, then x₃ = x₂ + (x₂ − x₁); 1,2,3 → 0,1,2
    if f₀ > f₂, then x₃ = x₀ − (x₁ − x₀); 3,0,1 → 0,1,2
iteration (search)
    if x₂ − x₁ > x₁ − x₀, then x₃ = x₁ + W(x₂ − x₁)
        if f₃ < f₁, then 0,1,3 → 0,1,2
        if f₃ > f₁, then 1,3,2 → 0,1,2
    if x₁ − x₀ > x₂ − x₁, then x₃ = x₁ − W(x₁ − x₀)
        if f₃ < f₁, then 3,1,2 → 0,1,2
        if f₃ > f₁, then 0,3,1 → 0,1,2
test convergence
```

Figure 5-6. Outline of golden-section search.

maximum $|f'| = 0.3$) can be specified. A limit on the maximum increment of x (as a fraction of the x value) can also be imposed. The process for the secant method is shown in figure 5-5.

5-2.4   Method of False Position

The method of false position is a derivative of the secant method, based on calculating the derivative with values that bracket the solution. The iteration starts with values of x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ have opposite signs. Then the derivative f' and new estimate x_{n+1} are

$$f'(x_n) \cong \frac{f(x_n) - f(x_k)}{x_n - x_k}$$
$$x_{n+1} = x_n - \lambda D^{-1} f(x_n)$$

using $k = n - 1$ or $k = n - 2$ such that $f(x_n)$ and $f(x_k)$ have opposite signs. The convergence is slower (roughly linear) than for the secant method, but by keeping the solution bracketed convergence is guaranteed. The process for the method of false position is shown in figure 5-5.

5-2.5   Golden-Section Search

The golden-section search method can be used to find the solution x that maximizes $f(x)$. The problem of maximizing $f(x)$ can be attacked by applying the secant method or method of false position to the derivative $f'(x) = 0$, but that approach is often not satisfactory as it depends on numerical evaluation of the second derivative. The golden-section search method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions f_0, f_1, f_2. The x value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$. Then a new value x_3 is selected in the interval x_0 to x_2; f_3 evaluated; and the new set of $x_0 < x_1 < x_2$ determined such that the maximum is still bracketed. The new value x_3 is a fraction $W = (3 - \sqrt{5})/2 \cong 0.38197$ from x_1 into the largest segment. The process for the golden-section search is shown in figure 5-6.

5-2.6   Curve Fit-Method

The curve fit method can be used to find the solution x that maximizes $f(x)$, by fitting the solution to a polynomial. If the function f is flat around the maximum and the inner loop tolerances are not tight

```
initialize
    evaluate f₀ at x₀,  f₁ at  x₁ = x₀ + Δx,  f₂ at  x₂ = x₁ + Δx
bracket maximum: while not  f₁ ≥ f₀, f₂
    if  f₂ > f₀,  then  x₃ = x₂ + (x₂ − x₁);  1,2,3 → 0,1,2
    if  f₀ > f₂,  then  x₃ = x₀ − (x₁ − x₀);  3,0,1 → 0,1,2
    fₘₐₓ = f₁
curve fit
    fₘₐₓ = f₁,  xₘₐₓ = x₁
    evaluate  f for  x = xₘₐₓ + nΔx  and  x = xₘₐₓ − nΔx
    least-squared error solution for polynomial coefficients
    solve polynomial for x at peak f
```

Figure 5-7. Outline of curve fit-method.

enough, the golden-section search can become erratic, particularly for best range and best endurance calculations. Curve fitting the evaluated points and then solving the curve for the maximum has the potential to improve the behavior. The curve-fit method begins with a set of three values $x_0 < x_1 < x_2$ and the corresponding functions f_0, f_1, f_2. The x value is incremented until the maximum is bracketed, $f_1 \geq f_0, f_2$, giving a course maximum $f_{\max}$ at $x_{\max}$. Next a set of x and f values are generated by incrementing x above and below $x_{\max}$, until find $f < r_{\text{fit}} f_{\max}$ (typically $r_{\text{fit}} = 0.98$ for best range). This set of points is fit to the cubic polynomial $f = c_3 z^3 + c_2 z^2 + c_1 z + c_0$, $z = x/x_{\max} - 1$ (or to a quadradic polynomial). Let $c^T = (c_0\, c_1\, c_2\, c_3)$ and $\xi^T = (1\, z\, z^2\, z^3)$. Then the least-squared-error solution for the coefficients is

$$c = \left(\sum_i \xi_i \xi_i^T \right)^{-1} \left(\sum_i f_i \xi_i \right)$$

where the sums are over the set of points to be fit. For a quadratic polynomial fit, the solution is then

$$x = x_{\max} \left(1 - \frac{c_1}{2c_2} \pm \sqrt{1 - r} \sqrt{\left(\frac{c_1}{2c_2} \right)^2 - \frac{c_0}{c_2}} \right)$$

where $r = 1$ for the maximum, or $r = 0.99$ for 99% best range. For a cubic polynomial fit, the maximum is at

$$z = -\frac{c_2}{3c_3} \left(1 - \sqrt{1 - \frac{3c_3 c_1}{c_2^2}} \right) \cong -\frac{c_1}{2c_2} \left(1 + \frac{1}{4} \frac{3c_3 c_1}{c_2^2} \right)$$

It is simplest to search the cubic for the peak (z where $df/dz = 0$), and then if necessary search for the 99% range point ($f = 0.99 f_{\text{peak}}$) The process for the golden-section search is shown in figure 5-7.

Cost

Costs are estimated using statistical models based on historical aircraft price and maintenance cost data, with appropriate factors to account for technology impact and inflation. The aircraft flyaway cost (C_{AC}, in \$) consists of airframe, mission equipment package (MEP), and flight control electronics (FCE) costs. The direct operating cost plus interest (DOC+I, in cents per available seat mile (ASM)) is the sum of maintenance cost (C_{maint}, in \$ per flight hour), flight crew salary and expenses, fuel and oil cost, depreciation, insurance cost, and finance cost. Inflation factors can be input, or internal factors used. Table 6-1 gives the internal inflation factors for DoD (ref. 1) and CPI (ref. 2). For years beyond the data in the table, optionally the inflation factor is extrapolated based on the last yearly increase.

6–1   CTM Rotorcraft Cost Model

The CTM rotorcraft cost model (refs. 3–5) gives an estimate of aircraft flyaway cost and direct operating cost plus interest. The basic statistical relationships for airframe purchase price and maintenance cost per hour are:

$$c_{AF} = 739.91\, K_{ET} K_{EN} K_{LG} K_R\, W_{AF}^{1.0619} (P/W_{AF})^{0.5887} N_{\mathrm{blade}}^{0.1465}$$
$$c_{\mathrm{maint}} = 0.49885\, W_E^{0.3746} P^{0.4635}$$

with $W_{AF} = W_E + \Delta W_{\mathrm{kit}} - W_{\mathrm{MEP}} - W_{\mathrm{FCE}}$, including airframe kits ΔW_{kit} (the wing and wing extension kits, and optionally the folding kit). The configuration factor $K_{\mathrm{config}} = K_{ET} K_{EN} K_{LG} K_R$ has the factors:

$K_{ET} =$	1.0 for turbine aircraft	0.557 for piston aircraft
$K_{EN} =$	1.0 for multi-engine aircraft	0.736 for single-engine aircraft
$K_{LG} =$	1.0 for retractable landing gear	0.884 for fixed landing gear
$K_R =$	1.0 for single main rotor	1.057 for twin main rotors, 1.117 for four main rotors

The term $C_{\mathrm{comp}} = r_{\mathrm{comp}} W_{\mathrm{comp}}$ accounts for additional costs for composite construction (negative for cost savings); W_{comp} is the composite structure weight, obtained as an input fraction of the component weight, with separate fractions for body, tail, pylon, and wing weight. The MEP and FCE costs are obtained from input cost-per-weight factors: $C_{\mathrm{MEP}} = r_{\mathrm{MEP}} W_{\mathrm{MEP}}$ and $C_{\mathrm{FCE}} = r_{\mathrm{FCE}} W_{\mathrm{FCE}}$. The statistical cost equations for cAF and c_{maint} are based on 1994 dollars and current technology levels. Including an inflation factor F_i and technology factors χ gives the unit flyaway cost C_{AC} and maintenance cost per flight hour C_{maint}:

$$C_{AC} = \chi_{AF}(F_i c_{AF}) + C_{\mathrm{comp}} + C_{\mathrm{MEP}} + C_{\mathrm{FCE}}$$
$$C_{\mathrm{maint}} = \chi_{\mathrm{maint}}(F_i c_{\mathrm{maint}})$$

In addition to technology, χ includes calibration and industry factors; for example, $\chi_{AF} = 0.87$ for U.S. Military (ref. 3). The statistical equation for c_{AF} predicts the price of 123 out of 128 rotorcraft within

20% (figures 6-1 and 6-2). These equations also serve to estimate turboprop airliner flyaway costs by setting $N_{\text{rotor}} = N_{\text{blade}} = 1$ and using additional factors 0.8754 (pressurized) or 0.7646 (unpressurized). The unit flyaway cost in \$/lb or \$/kg is

$$r_{AF} = (\chi_{AF}(F_i c_{AF}))/W_{AF}$$
$$r_{AC} = C_{AC}/(W_E + \Delta W_{\text{kit}})$$

for the airframe and the aircraft. Parameters are defined in table 6-2, including units as used in these equations.

The direct operating cost includes maintenance, fuel, crew, depreciation, insurance, and finance costs. The terms in the operating cost are:

$$C_{\text{fuel}} = G(W_{\text{fuel}}/\rho_{\text{fuel}})N_{\text{dep}}$$
$$C_{\text{crew}} = 2.84 F_i K_{\text{crew}} W_{MTO}^{0.4} B$$
$$C_{\text{dep}} = C_{AC}\frac{1+S}{D}(1-V)$$
$$C_{\text{ins}} = 0.0056\, C_{AC}$$
$$C_{\text{fin}} = C_{AC}\frac{1+S}{D}\frac{2L+1}{4}\frac{i}{100}$$

The crew factor $K_{\text{crew}} = 1$ corresponds to low cost, domestic airlines (1994 dollars). The fuel burn W_{fuel}, block time T_{miss}, and block range R_{miss} are obtained for a designated mission. The number of departures per year is $N_{\text{dep}} = B/T_{\text{miss}}$. The flight time per trip is $T_{\text{trip}} = T_{\text{miss}} - T_{NF}$. The flight hours per year are $T_F = T_{\text{trip}}N_{\text{dep}}$. The yearly operating cost C_{OP} and DOC+I are then:

$$C_{OP} = T_F C_{\text{maint}} + C_{\text{fuel}} + C_{\text{crew}} + C_{\text{dep}} + C_{\text{ins}} + C_{\text{fin}}$$
$$\text{DOC} + \text{I} = 100\, C_{OP}/\text{ASM}$$

where the available seat miles per year are $\text{ASM} = 1.1508 N_{\text{pass}} R_{\text{miss}} N_{\text{dep}}$.

6–2   References

1) "National Defense Budget Estimates for FY 1998/2011." Office of the Under Secretary of Defense (Comptroller), March 1997/2010. Department of Defense deflators, for Total Obligational Authority (TOA), Procurement.

2) "Consumer Price Index." U.S. Department Of Labor, Bureau of Labor Statistics, 2010. All Urban Consumers (CPI-U), U.S. city average, All items.

3) Harris, F.D., and Scully, M.P. "Rotorcraft Cost Too Much." Journal of the American Helicopter Society, Vol. 43, No. 1, January 1998.

4) Harris, F.D. "An Economic Model of U.S. Airline Operating Expenses." NASA CR 2005-213476, December 2005.

5) Coy, J.J. "Cost Analysis for Large Civil Transport Rotorcraft." American Helicopter Society Vertical Lift Aircraft Design Conference, San Francisco, California, January 2006.

Table 6-1. DoD and CPI inflation factors.

| year | inflation factors | | year | inflation factors | | year | inflation factors | |
	DoD	CPI		DoD	CPI		DoD	CPI
1911			1951	17.02	17.54	1991	93.99	91.90
1912			1952	16.26	17.88	1992	96.21	94.67
1913		6.68	1953	16.31	18.02	1993	98.16	97.50
1914		6.75	1954	16.02	18.15	1994	100.00	100.00
1915		6.82	1955	17.29	18.08	1995	101.70	102.83
1916		7.35	1956	17.58	18.35	1996	103.18	105.87
1917		8.64	1957	18.17	18.96	1997	104.32	108.30
1918		10.19	1958	18.02	19.50	1998	105.40	109.99
1919		11.67	1959	18.11	19.64	1999	106.81	112.42
1920		13.50	1960	18.34	19.97	2000	108.39	116.19
1921		12.08	1961	18.31	20.18	2001	109.92	119.50
1922		11.34	1962	18.84	20.38	2002	111.62	121.39
1923		11.54	1963	19.09	20.65	2003	113.95	124.16
1924		11.54	1964	19.76	20.92	2004	116.92	127.46
1925		11.81	1965	20.36	21.26	2005	120.10	131.78
1926		11.94	1966	21.97	21.86	2006	123.06	136.03
1927		11.74	1967	22.77	22.54	2007	125.56	139.91
1928		11.54	1968	24.03	23.48	2008	127.43	145.28
1929		11.54	1969	25.10	24.76	2009	128.95	144.76
1930		11.27	1970	25.76	26.18	2010	131.59	147.14
1931		10.26	1971	27.24	27.33	2011*	132.54	151.78
1932		9.24	1972	28.98	28.21	2012*	134.64	
1933		8.77	1973	31.34	29.96	2013*	136.92	
1934		9.04	1974	34.13	33.27	2014*	139.26	
1935		9.24	1975	38.10	36.30	2015*	141.70	
1936		9.38	1976	42.14	38.39	2016*	144.22	
1937		9.72	1977	43.75	40.89			
1938		9.51	1978	47.82	43.99			
1939		9.38	1979	53.02	48.99			
1940		9.45	1980	58.52	55.60			
1941		9.92	1981	63.80	61.34			
1942		11.00	1982	68.35	65.11			
1943		11.67	1983	71.92	67.21			
1944		11.88	1984	74.57	70.11			
1945		12.15	1985	76.86	72.60			
1946		13.16	1986	79.19	73.95			
1947		15.05	1987	81.87	76.65			
1948		16.26	1988	85.01	79.82			
1949		16.06	1989	88.19	83.67			
1950	15.37	16.26	1990	91.30	88.19			

* projected

Table 6-2. Cost model parameters.

parameter	definition	units
W_E	weight empty	lb
W_{MTO}	maximum takeoff weight	lb
N_{blade}	number of blades per rotor	
P	rated takeoff power (all engines)	hp
W_{MEP}	fixed useful load weight, mission equipment package	lb or kg
W_{FCE}	fixed useful load weight, flight control electronics	lb or kg
r_{MEP}	cost factor, mission equipment package	\$/lb or \$/kg
r_{FCE}	cost factor, flight control electronics	\$/lb or \$/kg
r_{comp}	additional cost for composite construction	\$/lb or \$/kg
F_i	inflation factor, relative 1994	
W_{fuel}	mission fuel burned	lb or kg
T_{miss}	mission time	hr
R_{miss}	mission range	nm
G	fuel cost	\$/gallon or \$/liter
B	available block hours	hr
S	spares per aircraft (fraction purchase price)	
D	depreciation period	yr
V	residual value (fraction)	
L	loan period	yr
i	interest rate	%
T_{NF}	non-flight time per trip	hr
N_{pass}	number of passengers	
ρ_{fuel}	fuel density (weight per volume)	lb/gal or kg/liter
N_{dep}	number of departures per year	
K_{crew}	crew factor	
T_F	flight hours per year	
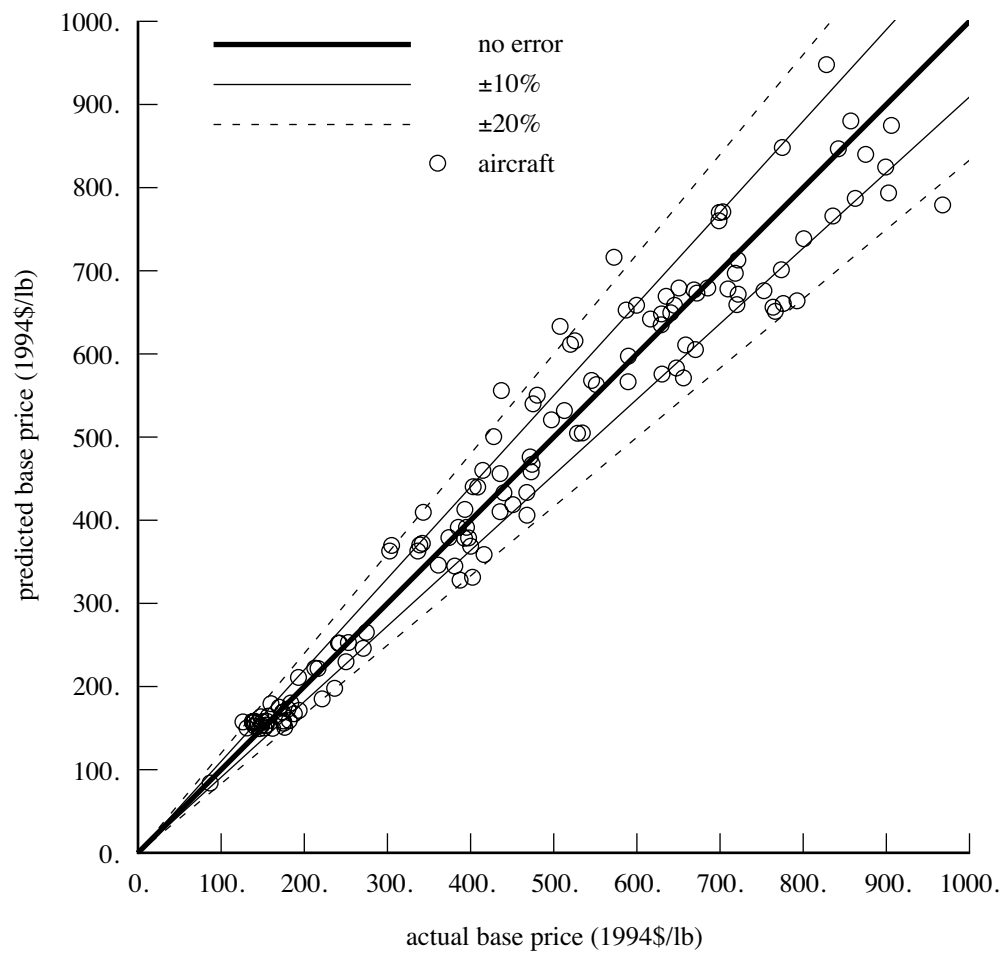
Figure 6-1. Statistical estimation of rotorcraft flyaway cost ($/lb).
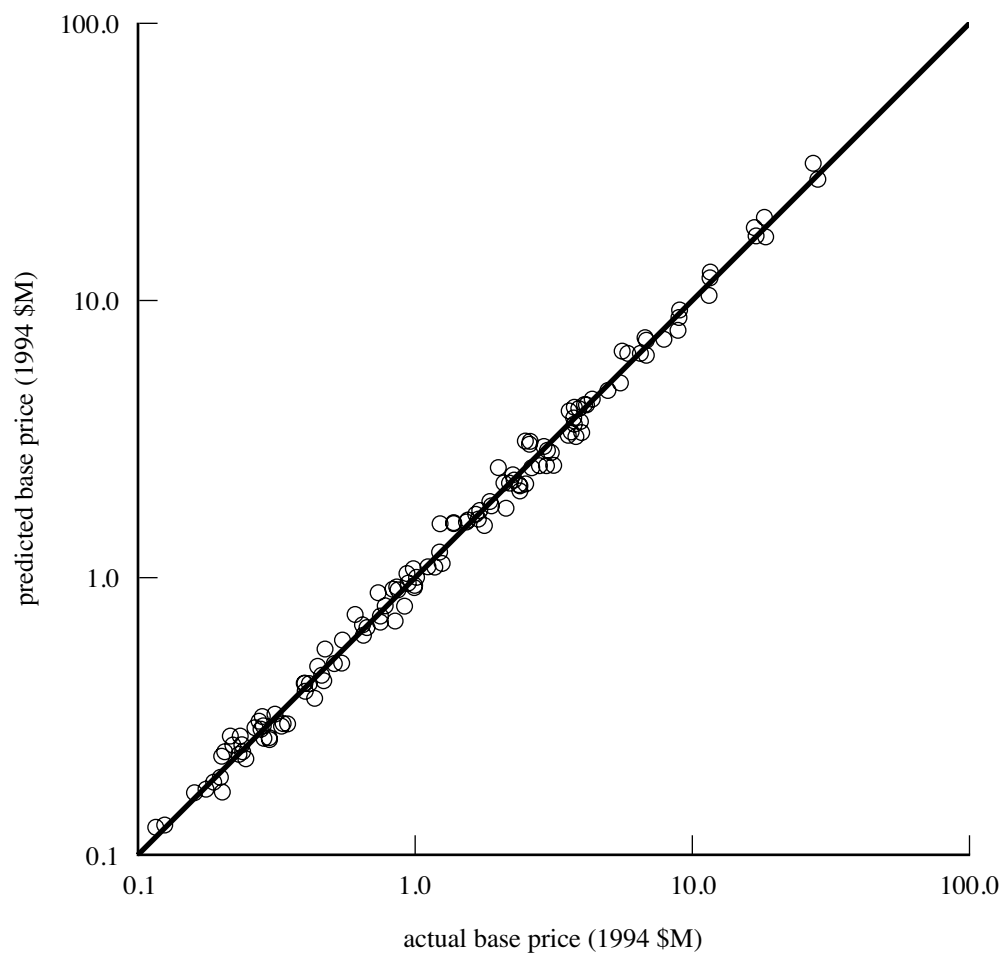
Figure 6-2. Statistical estimation of rotorcraft flyaway cost ($M).

Aircraft

The aircraft consists of a set of components, including rotors, wings, tails, fuselage, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered include: single-main-rotor and tail-rotor helicopter, tandem helicopter, and coaxial helicopter, and tiltrotor.

The following components form the aircraft.

a) Systems: The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment) for the aircraft.

b) Fuselage: There is one fuselage for the aircraft.

c) Landing Gear: There is one landing gear for the aircraft.

d) Rotors: The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the component can model tail rotors, propellers, proprotors, and ducted fans.

e) Forces: The force component is a simple model for a lift, propulsion, or control subsystem.

f) Wings: The aircraft can have one or more wings, or no wings.

g) Tails: The aircraft can have one or more horizontal or vertical tail surfaces, or no tails.

h) Fuel Tank: There is one fuel tank component for the aircraft. There can be one or more sizes of auxiliary fuel tanks.

i) Propulsion Groups: There are one or more propulsion groups. Each propulsion group is a set of components (rotors) and engine groups, connected by a drive system. The engine model describes a particular engine, used in one or more engine groups. The components define the power required. The engine groups define the power available.

j) Engine Groups: An engine group consists of one or more engines of a specific type. For each engine type an engine model is defined.

7–1 Loading

The aircraft disk loading is the ratio of the design gross weight and a reference rotor area: $DL = W_D/A_{\text{ref}}$. The reference area is a sum of specified fractions of the rotor areas, $A_{\text{ref}} = \sum f_A A$ (typically the projected area of the lifting rotors). The disk loading of a rotor is the ratio of a specified fraction of

the design gross weight and the rotor area:

$$(DL)_{\text{rotor}} = \frac{T}{A} = \frac{f_W W_D}{A} = \frac{f_W}{A/A_{\text{ref}}} \frac{W_D}{A_{\text{ref}}}$$

where probably $\sum_{\text{rotor}} f_W = 1$, and the lifting rotors are all rotors not designated antitorque or auxiliary-thrust. If there are N lifting rotors of the same area, with no overlap, then $f_A = 1$, $A_{\text{ref}} = NA$, $f_W = A/A_{\text{ref}} = 1/N$, and $(DL)_{\text{rotor}} = DL$. For rotors designated antitorque or auxiliary-thrust, the disk loading is calculated from the design rotor thrust: $(DL)_{\text{rotor}} = T_{\text{design}}/A$.

For coaxial rotors, the default reference area is the area of one rotor: $f_A = 1/2$, $A_{\text{ref}} = A$, $f_W = 1/2$, and $(DL)_{\text{rotor}} = 1/2 DL$. For tandem rotors, the default reference area is the projected area: $A_{\text{ref}} = (2 - m)A$, where mA is the overlap area ($m = 0$ for no overlap, $m = 1$ for coaxial). Then $f_A = \frac{2-m}{2}$, $f_W = 1/2$, and $(DL)_{\text{rotor}} = \frac{2-m}{2} DL$. Optionally, the reference area for tandem rotors can be total rotor area instead: $A_{\text{ref}} = 2A$.

The aircraft wing loading is the ratio of the design gross weight and a reference wing area: $WL = W_D/S_{\text{ref}}$. The reference area is a sum of the wing areas, $S_{\text{ref}} = \sum S$. The wing loading of an individual wing is the ratio of a specified fraction of the design gross weight and the wing area:

$$(WL)_{\text{wing}} = \frac{W}{S} = \frac{f_W W_D}{S} = \frac{f_W}{S/S_{\text{ref}}} \frac{W_D}{S_{\text{ref}}}$$

where probably $\sum_{\text{wing}} f_W = 1$. If there are N wings of the same area, then $f_W = S/S_{\text{ref}} = 1/N$, and $(WL)_{\text{wing}} = WL$.

The aircraft power loading is the ratio of the design gross weight and the total installed takeoff power: $W/P = W_D/\sum N_{\text{eng}} P\text{eng}$, where the sum is over all engine groups.

7–2   Controls

A set of aircraft controls c_{AC} are defined, and these aircraft controls are connected to the component controls. The connection to the component control c is typically of the form $c = ST c_{AC} + c_0$, where T is an input matrix and c_0 the component control for zero aircraft control. The connection (matrix T) is defined for a specified number of control system states (allowing change of control configuration with flight state). The factor S is available for internal scaling of the matrix. The control state and initial control values are specified for each flight state. Figure 7-1 illustrates the control relationships.

Typical (default) aircraft controls are the pilot's controls: collective stick, lateral and longitudinal cyclic sticks, pedal, and tilt. Units and sign convention of the pilot's controls are contained in the matrix T. For the single-main-rotor and tail-rotor configuration, it is often convenient for the collective and cyclic stick motion to equal the collective and cyclic pitch input of the main rotor, and the pedal motion to equal the collective pitch input of the tail rotor. The aircraft controls should be scaled to approximately the same amplitude, by appropriate definition of the T matrix and scale factor S.

These aircraft controls are available for trim of the aircraft. Any aircraft controls not selected for trim will remain fixed at the values specified for the flight state. Thus by defining additional aircraft controls, component controls can be specified as required for a flight state.

Each aircraft control variable c_{AC} can be zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override this value of the aircraft control. The input value is an initial value if the control is a trim variable.
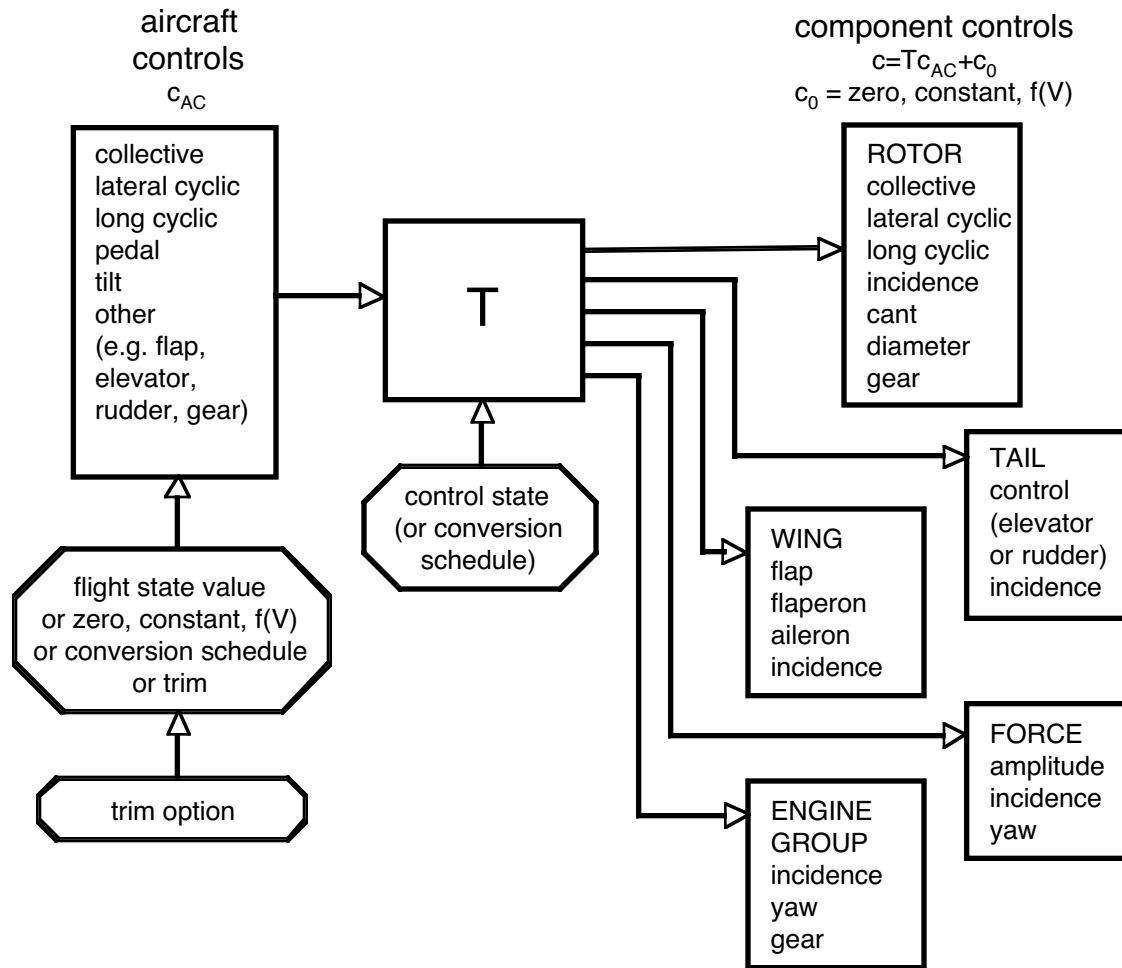
Figure 7-2. Aircraft and component controls.

Each component control variable c_0 (value for zero aircraft control) can be zero, constant, or a function of flight speed (piecewise linear input). Optionally the use of c_0 can be suppressed for a flight state. The component control from aircraft control (Tc_{AC}) is a fixed value, or a function of speed, or a linear function of another control (perhaps a trim variable).

The tilt control variable α_{tilt} is intended for nacelle tilt angle or conversion control, particularly for tiltrotors. The convention is $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. If α_{tilt} exists as a control, it can be zero, constant, or a function of flight speed (piecewise linear input).

An optional control conversion schedule is defined in terms of conversion speeds: hover and helicopter mode for speeds below $V_{C\text{hover}}$, cruise mode for speeds above $V_{C\text{cruise}}$, and conversion mode between. The nacelle angle is $\alpha_{\text{tilt}} = 90$ in helicopter mode, $\alpha_{\text{tilt}} = 0$ in airplane mode, and it varies linearly with speed in conversion mode. The tip speed is $V_{\text{tip}-\text{hover}}$ in helicopter and conversion mode, and $V_{\text{tip}-\text{cruise}}$ in airplane mode. Control states and drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion

schedule.

The flight speed used for control scheduling is usually the calibrated airspeed (CAS), hence variation with dynamic pressure. Velocity schedules are used for conversion, controls and motion, rotor tip speed, landing gear retraction, and trim targets. Optionally these velocity schedules use either calibrated airspeed V_{cal} or the true airspeed V.

The control matrices T can be defined based on the configuration. Let c_{AC0}, c_{ACc}, c_{ACs}, c_{ACp} be the pilot's controls (collective, lateral cyclic, longitudinal cyclic, and pedal). For the helicopter, the first rotor is the main rotor and the second rotor is the tail rotor; then

$$
\begin{pmatrix} T_{\text{Mcoll}} \\ T_{\text{Mlat}} \\ T_{\text{Mlng}} \\ T_{\text{Tcoll}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -r \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

where r is the main rotor direction of rotation ($r = 1$ for counter-clockwise rotation, $r = -1$ for clockwise rotation). For the tandem configuration, the first rotor is the front rotor and the second rotor is the rear rotor; then

$$
\begin{pmatrix} T_{\text{Fcoll}} \\ T_{\text{Flat}} \\ T_{\text{Rcoll}} \\ T_{\text{Rlat}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -r_F & 0 & -r_F \\ 1 & 0 & 1 & 0 \\ 0 & -r_R & 0 & r_R \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

For the coaxial configuration:

$$
\begin{pmatrix} T_{1\text{coll}} \\ T_{1\text{lat}} \\ T_{1\text{lng}} \\ T_{2\text{coll}} \\ T_{2\text{lat}} \\ T_{2\text{lng}} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_1 \\ 0 & -r_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & r_2 \\ 0 & -r_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

For the tiltrotor, the first rotor is the right rotor and the second rotor is the left rotor; then

$$
\begin{pmatrix} T_{\text{Rcoll}} \\ T_{\text{Rlng}} \\ T_{\text{Lcoll}} \\ T_{\text{Llng}} \\ T_{\text{ail}} \\ T_{\text{elev}} \\ T_{\text{rud}} \end{pmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c_{AC0} \\ c_{ACc} \\ c_{ACs} \\ c_{ACp} \end{pmatrix}
$$

with cyclic stick and pedal connected to rotor controls only for helicopter mode.

7–3   Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. In steady flight (including hover, level flight, climb and descent, and turns), equilibrium implies zero net force and moment on the aircraft. In general, there can be additional quantities that at equilibrium must equal target values. In practice, the trim solution can deal with a subset of these quantities. Usually it is at least necessary to achieve equilibrium in the aircraft lift and drag forces, as well as in yaw moment for torque balance. The basic purpose of the trim solution is to

determine the component states, including aircraft drag and rotor thrust, sufficient to evaluate the aircraft performance.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. The available trim quantities include:

> aircraft total force and moment; aircraft load factor;
> propulsion group power;
> power margin $P_{avPG} - P_{reqPG}$; torque margin $P_{DSlimit} - P_{reqPG}$;
> rotor force (lift, vertical, or propulsive);
> rotor thrust C_T/σ; rotor thrust margin $(C_T/\sigma)_{\max} - C_T/\sigma$;
> rotor flapping β_c, β_s; rotor hub moment, roll and pitch; rotor torque;
> wing force; wing lift coefficient C_L; wing lift margin $C_{L\max} - C_L$;
> tail force.

Targets for aircraft total force and total moment (including inertial loads in turns) are always zero. The available trim variables include:

> aircraft controls;
> aircraft orientation, θ (pitch), ϕ (roll);
> aircraft horizontal velocity V_h;
> aircraft vertical rate of climb V_c; aircraft sideslip angle ψ_V;
> aircraft angular rate, $\dot{\theta}$ (pullup), $\dot{\psi}$ (turn).

The aircraft orientation variables are the Euler angles of the body axes relative to inertial axes. The aircraft controls (appropriately scaled) are connected to the component controls.

A Newton–Raphson method is used for trim. The derivative matrix is obtained by numerical perturbation. A tolerance ϵ and a perturbation Δ are specified.

7–4   Geometry

The aircraft coordinate system has the x-axis forward, y-axis to the right, and z-axis down, measured from the center-of-gravity (figure 7-2). These aircraft axes are body axes (x is not aligned with the wind), the orientation determined by the convention used for the input geometry. The center-of-gravity is the appropriate origin for describing the motion of the aircraft and summing the forces and moments acting on the aircraft.

Layout of the geometry is typically in terms of station line (SL, positive aft), buttline (BL, positive right), and waterline (WL, positive up), measured relative to some arbitrary origin (figure 7-2). The x-y-z axes are parallel to the SL-BL-WL directions. One or more locations are defined for each component of the aircraft. Each component will at least have a location that is the point where component forces and moments act on the aircraft. Each location is input in fixed or scaled form. The fixed form input is SL/BL/WL (dimensional). The scaled form input is x/L (positive aft), y/L (positive right), and z/L (positive up), based on a reference length L, from a reference point. The reference length is the rotor radius or wing span of a designated component, or the fuselage length. The reference point can optionally be input, or the location (hub) of a designated rotor, or the location (center of action) of a

axes for description of
aircraft geometry
(arbitrary reference)

aircraft coordinate
system
(origin at CG)
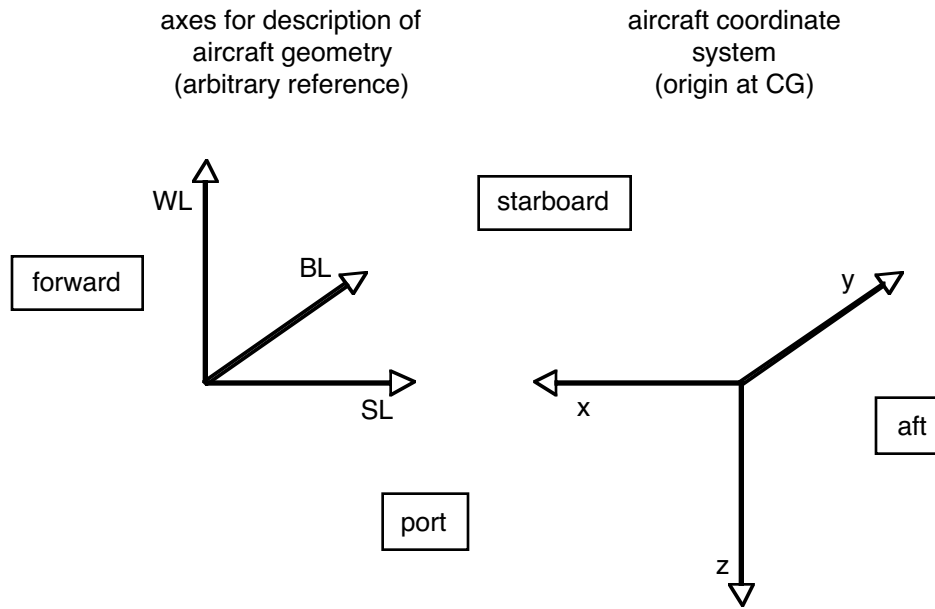
Figure 7-2. Aircraft geometry.

designated wing component, or the location (center of action) of the fuselage, or the location of the center-of-gravity. Fixed input can be used for the entire aircraft, or just for certain components.

From this fixed or scaled input and the current aircraft size, the actual geometry (x, y, z) can be calculated for each location. There are also options to calculate geometry from other parameters (such as tiltrotor span from rotor radius and clearance). This calculated geometry has the sign convention of the aircraft axes (x positive forward, y positive right, z positive down), but has the origin at the reference point (which may or may not be the center-of-gravity). All input uses the same sign convention; all internal calculations use the same sign conventions. Table 7-1 summarizes the conventions.

Table 7-1. Geometry conventions.

	layout	scaled input	calculated	motion and loads
origin	arbitrary	reference point	reference point	center-of-gravity
x	SL (+ aft)	x/L (+ aft)	x (+ forward)	x (+ forward)
y	BL (+ right)	y/L (+ right)	y (+ right)	y (+ right)
z	WL (+ up)	z/L (+ up)	z (+ down)	z (+ down)

The location of the aircraft center-of-gravity is specified for a baseline configuration. With tilting rotors, this location is in helicopter mode. For each flight state the aircraft center-of-gravity is calculated, from the baseline location plus any shift due to nacelle tilt, plus an input center-of-gravity increment. Alternatively, the aircraft center-of-gravity location for the flight state can be input. Any change of the center-of-gravity position with fuel burn during a mission is not automatically calculated, but could be accounted for using the flight state input.

The aircraft operating length and width are calculated from the component positions and dimensions: $\ell_{\text{total}} = x_{\max} - x_{\min}$ and $w_{\text{total}} = y_{\max} - y_{\min}$, where the maximum and minimum dimensions are for the fuselage and all rotors, wings, and tails. The corresponding footprint area is then $S_{\text{total}} = \ell_{\text{total}} w_{\text{total}}$.

7–5 Aircraft Motion

The aircraft velocity and orientation are defined by the following parameters: flight speed V; turn rate; orientation of the body frame relative to inertial axes (Euler angles); and orientation of the velocity frame relative to inertial axes (flight path angles). Aircraft conventions are followed for the direction and orientation of axes: the z-axis is down, the x-axis forward, and the y-axis to the right; and a yaw-pitch-roll sequence is used for the Euler angles. However, the airframe axes are body axes (fixed to the airframe, regardless of the flight direction) rather than wind axes (which have the x-axis in the direction of the flight speed). The orientation of the body frame F relative to inertial axes I is defined by yaw, pitch, and roll Euler angles, which are rotations about the z, y, and x axes, respectively:

$$C^{FI} = X_{\phi_F} Y_{\theta_F} Z_{\psi_F}$$

So yaw is positive to the right, pitch is positive nose up, and roll is positive to the right. The flight path is specified by the velocity V, in the positive x-axis direction of the velocity axes. The orientation of the velocity axes V relative to inertial axes I is defined by yaw (sideslip) and pitch (climb) angles:

$$C^{VI} = Y_{\theta_V} Z_{\psi_V} Z_{\psi_F}$$

Sideslip is positive for the aircraft moving to the right, and climb is positive for the aircraft moving up. Then

$$C^{FV} = C^{FI} C^{IV} = X_{\phi_F} Y_{\theta_F} Z_{-\psi_V} Y_{-\theta_V}$$

In straight flight, all these angles and matrices are constant. In turning flight at a constant yaw rate, the yaw angle is $\psi_F = \dot{\psi}_F t$; the turn radius is $R_T = V_h / \dot{\psi}_F$; and the nominal bank angle and load factor are $\tan \phi_F = \sqrt{n^2 - 1} = \dot{\psi}_F V_h / g$. Then the forward, sideward, and climb velocities are:

$$V_f = V \cos \theta_V \cos \psi_V = V_h \cos \psi_V$$
$$V_s = V \cos \theta_V \sin \psi_V = V_h \sin \psi_V$$
$$V_c = V \sin \theta_V = V_h \tan \theta_V$$

where $V_h = V \cos \theta_V$ is the horizontal velocity component. The velocity components in airframe axes are $v_{AC}^F = v^{FI/F} = C^{FV}(V\,0\,0)^T$ (aircraft velocity relative to the air). The calibrated airspeed is calculated from the true airspeed V:

$$V_{\text{cal}} = V\sqrt{\sigma} \sqrt{\frac{\left[\delta((1 + 0.2M^2)^{7/2} - 1) + 1\right]^{2/7} - 1}{0.2M^2\delta}} \cong V\sqrt{\sigma}\left[1 + \frac{1}{8}(1 - \delta)M^2 + \frac{3}{640}(1 - 10\delta + 9\delta^2)M^4\right]$$

where $\sigma = \rho/\rho_0$ is the density ratio, $\delta = p/p_0$ is the pressure ratio, and M is the Mach number. The aircraft angular velocity is

$$\omega_{AC}^F = \omega^{FI/F} = R\begin{pmatrix} \dot{\phi}_F \\ \dot{\theta}_F \\ \dot{\psi}_F \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta_F \\ 0 & \cos\phi_F & \sin\phi_F\cos\theta_F \\ 0 & -\sin\phi_F & \cos\phi_F\cos\theta_F \end{bmatrix} \begin{pmatrix} \dot{\phi}_F \\ \dot{\theta}_F \\ \dot{\psi}_F \end{pmatrix}$$

For steady state flight, $\dot{\theta}_F = \dot{\phi}_F = 0$; $\dot{\psi}_F$ is nonzero in a turn.

Accelerated flight is also considered, in terms of linear acceleration $a_{AC}^F = \dot{v}^{FI/F} = gn_L$ and pitch rate $\dot{\theta}_F$. The nominal pullup load factor is $n = 1 + \dot{\theta}_F V_h / g$. For accelerated flight, the instantaneous equilibrium of the forces and moments on the aircraft is evaluated, for specified acceleration and angular velocity; the equations of motion are not integrated to define a maneuver. Note that the fuselage and wing aerodynamic models do not include all roll and yaw moment terms needed for general unsteady flight (notably derivatives $L_v, L_p, L_r, N_v, N_p, N_r$).

The aircraft pitch and roll angles are available for trim of the aircraft. Any motion not selected for trim will remain fixed at the values specified for the flight state. The pitch and roll angles each can be zero, constant, or a function of flight speed (piecewise linear input). The flight state input can override this value of the aircraft motion. The input value is an initial value if the motion is a trim variable.

7–6   Loads and Performance

For each component, the power required and the net forces and moments acting on the aircraft can be calculated. The aerodynamic forces F and moments M are typically calculated in wind axes and then resolved into body axes (x, y, z), relative to the origin of the body axes (the aircraft center-of-gravity). The power and loads of all components are summed to obtain the aircraft power and loads. Typically the trim solution drives the net forces and moments on the aircraft to zero.

The aircraft equations of motion, in body axes F with origin at the aircraft center-of-gravity, are the equations of force and moment equilibrium:

$$m(\dot{v}^{FI/F} + \widetilde{\omega}^{FI/F} v^{FI/F}) = F^F + F_{\text{grav}}^F$$
$$I^F \dot{\omega}^{FI/F} + \widetilde{\omega}^{FI/F} I^F \omega^{FI/F} = M^F$$

where $m = W/g$ is the aircraft mass; the gravitational force is $F_{\text{grav}}^F = mC^{FI}g^I = mC^{FI}(0\,0\,g)^T$; and the moment of inertia matrix is

$$I^F = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

For steady flight, $\dot{\omega}^{FI/F} = \dot{v}^{FI/F} = 0$, and $\omega^{FI/F} = R(0\,0\,\dot{\psi}_F)^T$ is nonzero only in turns. For accelerated flight, $\dot{v}^{FI/F}$ can be nonzero, and $\omega^{FI/F} = R(0\,\dot{\theta}_F\,\dot{\psi}_F)^T$. The equations of motion are thus

$$m(a_{AC}^F + \widetilde{\omega}_{AC}^F v_{AC}^F) = F^F + F_{\text{grav}}^F$$
$$\widetilde{\omega}_{AC}^F I^F \omega_{AC}^F = M^F$$

The body axis load factor is $n = (C^{FI}g^I - (a_{AC}^F + \widetilde{\omega}_{AC}^F v_{AC}^F))/g$. The a_{AC}^F term is absent for steady flight. The forces and moments are the sum of loads from all components of the aircraft:

$$F^F = F_{\text{fus}}^F + \sum F_{\text{rotor}}^F + \sum F_{\text{force}}^F + \sum F_{\text{wing}}^F + \sum F_{\text{tail}}^F + \sum F_{\text{engine}}^F + F_{\text{tank}}^F$$
$$M^F = M_{\text{fus}}^F + \sum M_{\text{rotor}}^F + \sum M_{\text{force}}^F + \sum M_{\text{wing}}^F + \sum M_{\text{tail}}^F + \sum M_{\text{engine}}^F + M_{\text{tank}}^F$$

Forces and moments in inertial axes are also of interest ($F^I = C^{IF}F^F$ and $M^I = C^{IF}M^F$). A particular component can have more than one source of loads; for example, the rotor component produces hub forces and moments, but also includes hub and pylon drag. The equations of motion are $E_f = F^F + F_{\text{grav}}^F - F_{\text{inertial}}^F = 0$ and $E_m = M^F - M_{\text{inertial}}^F = 0$.

The component power required P_{comp} is evaluated for all components (rotors) of the propulsion group. The total power required for the propulsion group P_{reqPG} is obtained by adding the transmission

losses and accessory power. The power required for the propulsion group must be distributed to the engine groups. The fuel flow is calculated from the power required. The fuel flow of the propulsion group is obtained from the sum over the engine groups. The total fuel flow is the sum from all components of the aircraft: $\dot{w} = \sum \dot{w}_{reqEG} + \sum \dot{w}_{\text{force}}$.

7–7   Aerodynamics

Each component has a position z^F in aircraft axes F, relative to the reference point; and orientation of component axes B relative to aircraft axes given by the rotation matrix C^{BF}. It is expected that the component axes are (roughly) x forward and z down (or in negative lift direction). The aerodynamic model must be consistent with the convention for component orientation. Acting at the component are interference velocities v_{int}^F (velocity of air, in F axes), from all other components. Then the total component velocity relative to the air is

$$v^F = v_{AC}^F + \widetilde{\omega}_{AC}^F \Delta z^F - \sum v_{\text{int}}^F$$

where $\Delta z^F = z^F - z_{cg}^F$. Then $v^B = C^{BF} v^F$ is the velocity in component axes. The aerodynamic environment is defined in the component axes: velocity magnitude $v = |v^B|$, dynamic pressure $q = \frac{1}{2}\rho v^2$, angle-of-attack α, and sideslip angle β. The angle-of-attack and sideslip angle provide the transformation between airframe axes and velocity axes:

$$C^{BA} = Y_\alpha Z_{-\beta}$$

This is the conventional aircraft definition, corresponding to yaw-then-pitch of the airframe axes relative to the velocity vector. By definition, the velocity is along the x-axis in the A axes, $v^B = C^{BA}(v\,0\,0)^T$; from which the angle-of-attack and sideslip in terms of the components of v^B are obtained:

$$\alpha = \tan^{-1} v_3^B / v_1^B$$
$$\beta = \sin^{-1} v_2^B / |v^B|$$

This definition is not well behaved for $v_1^B = 0$ (it gives $\alpha = 90\,\text{sign}(v_3^B)$), so for sideward flight a pitch-then-yaw definition can be useful: $C^{BA} = Z_{-\beta} Y_\alpha$. Then

$$\alpha = \sin^{-1} v_3^B / |v^B|$$
$$\beta = \tan^{-1} v_2^B / v_1^B$$

which gives $\beta = 90\,\text{sign}(v_2^B)$ for $v_1^B = 0$.

The component aerodynamic model may include coefficient values for sideward flight, but not have equations for a continuous variation of the coefficients with sideslip angle. For such cases, sideward flight is defined as $|\beta| = 80$ to 100 degrees.

From v, q, α, and β, the aerodynamic model calculates the component force and moment, in wind axes acting at z^F:

$$F^A = \begin{pmatrix} -D \\ Y \\ -L \end{pmatrix} \qquad M^A = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

where D, Y, and L are the drag, side force, and lift; M_x, M_y, and M_z are the roll, pitch, and yaw moments. The aerodynamic loads in aircraft axes acting at the center-of-gravity are then:

$$F^F = C^{FB} C^{BA} F^A$$
$$M^F = C^{FB} C^{BA} M^A + \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$. In hover and low speed, the download is calculated: $F^I_z = k^T(C^{IF}F^F)$, the downward component of the aerodynamic force in inertial axes. Download can be expressed as a fraction of the total rotor vertical force, or as a fraction of gross weight. The aerodynamic model also calculates the interference velocities caused by this component at all other components: $v^F_{int} = C^{FB}v^B_{int}$.

Equations for the aerodynamics models are defined for all angles in radians. Input values of angles will, however, be in degrees.

The aircraft neutral point is calculated from the airframe aerodynamics with all controls set to zero. The neutral point is here defined as the longitudinal position about which the derivative of the pitch moment with lift is zero. Hence $SL_{na} = SL_{cg} - \Delta M/\Delta L$, with the change in lift and moment calculated from the loads at angles of attack of 0 and 5 deg.

7–8   Trailing-Edge Flaps

The lifting surfaces have controls in the form of trailing edge flaps: flap, flaperon, and aileron for wings; elevator or rudder for tails. The aerodynamic loads generated by flap deflection δ_f (radians) are estimated based on two-dimensional aerodynamic data (as summarized in refs. 1 and 2). Let $\ell_f = c_f/c$ be the ratio of the flap chord to the wing chord. The lift coefficient is $c_\ell = c_{\ell\alpha}(\alpha + \tau\eta\delta_f)$, where $\eta \cong 0.85 - 0.43\delta_f$ is an empirical correction for viscous effects (ref. 1, equation 3.54 and figure 3.36). Thin airfoil theory gives

$$\tau = 1 - \frac{\theta_f - \sin\theta_f}{\pi} \cong \left(\sin(\frac{\pi}{2}\ell_f)\right)^n$$

with $\theta_f = \cos^{-1}(2\ell_f - 1)$ (ref. 1, equation 3.56 and figure 3.35; ref. 2, equation 5.40). The last expression is an approximation that is a good fit to the thin airfoil theory result for $n = 1/2$, and a good approximation including the effects of real flow for $n = 2/3$ (ref. 2, figure 5.18); the last expression with $n = 2/3$ is used here. The increase of maximum lift coefficient caused by flap deflection is less than the increase in lift coefficient, so the stall angle-of-attack is decreased. Approximately

$$\frac{\Delta c_{\ell max}}{\Delta c_\ell} \cong (1 - \ell_f)(1 + \ell_f - 5\ell_f^2 + 3\ell_f^3)$$

(ref. 1, figure 3.37). Thin airfoil theory gives the moment coefficient increment about the quarter chord:

$$\Delta c_m = -0.85\left((1 - \ell_f)\sin\theta_f\right)\delta_f = -0.85\left((1 - \ell_f)2\sqrt{(1 - \ell_f)\ell_f}\right)\delta_f$$

(ref. 1, equation 3.57; ref. 2, equation 5.41); with the factor of 0.85 accounting for real flow effects (ref. 2, figure 5.19). The drag increment is estimated using

$$\Delta C_D = 0.9\,\ell_f^{1.38}\frac{S_f}{S}\sin^2\delta_f$$

for slotted flaps (ref. 1, equation 3.51). In summary, the section load increments are:

$$\Delta c_\ell = c_{\ell\alpha}\frac{c_f}{c}L_f\eta_f\delta_f$$
$$\Delta c_{\ell max} = X_f\,\Delta c_\ell$$
$$\Delta c_m = \frac{c_f}{c}M_f\delta_f$$

The decrease in angle-of-attack for maximum lift is

$$\Delta\alpha_{max} = -\frac{\Delta c_\ell - \Delta c_{\ell max}}{c_{\ell\alpha}} = -(1 - X_f)\frac{\Delta c_\ell}{c_{\ell\alpha}}$$

The coefficients

$$\eta_f = 0.85 - 0.43|\delta_f| = \eta_0 - \eta_1|\delta_f|$$

$$L_f = \frac{1}{\ell_f} \left(\sin(\frac{\pi}{2}\ell_f) \right)^{2/3}$$

$$X_f = (1 - \ell_f)(1 + \ell_f - 5\ell_f^2 + 3\ell_f^3)$$

$$M_f = -0.85 \frac{1}{\ell_f} \left((1 - \ell_f)2\sqrt{(1 - \ell_f)\ell_f} \right)$$

$$D_f = 0.9\,\ell_f^{1.38}$$

follow from the equations above.

For three-dimensional aerodynamic loads, these two-dimensional coefficients are corrected by using the three-dimensional lift-curve slope, and multiplying by the ratio of flap span to wing span b_f/b. Then the wing load increments caused by flap deflection, in terms of coefficients based on the wing area, are:

$$\Delta C_L = \frac{S_f}{S} C_{L\alpha} L_f \eta_f \delta_f$$

$$\Delta C_M = \frac{S_f}{S} M_f \delta_f$$

$$\Delta C_D = \frac{S_f}{S} D_f \sin^2 \delta_f$$

$$\Delta C_{L\max} = X_f \,\Delta C_L$$

$$\Delta \alpha_{\max} = -(1 - X_f) \frac{\Delta C_L}{C_{L\alpha}}$$

where S_f/S is the ratio of flap area to wing area.

7–9 Drag

Each component can contribute drag to the aircraft. A fixed drag can be specified, as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on an appropriate area S. There may also be other ways to define a scaled drag value. For fixed drag, the coefficient is $C_D = (D/q)/S$ (the aerodynamic model is formulated in terms of drag coefficient). For scaled drag, the drag area is $D/q = SC_D$. For all components, the drag $(D/q)_{\text{comp}}$ or $C_{D\text{comp}}$ is defined for forward flight or cruise; typically this is the minimum drag value. For some components, the vertical drag $((D/q)_{V\text{comp}}$ or $C_{DV\text{comp}})$ or sideward drag $((D/q)_{S\text{comp}}$ or $C_{DS\text{comp}})$ is defined. For some components, the aerodynamic model includes drag due to lift, angle-of-attack, or stall.

Table 7-2 summarizes the component contributions to drag, and the corresponding reference areas. If no reference area is indicated, then the input is only drag area D/q. An appropriate drag reference area is defined for each component, and either input or calculated. Wetted area is calculated for each component, even if it is not the reference area. The component wetted areas are summed to obtain the aircraft wetted area. Some of the weight models also require the wetted area. The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

Optionally the aircraft drag can be fixed. The quantity specified is the sum (over all components) of the drag area D/q (minimum drag, excluding drag due to lift and angle-of-attack), without accounting for interference effects on dynamic pressure. The input parameter can be D/q; or the drag can be scaled, specified as a drag coefficient based on the rotor disk area, so $D/q = A_{\text{ref}}C_D$ (A_{ref} is the reference rotor

disk area); or the drag can be estimated based on the gross weight, $D/q = k(W_{MTO}/1000)^{2/3}$ (W_{MTO} is the maximum takeoff gross weight; units of k are ft²/k-lb$^{2/3}$ or m²/Mg$^{2/3}$). Based on historical data, the drag coefficient $C_D = 0.02$ for old helicopters, $C_D = 0.008$ for current low drag helicopters. Based on historical data, $k = 9$ for old helicopters, $k = 2.5$ for current low drag helicopters, $k = 1.6$ for current tiltrotors, and $k = 1.4$ for turboprop aircraft (English units). If the aircraft drag is input, then the fuselage contingency drag is adjusted so the total aircraft D/q equals the input value.

Optionally the aircraft vertical drag (download fraction) can be fixed. The quantity specified is the sum over all components of the vertical drag area $(D/q)_V$. The input parameter can be $(D/q)_V$, or $k = (D/q)_V/A_{\text{ref}}$ (A_{ref} is reference rotor disk area). Approximating the dynamic pressure in the wake as $q = \frac{1}{2}\rho(2v_h)^2 = T/A_{\text{ref}}$, the download fraction is $DL/T = q(D/q)_V/T = k$. If the aircraft vertical drag is input, then the fuselage contingency vertical drag is adjusted so the total aircraft $(D/q)_V$ equals the input value.

The nominal drag areas of the components and the aircraft are part of the aircraft description and are used when the aircraft drag is fixed. The nominal drag area is calculated for low speed helicopter flight, for high speed cruise flight, and for vertical flight. An incidence angle i is specified for the rotors, wings, and nacelles, to be used solely to calculate the nominal helicopter and vertical drag areas. The convention is that $i = 0$ if the component does not tilt. Table 7-3 summarizes the contributions to the nominal drag areas, with D for the drag in normal flow and D_V for the drag in vertical flow. While vertical drag parameters are part of the aerodynamic model for the hub, pylon, and nacelle, aerodynamic interference at the rotor and at the engine group is not considered, so these terms do not contribute to download. In the context of download, only the fuselage, wing, tail, and contingency contribute to the nominal vertical drag.

From the input and the current aircraft size, the drag areas D/q and coefficients C_D are calculated. The aerodynamic analysis is usually in terms of coefficients. If the aircraft drag is fixed for the aircraft model, then the fuselage contingency drag is set:

$$(D/q)_{\text{cont}} = (D/q)_{\text{fixed}} - \sum(D/q)_{\text{comp}}$$

and similarly for fixed vertical drag. Note that this adjustment ignores changes caused by interference in the dynamic pressure and the velocity direction, which will affect the actual component drag.

The component aerodynamic model calculates the drag, typically from a drag coefficient C_D, a reference area, and the air velocity of the component. The drag force is then $D = \sum q_{\text{comp}} S_{\text{ref}} C_D$, where the dynamic pressure q_{comp} includes interference. From the aerodynamic forces and moments in wind axes, the total force and moment in body axes (F^F and M^F) are calculated. For reference, the aircraft total drag and total drag area are

$$D_{AC} = \sum e_d^T F_{\text{aero}}^F$$
$$(D/q)_{AC} = D_{AC}/q$$

where the aircraft velocity (without interference) gives the direction $e_d = -v_{AC}^F/|v_{AC}^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v_{AC}^F|^2$; and F_{aero}^F is the component aerodynamic force. An overall skin friction drag coefficient is then $C_{D\,AC} = (D/q)_{AC\text{wet}}/S_{AC}$, based on the aircraft wetted area $S_{AC} = \sum S_{\text{wet}}$ and excluding drag terms not associated with skin friction (specifically landing gear, rotor hub, and contingency drag).

Table 7-2. Component contributions to drag.

component	drag contribution	reference area
fuselage	fuselage	fuselage wetted area
	fuselage vertical	fuselage projected area
	fittings and fixtures	fuselage wetted area
	rotor-body interference	fuselage wetted area
	contingency (aircraft)	—
	payload increment (flight state)	—
landing gear	landing gear	—
rotor	hub, hub vertical	rotor disk area
	pylon, pylon vertical	pylon wetted area
	spinner	spinner wetted area
wing	wing, wing vertical	wing planform area
	wing-body interference	wing planform area
tail	tail, tail vertical	tail planform area
engine	nacelle, nacelle vertical	nacelle wetted area
	momentum drag	—
fuel tank	auxiliary tank (flight state)	—

Table 7-3. Component contributions to nominal drag area.

component	drag contribution	cruise	helicopter	vertical
fuselage	fuselage	D	D	D_V
	fittings and fixtures	D	D	D
	rotor-body int	D	D	D
landing gear	landing gear	D	D	0
	retractable	0	D	0
rotor	hub	D	$D\cos^2 i + D_V \sin^2 i$	0
	pylon	D	$D\cos^2 i + D_V \sin^2 i$	0
	spinner	D	D	0
wing	wing	D	$D\cos^2 i + D_V \sin^2 i$	$D\sin^2 i + D_V \cos^2 i$
	wing-body int	D	D	D
tail	horizontal tail	D	D	$D_V \cos^2 \phi$
tail	vertical tail	D	D	$D_V \sin^2 \phi$
engine	nacelle	D	$D\cos^2 i + D_V \sin^2 i$	0
	contingency	D	D	D_V

7–10   Performance Indices

The following performance indices are calculated for the aircraft. The aircraft hover figure of merit is $M = W\sqrt{W/2\rho A_{\text{ref}}}/P$. The aircraft effective drag is $D_e = P/V$, hence the effective lift-to-drag ratio is $L/D_e = WV/P$. The aircraft power loading is W/P (lb/hp or kg/kW). Isolated rotor performance indices are described in Chapter 11.

7–11   Weights

The design gross weight W_D is a principal parameter defining the aircraft, usually determined by the sizing task for the design conditions and missions. The aircraft weight statement defines the empty weight, fixed useful load, and operating weight for the design configuration. The aircraft weight statement is the sum of the weight statements for all the aircraft components, the component weight determined by input or by parametric calculations with technology factors. The definition of the weight terms is as follows.

$$
\begin{aligned}
\text{gross weight} \qquad & W_G = W_E + W_{UL} = W_O + W_{\text{pay}} + W_{\text{fuel}} \\
\text{operating weight} \qquad & W_O = W_E + W_{FUL} \\
\text{useful load} \qquad & W_{UL} = W_{FUL} + W_{\text{pay}} + W_{\text{fuel}}
\end{aligned}
$$

where W_E is the weight empty; W_{FUL} the fixed useful load; W_{pay} the payload weight; and W_{fuel} the usable fuel weight. The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. If the weight empty is input, then the contingency weight is adjusted so W_E equals the required value. If the design gross weight is input, then the payload or fuel weight must be fallout.

The structural design gross weight W_{SD} and maximum takeoff weight W_{MTO} can be input, or specified as an increment d plus a fraction f of a weight W:

$$
W_{SD} = d_{SDGW} + f_{SDGW}W = \begin{cases} d_{SDGW} + f_{SDGW}W_D \\ d_{SDGW} + f_{SDGW}(W_D - W_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel-cap}}) \end{cases}
$$

$$
W_{MTO} = d_{WMTO} + f_{WMTO}W = \begin{cases} d_{WMTO} + f_{WMTO}W_D \\ d_{WMTO} + f_{WMTO}(W_D - W_{\text{fuel}} + W_{\text{fuel-cap}}) \end{cases}
$$

This convention allows the weights to be input directly ($f = 0$), or scaled with W_D. For W_{SD}, W is the design gross weight W_D, or W_D adjusted for a specified fuel state (input fraction of fuel capacity). Alternatively, W_{SD} can be calculated as the gross weight at a designated sizing flight condition. The structural design gross weight is used in the weight estimation. For W_{MTO}, W is the design gross weight W_D, or W_D adjusted for maximum fuel capacity. Alternatively, W_{MTO} can be calculated as the maximum gross weight possible at a designated sizing flight condition. The maximum takeoff weight is used in the cost model, in the scaled aircraft and hub drag, and in the weight estimation.

The design ultimate load factor n_{zult} at the structural design gross weight W_{SD} is specified, in particular for use in the component weight estimates. The structural design gross weight W_{SD} and design ultimate load factor n_{zult} are used for the fuselage, rotor, and wing weight estimations. The maximum takeoff weight W_{MTO} is used for the cost and drag (scaled aircraft and hub), and for the weights (system, fuselage, landing gear, and engine group).

The gross weight W_G is specified for each flight condition and mission, perhaps in terms of the design gross weight W_D. For a each flight state, the fixed useful load may be different from the design

configuration because of changes in auxiliary fuel tank weight or kit weights or increments in crew or equipment weights. Thus the fixed useful load weight is calculated for the flight state; and from it the useful load weight and operating weight are calculated. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) complete the weight information for the flight state.

For each weight group, fixed (input) weights can be specified; or weight increments dW added to the results of the parametric weight model. The parametric weight model includes technology factors χ. Thus typically a component or element weight is obtained from $W = \chi W_{\mathrm{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

For scaled weights of all components, the AFDD weight models are implemented. The user can incorporate custom weight models as well.

The operating weight is composed of scaled and fixed weights, so the design gross weight can be written $W_D = W_O + W_{\mathrm{pay}} + W_{\mathrm{fuel}} = W_{O\mathrm{fixed}} + W_{O\mathrm{scaled}} + W_{\mathrm{pay}} + W_{\mathrm{fuel}}$. The growth factor is the change in gross weight due to a change in payload:

$$\frac{\partial W_D}{\partial W_{\mathrm{pay}}} = 1 + \frac{\partial W_{O\mathrm{scaled}}}{\partial W_{\mathrm{pay}}} + \frac{\partial W_{\mathrm{fuel}}}{\partial W_{\mathrm{pay}}} = 1 + \left(\frac{\partial W_{O\mathrm{scaled}}}{\partial W_D} + \frac{\partial W_{\mathrm{fuel}}}{\partial W_D} \right) \frac{\partial W_D}{\partial W_{\mathrm{pay}}}$$

$$\cong 1 + \left(\frac{W_{O\mathrm{scaled}}}{W_D} + \frac{W_{\mathrm{fuel}}}{W_D} \right) \frac{\partial W_D}{\partial W_{\mathrm{pay}}} = \frac{1}{1 - \phi_{O\mathrm{scaled}} - \phi_{\mathrm{fuel}}}$$

in terms of the weight fractions $\phi = W/W_D$.

7–12   Weight Statement

Aircraft weight information is stored in a data structure that follows SAWE RP8A Group Weight Statement format (ref. 3), as outlined in figure 7-3. The asterisks designate extensions of RP8A for the purposes of this analysis. Typically only the lowest elements of the hierarchy are specified; higher elements are obtained by summation. Fixed (input) weight elements are identified in the data structure. A weight statement data structure exists for each component. The aircraft weight statement is the sum of the structures from all components.

7–13   References

1) McCormick, B.W. *Aerodynamics, Aeronautics, and Flight Mechanics*. New York: John Wiley & Sons, Second Edition, 1995.

2) Kuethe, A.M., and Chow, C.-Y. *Foundations of Aerodynamics*. New York: John Wiley & Sons, Fifth Edition, 1998.

3) "Weight and Balance Data Reporting Forms for Aircraft (including Rotorcraft), Revision A." Society of Allied Weight Engineers, Recommended Practice Number 8, June 1997.

WEIGHT EMPTY
    STRUCTURE
        wing group
            basic structure
            secondary structure
                fairings (*), fittings (*), fold/tilt (*)
            control surfaces
        rotor group
            blade assembly
            hub & hinge
                basic (*), fairing/spinner (*), blade fold (*), shaft (*)
        empennage group
            horizontal tail (*)
                basic (*), fold (*)
            vertical tail (*)
                basic (*), fold (*)
            tail rotor (*)
                blades, hub & hinge, rotor/fan duct & rotor supts
        fuselage group
            basic (*)
            wing & rotor fold/retraction (*)
            tail fold/tilt (*)
            marinization (*)
            pressurization (*)
            crashworthiness (*)
        alighting gear group
            basic (*), retraction (*), crashworthiness (*)
        engine section or nacelle group
            engine support (*), engine cowling (*), pylon support (*)
        air induction group
    PROPULSION GROUP
        engine system
            engine
            exhaust system
            accessories (*)
        propeller/fan installation
            blades (*), hub & hinge (*), rotor/fan duct & supports (*)
        fuel system
            tanks and support
            plumbing
        drive system
            gear boxes
            transmission drive
            rotor shaft
            rotor brake (*)
            clutch (*)
            gas drive

Figure 7-3a. Weight statement (* indicates extension of RP8A).

SYSTEMS AND EQUIPMENT
    flight controls group
        cockpit controls
        automatic flight control system
        system controls
            fixed wing systems
                non-boosted (*), boost mechanisms (*)
            rotary wing systems
                non-boosted (*), boost mechanisms (*), boosted (*)
            conversion systems
                non-boosted (*), boost mechanisms (*)
    auxiliary power group
    instruments group
    hydraulic group
        fixed wing (*), rotary wing (*), conversion (*)
        equipment (*)
    pneumatic group
    electrical group
        aircraft (*), anti-icing (*)
    avionics group (mission equipment)
    armament group
        armament provisions (*), armor (*)
    furnishings & equipment group
    environmental control group
    anti-icing group
    load & handling group
VIBRATION (*)
CONTINGENCY
FIXED USEFUL LOAD
    crew
    fluids (oil, unusable fuel) (*)
    auxiliary fuel tanks
    other fixed useful load (*)
    equipment increment (*)
    folding kit (*)
    wing extension kit (*)
    wing kit (*)
    other kit (*)
PAYLOAD
USABLE FUEL
    standard tanks (*)
    auxiliary tanks (*)

OPERATING WEIGHT = weight empty + fixed useful load
USEFUL LOAD = fixed useful load + payload + usable fuel

GROSS WEIGHT = weight empty + useful load
GROSS WEIGHT = operating weight + payload + usable fuel

Figure 7-3b. Weight statement (* indicates extension of RP8A).

Chapter 8

Systems

The systems component contains weight information (fixed useful load, vibration, contingency, and systems and equipment).

8–1   Weights

The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. The vibration control weight can be input, or specified as a fraction of weight empty: $W_{\text{vib}} = f_{\text{vib}} W_E$. The contingency weight can be input, or specified as a fraction of weight empty: $W_{\text{cont}} = f_{\text{cont}} W_E$. However, if the weight empty is input, then the contingency weight is adjusted so W_E equals the required value. The weights of all components are evaluated and summed, producing the aircraft weight empty less vibration and contingency weight, W_X. Then:

a) Fixed weight empty: W_{vib} input or $W_{\text{vib}} = f_{\text{vib}} W_E$; $W_{\text{cont}} = W_E - (W_X + W_{\text{vib}})$.
b) Both fractional: $W_E = W_X/(1 - f_{\text{vib}} - f_{\text{cont}})$, $W_{\text{vib}} = f_{\text{vib}} W_E$, $W_{\text{cont}} = f_{\text{cont}} W_E$.
c) Only vibration weight fractional: W_{cont} input, $W_E = (W_X + W_{\text{cont}})/(1 - f_{\text{vib}})$, $W_{\text{vib}} = f_{\text{vib}} W_E$.
d) Only contingency weight fractional: W_{vib} input, $W_E = (W_X + W_{\text{vib}})/(1 - f_{\text{cont}})$, $W_{\text{cont}} = f_{\text{cont}} W_E$.
e) Both input: W_{vib} and W_{cont} input, $W_E = W_X + W_{\text{vib}} + W_{\text{cont}}$.

Finally, the operating weight $W_O = W_E + W_{FUL}$ is recalculated.

The fixed useful load W_{FUL} consists of crew (W_{crew}), trapped fluids (oil and unusable fuel, W_{trap}), auxiliary fuel tanks (W_{auxtank}), equipment increment, kits (folding, wing, wing extension, other), and other fixed useful load ($W_{FUL\text{other}}$). W_{crew}, W_{trap}, and $W_{FUL\text{other}}$ are input. For a each flight state, the fixed useful load may be different from the design configuration, because of changes in auxiliary fuel tank weight, kit weight, and crew or equpment weight increments.

Folding weights can be calculated in several weight groups, including wing, rotor, empennage, and fuselage. These weights are the total weights for folding and the impact of folding on the group. A fraction f_{foldkit} of these weights can be in a kit, hence optionally removable. Thus of the total folding weight, the fraction f_{foldkit} is a kit weight in the fixed useful load of the weight statement, while the remainder is kept in the component group weight.

Systems and equipment includes the following fixed (input) weights: auxiliary power group, instruments group, pneumatic group, electrical group (aircraft), avionics group (mission equipment), armament group (armor and armament provisions), furnishings and equipment group, environmental control group, and load and handling group. Systems and equipment includes the following scaled weights: flight controls group, hydraulic group, electrical group (anti-icing), and anti-icing group.

Flight controls group includes the following fixed (input) weights: cockpit controls and automatic flight control system. Flight controls group includes the following scaled weights: fixed wing systems, rotary wing systems, and conversion or thrust vectoring systems. Rotary wing flight control weights can be calculated for the entire aircraft (using rotor parameters such as chord and tip speed for a designated rotor), an approach that is consistent with parametric weight equations developed for conventional two-rotor configurations. Alternatively, rotary wing flight control weights can be calculated separately for each rotor and then summed. The fixed wing flight controls and the conversion controls can be absent.

Fuselage

There is one fuselage component for the aircraft.

9–1   Geometry

The fuselage length ℓ_{fus} can be input or calculated. The calculated length depends on the longitudinal positions of all components. Let x_{max} and x_{min} be the maximum (forward) and minimum (aft) position of all rotors, wings, and tails. Then the calculated fuselage length is

$$\ell_{\text{fus}} = \ell_{\text{nose}} + (x_{\text{max}} - x_{\text{min}}) + \ell_{\text{aft}}$$

The nose length ℓ_{nose} (distance forward of hub) and aft length ℓ_{nose} (distance aft of hub) are input, or calculated as $\ell_{\text{nose}} = f_{\text{nose}}R$ and $\ell_{\text{aft}} = f_{\text{aft}}R$. Typically $f_{\text{aft}} = 0$ or negative for the main rotor and tail rotor configuration, and $f_{\text{aft}} = 0.75$ for the coaxial configuration. The fuselage width w_{fus} is input.

The fuselage wetted area S_{wet} (reference area for drag coefficients) and projected area S_{proj} (reference area for vertical drag) are input (excluding or including the tail boom terms); or calculated from the nose length:

$$S_{\text{wet}} = f_{\text{wet}} \left(2\ell_{\text{nose}}h_{\text{fus}} + 2\ell_{\text{nose}}w_{\text{fus}} + 2h_{\text{fus}}w_{\text{fus}}\right) + C_{\text{boom}}R$$
$$S_{\text{proj}} = f_{\text{proj}} \left(\ell_{\text{nose}}w_{\text{fus}}\right) + w_{\text{boom}}R$$

using input fuselage height h_{fus}, and factors f_{wet} and f_{proj}; or calculated from the fuselage length:

$$S_{\text{wet}} = f_{\text{wet}} \left(2\ell_{\text{fus}}h_{\text{fus}} + 2\ell_{\text{fus}}w_{\text{fus}} + 2h_{\text{fus}}w_{\text{fus}}\right) + C_{\text{boom}}R$$
$$S_{\text{proj}} = f_{\text{proj}} \left(\ell_{\text{fus}}w_{\text{fus}}\right) + w_{\text{boom}}R$$

Using the nose length and the tail boom area is probably best for a single-main-rotor and tail-rotor helicopter. Here C_{boom} is the effective tail boom circumference (boom wetted area divided by rotor radius), and w_{boom} is the effective tail boom width (boom vertical area divided by rotor radius).

The fuselage contribution to the aircraft operating length is $x_{\text{fus}} + f_{\text{ref}}\ell_{\text{fus}}$ (forward) and $x_{\text{fus}} - (1 - f_{\text{ref}})\ell_{\text{fus}}$ (aft). Here f_{ref} is the position of the fuselage aerodynamic reference location aft of the nose, as a fraction of the fuselage length. If the fuselage length is input, then f_{ref} is input; if the fuselage length is calculated, then $f_{\text{ref}} = (x_{\text{max}} + \ell_{\text{nose}} - x_{\text{fus}})/\ell_{\text{fus}}$.

9–2   Control and Loads

The fuselage has a position z^F, where the aerodynamic forces act; and the component axes are aligned with the aircraft axes, $C^{BF} = I$. The fuselage has no control variables.

9–3   Aerodynamics

The aerodynamic velocity of the fuselage relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack α_{fus}, sideslip angle β_{fus} (hence C^{BA}), and dynamic pressure q are calculated from v^B. The reference area for the fuselage forward flight drag is the fuselage wetted area S_{wet}, which is input or calculated as described above. The reference area for the fuselage vertical drag is the fuselage projected area S_{proj}, which is input or calculated as described above.

9-3.1   Drag

The drag area or drag coefficient is defined for forward flight, vertical flight, and sideward flight. In addition, the forward flight drag area or drag coefficient is defined for fixtures and fittings, and for rotor-body interference. The effective angle-of-attack is $\alpha_e = \alpha_{\text{fus}} - \alpha_{D\text{min}}$, where $\alpha_{D\text{min}}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle-of-attack term. Thus if $|\alpha_e| \leq \alpha_t$

$$C_D = C_{D0}\left(1 + K_d|\alpha_e|^{X_d}\right)$$

and otherwise

$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d}\right)$$

$$C_D = C_{Dt} + \left(\frac{S_{\text{proj}}}{S_{\text{wet}}}C_{DV} - C_{Dt}\right)\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

and similarly for the transition of payload drag $(D/q)_{\text{pay}}$ and contingency drag $(D/q)_{\text{cont}}$. Optionally there might be no angle-of-attack variation at low angles ($K_d = 0$), or quadratic variation ($X_d = 2$). With an input transition angle, there will be a jump in the slope of the drag coefficient at α_t. For a smooth transition, the transition angle that matches slopes as well as coefficients is found by solving

$$\left(\frac{2X_d}{\pi} - 1\right)\alpha_t^{X_d} - X_d\alpha_t^{X_d - 1} + \frac{(S_{\text{proj}}/S_{\text{wet}})C_{DV} - C_{D0}}{K_dC_{D0}} = 0$$

This calculation of the transition angle is only implemented with quadratic variation, for which

$$\alpha_t = \frac{1}{a}\left(1 + \sqrt{1 - a\frac{(S_{\text{proj}}/S_{\text{wet}})C_{DV} - C_{D0}}{K_dC_{D0}}}\right)$$

with $a = (4/\pi) - 1$; α_t is however required to be between 15 and 45 deg. For sideward flight ($v_x^B = 0$) the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate between sideward and vertical coefficients:

$$C_D = C_{DS}\cos^2\phi_v + \frac{S_{\text{proj}}}{S_{\text{wet}}}C_{DV}\sin^2\phi_v$$

Then the drag force is

$$D = qS_{\text{wet}}\left(C_D + C_{D\text{fit}} + \sum C_{D\text{rb}}\right) + q\left((D/q)_{\text{pay}} + (D/q)_{\text{cont}}\right)$$

including drag coefficient for fixtures and fittings $C_{D\text{fit}}$ and rotor-body interference $C_{D\text{rb}}$ (summed over all rotors); drag area of the payload (specified for flight state); and contingency drag area.

9-3.2   Lift and Pitch Moment

The fuselage lift and pitch moment are defined in fixed form (L/q and M/q), or scaled form (C_L and C_M, based on the fuselage wetted area and fuselage length). The effective angle-of-attack is $\alpha_e = \alpha_{\text{fus}} - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. Let $\alpha_{\max}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. If $|\alpha_e| \leq \alpha_{\max}$

$$C_L = C_{L\alpha}\alpha_e$$
$$C_M = C_{M0} + C_{M\alpha}\alpha_e$$

and otherwise

$$C_L = C_{L\alpha}\alpha_{\max}\text{sign}\alpha_e \left(\frac{\pi/2 - |\alpha_e|}{\pi/2 - |\alpha_{\max}|} \right)$$
$$C_M = (C_{M0} + C_{M\alpha}\alpha_{\max}\text{sign}\alpha_e) \left(\frac{\pi/2 - |\alpha_e|}{\pi/2 - |\alpha_{\max}|} \right)$$

for zero lift and moment at 90 deg angle-of-attack. In sideward flight, these coefficients are zero. Then $L = qS_{\text{wet}}C_L$ and $M = qS_{\text{wet}}\ell_{\text{fus}}C_M$ are the lift and pitch moment.

9-3.3   Side Force and Yaw Moment

The fuselage side force and yaw moment are defined in fixed form (Y/q and N/q), or scaled form (C_Y and C_N, based on the fuselage wetted area and fuselage length). The effective sideslip angle is $\beta_e = \beta_{\text{fus}} - \beta_{zy}$, where β_{zy} is the angle of zero side force; in reverse flow ($|\beta_e| > 90$), $\beta_e \leftarrow \beta_e - 180\,\text{sign}\beta_e$. Let $\beta_{\max}$ be the sideslip angle increment (above or below zero side force angle) for maximum side force. If $|\beta_e| \leq \beta_{\max}$

$$C_Y = C_{Y\beta}\beta_e$$
$$C_N = C_{N0} + C_{N\beta}\beta_e$$

and otherwise

$$C_Y = C_{Y\beta}\beta_{\max}\text{sign}\beta_e \left(\frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\max}|} \right)$$
$$C_N = (C_{N0} + C_{N\beta}\beta_{\max}\text{sign}\beta_e) \left(\frac{\pi/2 - |\beta_e|}{\pi/2 - |\beta_{\max}|} \right)$$

for zero side force and yaw moment at 90 deg sideslip angle. Then $Y = qS_{\text{wet}}C_Y$ and $N = qS_{\text{wet}}\ell_{\text{fus}}C_N$ are the side force and yaw moment. The roll moment is zero.

9–4   Weights

The fuselage group consists of the basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure.

Landing Gear

There is one landing gear component for the aircraft. The landing gear can be located on the body or on the wing. The landing gear can be fixed or retractable; a gear retraction speed is specified (CAS), or the landing gear state can be specified in the flight state.

10–1   Geometry

The landing gear has a position z^F, where the aerodynamic forces act. The component axes are aligned with the aircraft axes, $C^{BF} = I$. The landing gear has no control variables. The height of the bottom of the landing gear above ground level, h_{LG}, is specified in the flight state. The landing gear position z^F is a distance d_{LG} above the bottom of the gear.

10-1.1   Drag

The drag area is specified for landing gear extended, $(D/q)_{LG}$. The velocity relative to the air at z^F gives the drag direction $e_d = -v^F/|v^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v^F|^2$ (no interference). Then

$$F^F = e_d\, q(D/q)_{LG}$$

is the total drag force.

10–2   Weights

The alighting gear group consists of basic structure, retraction, and crashworthiness structure.

Rotor

The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the rotor component can model tail rotors, propellers, proprotors, ducted fans, thrust vectoring rotors, and auxiliary-thrust rotors. The principal configuration designation (main rotor, tail rotor, or propeller) is identified for each rotor component, and in particular determines where the weights are put in the weight statement (summarized in table 11-1). Each configuration can possibly have a separate performance or weight model, which is separately specified. Antitorque rotors and auxiliary-thrust rotors can be identified, for special sizing options. Other configuration features are variable diameter and ducted fan.

Multi-rotor systems (such as coaxial or tandem configuration) are modeled as a set of separate rotors, in order to accommodate the description of the position, orientation, controls, and loads. Optionally the location of the center of the rotor system can be specified, and the rotor locations calculated based on input separation parameters. The performance calculation for twin rotor systems can include the mutual influence of the induced velocity on the power.

The main rotor size is defined by the radius R or disk loading W/A, thrust-weighted solidity σ, hover tip speed V_{tip}, and blade loading $C_W/\sigma = W/\rho A V_{\text{tip}}^2 \sigma$. With more than one main rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight, $W = f_W W_D$. The air density ρ for C_W/σ is obtained from a specified takeoff condition. If the rotor radius is fixed for the sizing task, three of (R or W/A), C_W/σ, V_{tip}, σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. If the sizing task determines the rotor radius (R and W/A), then two of C_W/σ, V_{tip}, σ are input, and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others.

For antitorque and auxiliary-thrust rotors, three of (R or W/A), C_W/σ, V_{tip}, σ are input, and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. The disk loading and blade loading are based on fT, where f is an input factor and T is the maximum thrust from designated design conditions. Optionally the tail rotor radius can be scaled with the main rotor radius: $R = f R_{\text{mr}}(0.1348 + 0.0071 W/A)$, where f is an input factor and the units of disk loading W/A are lb/ft^2. Figure 11-1 is the basis for this scaling.

Table 11-1. Principal configuration designation.

configuration	weight statement	weight model	performance model
main rotor	rotor group	rotor	rotor
tail rotor	empennage group	tail rotor	rotor
propeller	propulsion group	rotor, aux thrust	rotor
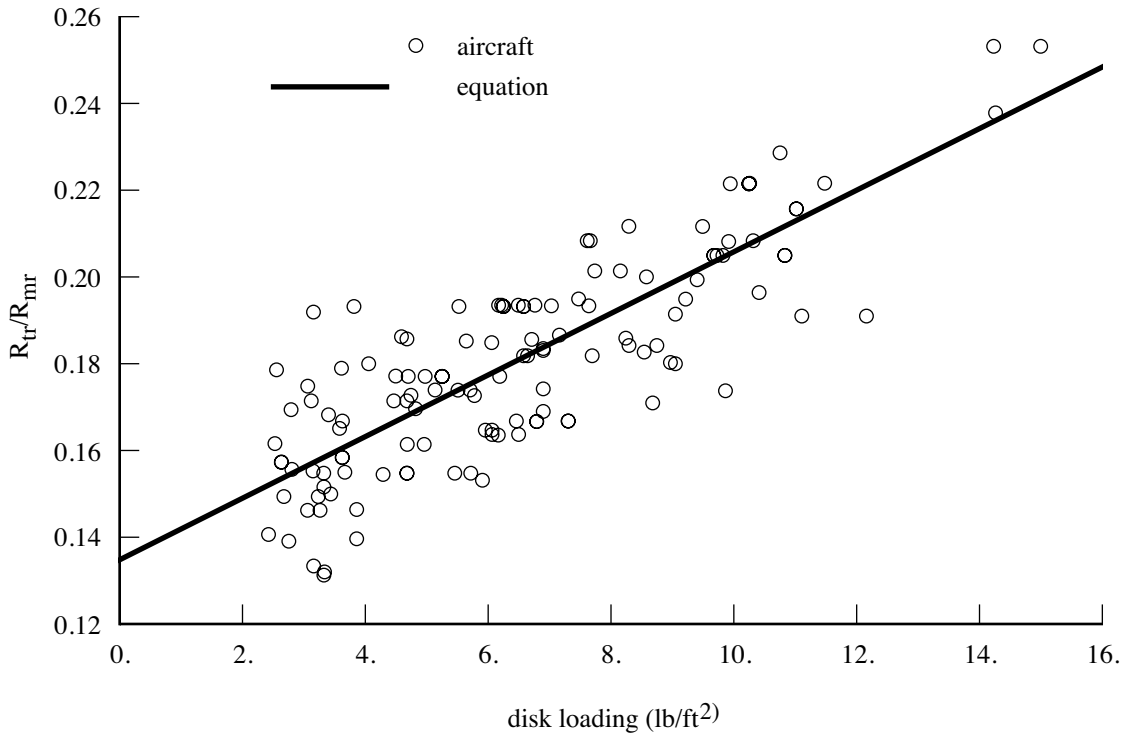
Figure 11-1. Tail rotor radius scaling.

11–1   Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\text{tip}-\text{ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$. For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio calculated ($r = \Omega_{\text{dep}}/\Omega_{\text{prim}}$, $\Omega = V_{\text{tip}-\text{ref}}/R$). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio calculated from the specification engine turbine speed $\Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}}$ and the reference tip speed of the primary rotor ($r = \Omega_{\text{spec}}/\Omega_{\text{prim}}$, $\Omega_{\text{prim}} = V_{\text{tip}-\text{ref}}/R$). The latter option means the specification engine turbine speed N_{spec} corresponds to $V_{\text{tip}-\text{ref}}$ for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each

propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius the rotational speed of the primary rotor is obtained ($\Omega_{\mathrm{prim}} = V_{\mathrm{tip}}/R$); from the gear ratios, the rotational speed of dependent rotors ($\Omega_{\mathrm{dep}} = r\Omega_{\mathrm{prim}}$) and the engine groups ($N = (60/2\pi)r_{\mathrm{eng}}\Omega_{\mathrm{prim}}$) are obtained; and from the rotor radius, the tip speed of the dependent rotor ($V_{\mathrm{tip}} = \Omega_{\mathrm{dep}}R$) is obtained. The flight state specification of the tip speed can be an input value; the reference tip speed; a function of flight speed or a conversion schedule; or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor f_{gear} on the gear ratio when the speeds of the dependent rotors and engines are evaluated. The factor f_{gear} is a component control, which can be connected to an aircraft control and thus set for each flight state.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below $V_{C\mathrm{hover}}$, cruise mode for speeds above $V_{C\mathrm{cruise}}$, and conversion mode for speeds between $V_{C\mathrm{hover}}$ and $V_{C\mathrm{cruise}}$. The tip speed is $V_{\mathrm{tip-hover}}$ in helicopter and conversion mode, and $V_{\mathrm{tip-cruise}}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from an input $C_T/\sigma = t_0 - \mu t_1$, so $V_{\mathrm{tip}} = \sqrt{T/\rho A\sigma t_0 + (Vt_1/2t_0)^2} + (Vt_1/2t_0)$; or from $\mu = V/V_{\mathrm{tip}}$, so $V_{\mathrm{tip}} = V/\mu$; or from $M_{at} = M_{\mathrm{tip}}\sqrt{(1+\mu)^2 + \mu_z^2}$, so $V_{\mathrm{tip}} = \sqrt{(c_s M_{at})^2 - V_z^2} - V$.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed N_{spec}. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

11–2   Geometry

The rotor rotation direction is described by the parameter r: $r = 1$ for counter-clockwise rotation and $r = -1$ for clockwise rotation (as viewed from the positive thrust side of the rotor).

The rotor solidity and blade mean chord are related by $\sigma = Nc/\pi R$; usually thrust-weighted values are used, but geometric values are also required by the analysis. The mean chord is the average of the chord over the rotor blade span, from root cutout to tip. The thrust-weighted chord is the average of the chord over the rotor blade span r, from root cutout to tip, weighted by r^2. A general blade chord distribution is specified as $c(r) = c_{\mathrm{ref}}\hat{c}(r)$, where c_{ref} is the thrust-weighted chord. Linear taper is specified in terms of a taper ratio $t = c_{\mathrm{tip}}/c_{\mathrm{root}}$, or in terms of the ratio of thrust-weight and geometric chords, $f = \sigma_t/\sigma_g = c_{.75R}/c_{.50R}$.

The rotor hub is at position z_{hub}^F. Optionally, a component of the position can be calculated, superseding the location input. The calculated geometry depends on the configuration. For a coaxial rotor, the rotor separation is $s = |k^T C^{SF}(z_{\mathrm{hub1}}^F - z_{\mathrm{hub2}}^F)/(2R)|$ (fraction rotor diameter), or the hub

locations are calculated from the input separation s, and the input location midway between the hubs:

$$z^F_{\text{hub}} = z^F_{\text{center}} \pm C^{FS} \begin{pmatrix} 0 \\ 0 \\ sR \end{pmatrix}$$

For a tandem rotor, the rotor longitudinal overlap is $o = \Delta\ell/(2R) = 1 - \ell/(2R)$ (fraction rotor diameter), or the hub locations are calculated from the input overlap o, and the input location midway between the hubs:

$$x_{\text{hub}} = x_{\text{center}} \pm R(1 - o)$$

For a tail rotor, the longitudinal position can be calculated from the main rotor radius R, tail rotor radius R_{tr}, and tail-rotor/main-rotor clearance d_{tr}:

$$x_{\text{hub}tr} = x_{\text{hub}mr} - (R_{mr} + d_{tr} + R_{tr})$$

For a tiltrotor, the lateral position can be calculated from the rotor radius R (cruise value for variable-diameter rotor), fuselage/rotor clearance d_{fus}, and fuselage width w_{fus}:

$$y_{\text{hub}} = \pm \left(fR + d_{\text{fus}} + {}^1\!/\!_2 w_{\text{fus}} \right)$$

with the pivot, pylon, and nacelle center-of-gravity lateral positions adjusted to keep the same relative position to the hub. The calculated clearance between the rotor and fuselage is $d_{\text{fus}} = |y_{\text{hub}}| - (R + {}^1\!/\!_2 w_{\text{fus}})$. Alternatively for a tiltrotor, the lateral position can be calculated from the wing span, $y_{\text{hub}} = \pm b/2$, so the rotors are at the wing tips, or from a designated wing panel edge, $y_{\text{hub}} = \pm \eta_p(b/2)$.

For twin rotors (tandem, side-by-side, or coaxial), the overlap is $o = \Delta\ell/(2R) = 1 - \ell/(2R)$ (fraction of diameter; 0 for no overlap and 1 for coaxial), where the hub-to-hub separation is $\ell = [(x_{\text{hub}1} - x_{\text{hub}2})^2 + (y_{\text{hub}1} - y_{\text{hub}2})^2]^{1/2}$ ($\ell = 2R$ for no overlap and $\ell = 0$ for coaxial). The overlap area is mA, with A the area of one rotor disk and

$$m = \frac{2}{\pi} \left[\cos^{-1}(\ell/2R) - (\ell/2R)\sqrt{1 - (\ell/2R)^2} \right]$$

The vertical separation is $s = |z_{\text{hub}1} - z_{\text{hub}2}|/(2R)$.

The reference areas for the component drag coefficients are the rotor disk area $A = \pi R^2$ (for hub drag), pylon wetted area S_{pylon}, and spinner wetted area S_{spin}. The pylon wetted area is input, or calculated from the drive system (gear box and rotor shaft) weight, or from the drive system plus engine system (engine, exhaust, and accessories) weight:

$$S_{\text{pylon}} = k\big(w/N_{\text{rotor}}\big)^{2/3}$$

where $w = W_{gbrs}$ or $w = W_{gbrs} + \sum W_{ES}$ and the units of k are ft²/lb²/³ or m²/kg²/³. The pylon area is included in the aircraft wetted area if the pylon drag coefficient is nonzero. The spinner wetted area is input, or calculated from the spinner frontal area:

$$S_{\text{spin}} = k(\pi R^2_{\text{spin}})$$

where R_{spin} is the spinner radius, which is specified as a fraction of the rotor radius.

The rotor contribution to the aircraft operating length and width is calculated from the locus of the rotor disk: $z_{\text{disk}} = z_{\text{hub}} + RC^{FS}(\cos\psi \ \sin\psi \ 0)^T$. The longitudinal distance from the hub position is

$\Delta x = R(a\cos\psi + b\sin\psi)$, so the maximum distance is $\Delta x = \pm R\sqrt{a^2 + b^2}$. The lateral distance from the hub position is $\Delta y = R(c\cos\psi + d\sin\psi)$, so the maximum distance is $\Delta y = \pm R\sqrt{c^2 + d^2}$.

11–3   Control and Loads

The rotor controls consist of collective, lateral cyclic, longitudinal cyclic, and perhaps shaft incidence (tilt) and cant angles. Rotor cyclic control can be defined in terms of tip-path plane or no-feathering plane command. The collective control variable is the rotor thrust amplitude or the collective pitch angle.

The relationship between tip-path plane tilt and hub moment is $M = \frac{N}{2}I_b\Omega^2(\nu^2 - 1)\beta = K_{\text{hub}}\beta$, where N is the number of blades, Ω the rotor speed, and ν the dimensionless fundamental flap frequency. The flap moment of inertia I_b is obtained from the Lock number: $\gamma = \rho a c R^4/I_b$, for SLS density ρ and lift curve slope $a = 5.7$ (or from the blade weight, or from an autorotation index). The flap frequency and Lock number are specified for hover radius and rotational speed. The flap frequency and hub stiffness are required for the radius and rotational speed of the flight state. For a hingeless rotor, the blade flap spring is $K_{\text{flap}} = I_b\Omega^2(\nu^2 - 1)$, obtained from the hover quantities; then $K_{\text{hub}} = \frac{N}{2}K_{\text{flap}}$ and

$$\nu^2 = 1 + \frac{K_{\text{flap}}}{I_b\Omega^2}$$

For an articulated rotor, the hinge offset is $e = Rx/(1+x)$, $x = \frac{2}{3}(\nu^2 - 1)$ from the hover quantities; then

$$\nu^2 = 1 + \frac{3}{2}\frac{e/R}{1 - e/R}$$

and $K_{\text{hub}} = \frac{N}{2}I_b\Omega^2(\nu^2 - 1)$, using I_b from γ (and scaled with R for a variable diameter rotor) and Ω for the flight state.

Optionally the rotor can have a variable diameter. The rotor diameter is treated as a control, allowing it to be connected to an aircraft control and thus set for each flight state. The basic variation can be specified based on the conversion schedule, or input as a function of flight speed (piecewise linear input). For the conversion schedule, the rotor radius is R_{hover} for speeds below $V_{C\text{hover}}$, $R_{\text{cruise}} = fR_{\text{hover}}$ for speeds above $V_{C\text{cruise}}$, and linear with flight speed in conversion mode. During the diameter change, the chord, chord radial distribution, and blade weight are assumed fixed; hence solidity scales as $\sigma \sim 1/R$, blade flap moment of inertia as $I_b \sim R^2$, and Lock number as $\gamma \sim R^2$.

11-3.1   Control Variables

The collective control variable is direct command of rotor thrust magnitude T or C_T/σ (in shaft axes), from which the collective pitch angle can be calculated; or rotor collective pitch angle $\theta_{0.75}$, from which the thrust and inflow can be calculated.

Shaft tilt control variables are incidence (tilt) and cant angles, acting at a pivot location.

Tip-path plane command is direct control of the tip-path plane tilt, hence tilt of the thrust vector. This control mode requires calculation of rotor cyclic pitch angles from the flapping. The control variables are longitudinal tilt β_c (positive forward) and lateral tilt β_s (positive toward retreating side). Alternatively, the cyclic control can be specified in terms of hub moment or lift offset, if the blade flap frequency is greater than 1/rev. The relationship between tip-path plane tilt and hub moment is $M = K_{\text{hub}}\beta$, and between moment and lift offset is $M = o(TR)$. Thus the flapping is

$$\begin{pmatrix} \beta_s \\ \beta_c \end{pmatrix} = \frac{1}{K_{\text{hub}}}\begin{pmatrix} rM_x \\ -M_y \end{pmatrix} = \frac{TR}{K_{\text{hub}}}\begin{pmatrix} o_x \\ -o_y \end{pmatrix}$$

for hub moment command or lift offset command, respectively.

No-feathering plane command is control of rotor cyclic pitch angles, usually producing tilt of the thrust vector. This control mode requires calculation of rotor tip-path plane tilt from the cyclic control, including the influence of inflow. The control variables are longitudinal cyclic pitch angle θ_s (positive aft) and lateral cyclic pitch angle θ_c (positive toward retreating side).

11-3.2   Aircraft Controls

Each control can be connected to the aircraft controls c_{AC}: $c = c_0 + ST c_{AC}$, with c_0 zero, constant, or a function of flight speed (piecewise linear input). The factor S can be introduced to automatically scale the collective matrix: $S = a/6 = 1/60$ if the collective control variable is C_T/σ; $S = \rho V_{\text{tip}}^2 A_{\text{blade}}(a/6)$ if the collective control variable is rotor thrust T; $S = 1$ if the collective control variable is pitch angle $\theta_{0.75}$. For cyclic matrices, $S = 1$ with no-feathering plane command, and $S = -1$ for tip-path plane command.

11-3.3   Rotor Axes and Shaft Tilt

The rotor hub is at position z_{hub}^F, where the rotor forces and moments act; the orientation of the rotor shaft axes relative to the aircraft axes is given by the rotation matrix C^{SF}. The pivot is at position z_{pivot}^F. The hub or shaft axes S have origin at the hub node; the z-axis is the shaft, positive in the positive thrust direction; and the x-axis downstream or up. The rotor orientation is specified by selecting a nominal direction in body axes (positive or negative x, y, or z-axis) for the positive thrust direction; the other two axes are then the axes of control. For a main rotor the nominal direction would be the negative z-axis; for a tail rotor it would be the lateral axis (ry-axis, depending on the direction of rotation of the main rotor); and for a propeller the nominal direction would be the positive x-axis. This selection defines a rotation matrix W from F to S axes. The hub and pivot axes have a fixed orientation relative to the body axes:

$$\text{hub incidence and cant:} \qquad C^{HF} = U_{\theta_h} V_{\phi_h}$$
$$\text{pivot dihedral, pitch, and sweep:} \qquad C^{PF} = X_{\phi_h} Y_{\theta_h} Z_{\psi_p}$$

where U and V depend on the nominal direction, as described in table 11-2. The shaft control consists of incidence and cant about the pivot axes, from reference angles i_{ref} and c_{ref}:

$$C_{\text{cont}} = U_{i - i_{\text{ref}}} V_{c - c_{\text{ref}}}$$

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise, and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The rotor shaft incidence angle is then connected to α_{tilt} by defining the matrix T_i appropriately. For the locations and orientation input in helicopter mode, $i_{\text{ref}} = 90$. Thus the orientation of the shaft axes relative to the body axes is:

$$C^{SF} = W C^{HF} C^{FP} C_{\text{cont}} C^{PF}$$

or just $C^{SF} = W C^{HF}$ with no shaft control. From the pivot location z_{pivot}^F and the hub location for the reference shaft control z_{hub0}^F, the hub location in general is

$$z_{\text{hub}}^F = z_{\text{pivot}}^F + (C^{FP} C_{\text{cont}} C^{PF})^T (z_{\text{hub0}}^F - z_{\text{pivot}}^F)$$

Similarly, the pylon location and nacelle center-of-gravity location can be calculated for given shaft control. The shift in the aircraft center-of-gravity produced by nacelle tilt is

$$W(z_{cg}^F - z_{cg0}^F) = W_{\text{move}}(z_{\text{nac}}^F - z_{\text{nac0}}^F) = W_{\text{move}}(C^{FP} C_{\text{cont}}^T C^{PF} - I)(z_{\text{nac0}}^F - z_{\text{pivot}}^F)$$

where W is the gross weight and W_{move} the weight moved. Table 11-2 summarizes the geometry options.

Table 11-2. Rotor shaft axes.

	nominal thrust z^S-axis		x^S-axis		W	incidence + for T	cant + for T	$U_{\theta_h} V_{\phi_h}$
main rotor	$-z^F$	up	$-x^F$	aft	Y_{180}	aft	right	$Y_\theta X_\phi$
	z^F	down	$-x^F$	aft	Z_{180}	aft	right	$Y_{-\theta} X_{-\phi}$
propeller	x^F	forward	$-z^F$	up	Y_{90}	up	right	$Y_\theta Z_\phi$
	$-x^F$	aft	$-z^F$	up	$Z_{180} Y_{-90}$	up	right	$Y_{-\theta} Z_{-\phi}$
tail rotor $(r = 1)$	y^F	right	$-x^F$	aft	$Z_{180} X_{-90}$	aft	up	$Z_\theta X_{-\phi}$
tail rotor $(r = -1)$	$-y^F$	left	$-x^F$	aft	$Z_{180} X_{90}$	aft	up	$Z_{-\theta} X_\phi$

$$Y_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Z_{180} Y_{-90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$Z_{180} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Z_{180} X_{-90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Y_{90} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad Z_{180} X_{90} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

11-3.4   Hub Loads

The rotor controls give the thrust magnitude and the tip-path plane tilt angles β_c and β_s, either directly or from the collective and cyclic pitch. The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z, x, y-axis directions, respectively). The hub pitch and roll moments are proportional to the flap angles. The hub torque is obtained from the shaft power P_{shaft} and rotor speed Ω. The force and moment acting on the hub, in shaft axes, are then:

$$F^S = \begin{pmatrix} H \\ Y \\ T \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f_B T \end{pmatrix}$$

$$M^S = \begin{pmatrix} M_x \\ M_y \\ -rQ \end{pmatrix} = \begin{pmatrix} K_{\text{hub}} (r\beta_s) \\ K_{\text{hub}} (-\beta_c) \\ -r P_{\text{shaft}}/\Omega \end{pmatrix}$$

The force includes a term proportional to the rotor thrust and an input blockage factor $f_B = \Delta T/T \geq 0$. This term accounts for blockage or download, as an alternative to including the drag of the fuselage or a lifting surface in the aircraft trim. For example, f_B can model the tail rotor blockage caused by operation near the vertical tail. The rotor loads in aircraft axes acting at the center-of-gravity are then:

$$F^F = C^{FS} F^S$$

$$M^F = C^{FS} M^S + \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F_{\text{hub}} - z^F_{cg}$.

The wind axis lift L and drag X are calculated from the net rotor hub force F^F and the rotor velocity v^F. The velocity relative to the air gives the propulsive force direction $e_p = v^F/|v^F|$ (no interference) and the velocity magnitude $V = |v^F|$. The drag and lift components of the force are $X = -e_p^T F^F$ and $L = |(I - e_p e_p^T) F^F|$, respectively. Thus $XV = -(v^F)^T F^F$ and $L^2 = |F^F|^2 - |X|^2$. The rotor contribution to vertical force is the z-axis component of the force in inertial axes, $F_V = -k^T C^{IF} F^F$.

11–4 Aerodynamics

The rotor velocity relative to the air is $v^F = v_{AC}^F + \widetilde{\omega}_{AC}^F \Delta z^F$ in aircraft axes. The velocities in shaft axes are

$$v^S = C^{SF} v^F = \Omega R \begin{pmatrix} -\mu_x \\ r\mu_y \\ \mu_z \end{pmatrix} \qquad \omega^S = C^{SF} \omega_{AC}^F = \Omega \begin{pmatrix} r\dot{\alpha}_x \\ \dot{\alpha}_y \\ r\dot{\alpha}_z \end{pmatrix}$$

where ΩR is the rotor tip speed. The advance ratio μ, inflow ratio λ, and shaft angle-of-attack α are defined as

$$\mu = \sqrt{\mu_x^2 + \mu_y^2}$$
$$\lambda = \lambda_i + \mu_z$$
$$\alpha = \tan^{-1}(\mu_z/\mu)$$

The blade velocity relative to the air has the maximum amplitude (advancing tip velocity) of $\mu_{at} = \sqrt{(1+\mu)^2 + \mu_z^2}$, from which the advancing tip Mach number is $M_{at} = M_{\text{tip}} \mu_{at}$, using the tip Mach number $M_{\text{tip}} = (\Omega R)/c_s$. The rotor thrust coefficient is defined as $C_T = T/\rho A(\Omega R)^2$. The dimensionless ideal induced velocity λ_i is calculated from μ, μ_z, and C_T; then the dimensional velocity is $v_i = \Omega R \lambda_i$. The ideal induced power is then $P_{\text{ideal}} = Tv_i$. Note that for these inflow velocities, the subscript "i" denotes "ideal."

11-4.1   Ideal Inflow

The ideal wake-induced velocity is obtained from the momentum theory result of Glauert:

$$\lambda_i = \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}} = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}}$$

where $\lambda = \lambda_i + \mu_z$, $\lambda_h^2 = |C_T|/2$ (λ_h is always positive), and $s = \text{sign}\, C_T$. This expression is generalized to

$$\lambda_i = \lambda_h\, s\, F(\mu/\lambda_h, s\mu_z/\lambda_h)$$

If μ is zero, the equation for λ_i can be solved analytically. Otherwise, for non-axial flow, the equation is written as follows:

$$\lambda = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}} + \mu_z$$

Using λ instead of λ_i as the independent variable simplifies implementation of the ducted fan model. A Newton–Raphson solution for λ gives:

$$\widehat{\lambda}_{in} = \frac{s\lambda_h^2}{\sqrt{\lambda_n^2 + \mu^2}}$$

$$\lambda_{n+1} = \lambda_n - \frac{\lambda_n - \mu_z - \widehat{\lambda}_{in}}{1 + \widehat{\lambda}_{in}\lambda_n/(\lambda_n^2 + \mu^2)} f$$

A relaxation factor of $f = 0.5$ is used to improve convergence. Three or four iterations are usually sufficient, using

$$\lambda \cong \frac{s\lambda_h^2}{\sqrt{(s\lambda_h + \mu_z)^2 + \mu^2}} + \mu_z$$

to start the solution. To eliminate the singularity of the momentum theory result at ideal autorotation, the expression

$$\lambda = \mu_z \left[\frac{0.373\mu_z^2 + 0.598\mu^2}{\lambda_h^2} - 0.991 \right]$$

is used when

$$1.5\mu^2 + (2s\mu_z + 3\lambda_h)^2 < \lambda_h^2$$

The equation $\lambda = \mu_z(a\mu_z^2 - b\lambda_h^2 + c\mu^2)/\lambda_h^2$ is an approximation for the induced power measured in the turbulent-wake and vortex-ring states. Matching this equation to the axial-flow momentum theory result at $\mu_z = -2\lambda_h$ and $\mu_z = -\lambda_h$ gives $a = \sqrt{5}/6 = 0.3726780$ and $b = (4\sqrt{5} - 3)/6 = 0.9907120$. Then matching to the forward-flight momentum theory result at $(\mu = \lambda_h, \mu_z = -1.5\lambda_h)$ gives $c = 0.5980197$. For axial flow ($\mu = 0$) the solution is:

$$\lambda = \begin{cases} \dfrac{\mu_z}{2} + s\sqrt{\left(\dfrac{\mu_z}{2}\right)^2 + \lambda_h^2} & -\lambda_h < s\mu_z \\[3mm] \mu_z \left[\dfrac{0.373\mu_z^2}{\lambda_h^2} - 0.991 \right] & -2\lambda_h < s\mu_z < -\lambda_h \\[3mm] \dfrac{\mu_z}{2} - s\sqrt{\left(\dfrac{\mu_z}{2}\right)^2 - \lambda_h^2} & s\mu_z < -2\lambda_h \end{cases}$$

Note that λ_i and v_i are the ideal induced velocities; additional factors are required for the wake induced velocity or induced power calculations.

11-4.1.1   Ducted Fan

Rotor momentum theory can be extended to the case of a ducted fan. Consider a rotor system with disk area A, operating at speed V, with an angle α between V and the disk plane. The induced velocity at the rotor disk is v, and in the far wake $w = f_W v$. The far wake area is $A_\infty = A/f_A$. The axial velocity at the fan is $f_{Vz}V_z$, with f_{Vz} accounting for acceleration or deceleration through the duct. The edgewise velocity at the fan is $f_{Vx}V_x$, with $f_{Vx} = 1.0$ for wing-like behavior, or $f_{Vx} = 0$ for tube-like behavior of the flow. The total thrust (rotor plus duct) is T, and the rotor thrust is $T_{\text{rotor}} = f_T T$. For this model, the duct aerodynamics are defined by the thrust ratio f_T or far wake area ratio f_A, plus the fan velocity ratio f_V. The mass flux through the rotor disk is $\dot{m} = \rho A U = \rho A_\infty U_\infty$, where U and U_∞ are respectively the total velocity magnitudes at the fan and in the far wake:

$$U^2 = (f_{Vx}V\cos\alpha)^2 + (f_{Vz}V\sin\alpha + v)^2$$
$$U_\infty^2 = (V\cos\alpha)^2 + (V\sin\alpha + w)^2$$

Mass conservation ($f_A = A/A_\infty = U_\infty/U$) relates f_A and f_W. Momentum and energy conservation give

$$T = \dot{m}w = \rho A U_\infty w/f_A = \rho A U f_W v$$
$$P = \frac{1}{2}\dot{m}w\,(2V\sin\alpha + w) = T\left(V\sin\alpha + \frac{w}{2}\right)$$

With these expressions, the span of the lifting system in forward flight is assumed equal to the rotor diameter $2R$. Next it is required that the power equals the rotor induced and parasite loss:

$$P = T_{\text{rotor}}(f_{Vz}V\sin\alpha + v) = Tf_T(f_{Vz}V\sin\alpha + v)$$

In axial flow, this result can be derived from Bernoulli's equation for the pressure in the wake. In forward flight, any induced drag on the duct is being neglected. From these two expressions for power, $V_z + f_W v/2 = f_T(f_{Vz}V_z + v)$ is obtained, relating f_T and f_W. With no duct ($f_T = f_{Vx} = f_{Vz} = 1$), the far wake velocity is always $w = 2v$, hence $f_W = 2$. With an ideal duct ($f_A = f_{Vx} = f_{Vz} = 1$), the far wake velocity is $f_W = 1$. In hover (with or without a duct), $f_W = f_A = 2f_T$, and $v = \sqrt{2/f_W}v_h$. The rotor ideal induced power is $P_{\text{ideal}} = Tw/2 = f_D Tv$, introducing the duct factor $f_D = f_W/2$.

For a ducted fan, the thrust C_T is calculated from the total load (rotor plus duct). To define the duct effectiveness, either the thrust ratio $f_T = T_{\text{rotor}}/T$ or the far wake area ratio $f_A = A/A_\infty$ is specified (and the fan velocity ratio f_V). The wake-induced velocity is obtained from the momentum theory result for a ducted fan: $\lambda_h^2 = (f_W \lambda_i/2)\sqrt{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}$. If the thrust ratio f_T is specified, this can be written

$$f_{Vz}\mu_z + \lambda_i = \frac{s\lambda_h^2/f_T}{\sqrt{(f_{Vz}\mu_z + \lambda_i)^2 + (f_{Vx}\mu)^2}} + \frac{\mu_z}{f_T}$$

In this form, λ_i can be determined using the free-rotor expressions given previously: replacing λ_h^2, μ_z, μ, λ with respectively λ_h^2/f_T, μ_z/f_T, $f_{Vx}\mu$, $f_{Vz}\mu_z + \lambda_i$. Then from λ_i the velocity and area ratios are obtained:

$$f_W = 2\left(f_T - (1 - f_T f_{Vz})\frac{\mu_z}{\lambda_i}\right)$$

$$f_A = \sqrt{\frac{\mu^2 + (\mu_z + f_W \lambda_i)^2}{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}}$$

If instead the area ratio f_A is specified, it is simplest to first solve for the far wake velocity $f_W \lambda_i$:

$$\mu_z + f_W \lambda_i = \frac{s\lambda_h^2 2f_A}{\sqrt{(\mu_z + f_W \lambda_i)^2 + \mu^2}} + \mu_z$$

In this form, $f_W \lambda_i$ can be determined using the free-rotor expressions given previously: replacing λ_h^2, λ with respectively $\lambda_h^2 2f_A$, $\mu_z + f_W \lambda_i$. The induced velocity is

$$(f_{Vz}\mu_z + \lambda_i)^2 = \frac{1}{f_A^2}\left[\mu^2 + (\mu_z + f_W \lambda_i)^2\right] - (f_{Vx}\mu)^2$$

The velocity ratio is $f_W = (f_W \lambda_i)/\lambda_i$, and

$$f_T = \frac{\mu_z + f_W \lambda_i/2}{f_{Vz}\mu_z + \lambda_i}$$

is the thrust ratio. However, physical problems and convergence difficulties are encountered with this approach in descent, if an arbitrary value of f_T is permitted. From the expression for f_T, f_T should approach $1/f_{Vz}$ at high rates of climb or descent. To avoid problems with an arbitrary value of f_T, it is assumed that the input value of f_T defines the velocity ratio $f_W = 2f_T$ in descent. So in descent μ_z is not replaced by μ_z/f_T.

11-4.1.2   Ground Effect

The wake-induced velocity is reduced when the rotor disk is in the proximity of the ground plane. Ground effect in hover can be described in terms of the figure of merit $M = (T^{3/2}/\sqrt{2\rho A})/P$ as a function of scaled rotor height above the ground, $z_g/D = z_g/2R$. Usually test data are given as the ratio of the thrust to OGE thrust, for constant power: $T/T_\infty = (M/M_\infty)^{2/3} = \kappa_g \geq 1$. The effect on power at

constant thrust is then $P = P_\infty f_g$, where $f_g = \kappa_g^{-3/2} \le 1$. Ground effect is generally negligible at heights above $z_g/D = 1.5$ and at forward speeds above $\mu = 3\lambda_h$.

The ground plane is assumed to be perpendicular to the inertial frame z-axis. The ground normal (directed downward) is $k_g^F = C^{FI}k$ in airframe axes, or $k_g^S = C^{SF}k_g^F$ in rotor shaft axes. The height of the landing gear above ground level, h_{LG}, is specified in the flight state. The height of the rotor hub above ground level is then

$$z_g = h_{LG} - (k_g^F)^T (z_{\text{hub}}^F - z_{LG}^F) + d_{LG}$$

where z_{LG}^F is the position of the landing gear in the airframe; and d_{LG} is the distance from the bottom of the gear to the location z_{LG}. From the velocity

$$v^S = \begin{pmatrix} \mu_x \\ -r\mu_y \\ -\lambda \end{pmatrix}$$

the angle ϵ between the ground normal and the rotor wake is evaluated: $\cos\epsilon = (k_g^S)^T v^S / |v^S|$ ($\epsilon = 0$ for hover, $\epsilon = 90$ deg in forward flight). Note that if the rotor shaft is vertical, then $\cos\epsilon = \lambda/\sqrt{\mu^2 + \lambda^2}$ (see ref. 1). The expressions for ground effect in hover are generalized to forward flight by using $(z_g/\cos\epsilon)$ in place of z_g. No ground effect correction is applied if the wake is directed upward ($\cos\epsilon \le 0$), or if $z_g/\cos\epsilon > 1.5D$. From $z_g/D\cos\epsilon$, the ground effect factor $f_g = \kappa_g^{-3/2}$ is calculated. Then

$$(\lambda_i)_{\text{IGE}} = f_g\,(\lambda_i)_{\text{OGE}}$$

is the effective ideal induced velocity.

Several empirical ground effect models are implemented: from Cheeseman and Bennett (ref. 1, basic model and using blade-element (BE) theory to incorporate influence of thrust), from Law (ref. 2), from Hayden (ref. 3), and a curve fit of the interpolation from Zbrozek (ref. 4):

$$f_g = \begin{cases} \left[1 - \dfrac{1}{(4z_g/R)^2}\right]^{3/2} & \text{Cheeseman and Bennett} \\[3ex] \left[1 + 1.5\,\dfrac{\sigma a\lambda_i}{4C_T}\dfrac{1}{(4z_g/R)^2}\right]^{-3/2} & \text{Cheeseman and Bennett (BE)} \\[3ex] \left[\dfrac{1.0991 - 0.1042/(z_g/D)}{1 + (C_T/\sigma)(0.2894 - 0.3913/(z_g/D))}\right]^{3/2} & \text{Law} \\[3ex] \left[0.9926 + \dfrac{0.03794}{(z_g/2R)^2}\right]^{-1} & \text{Hayden} \\[3ex] \left[0.9122 + \dfrac{0.0544}{(z_g/R)\sqrt{C_T/\sigma}}\right]^{-3/2} & \text{Zbrozek} \end{cases}$$

These equations break down at small height above the ground, and so are restricted to $z_g/D \ge 0.15$; however, the data base for ground effect extends only to about $z/D = 0.3$. Also, $f_g \le 1$ is required. Figure 11-2 shows $T/T_\infty = \kappa_g = f_g^{-2/3}$ as a function of z/R for these models ($C_T/\sigma = 0.05, 0.10, 0.15$), compared with test data from several sources.

11-4.1.3 Inflow Gradient

As a simple approximation to nonuniform induced velocity distribution, a linear variation over the disk is used: $\Delta\lambda = \lambda_x r \cos\psi + \lambda_y r \sin\psi$. There are contributions to $\Delta\lambda$ from forward flight and from
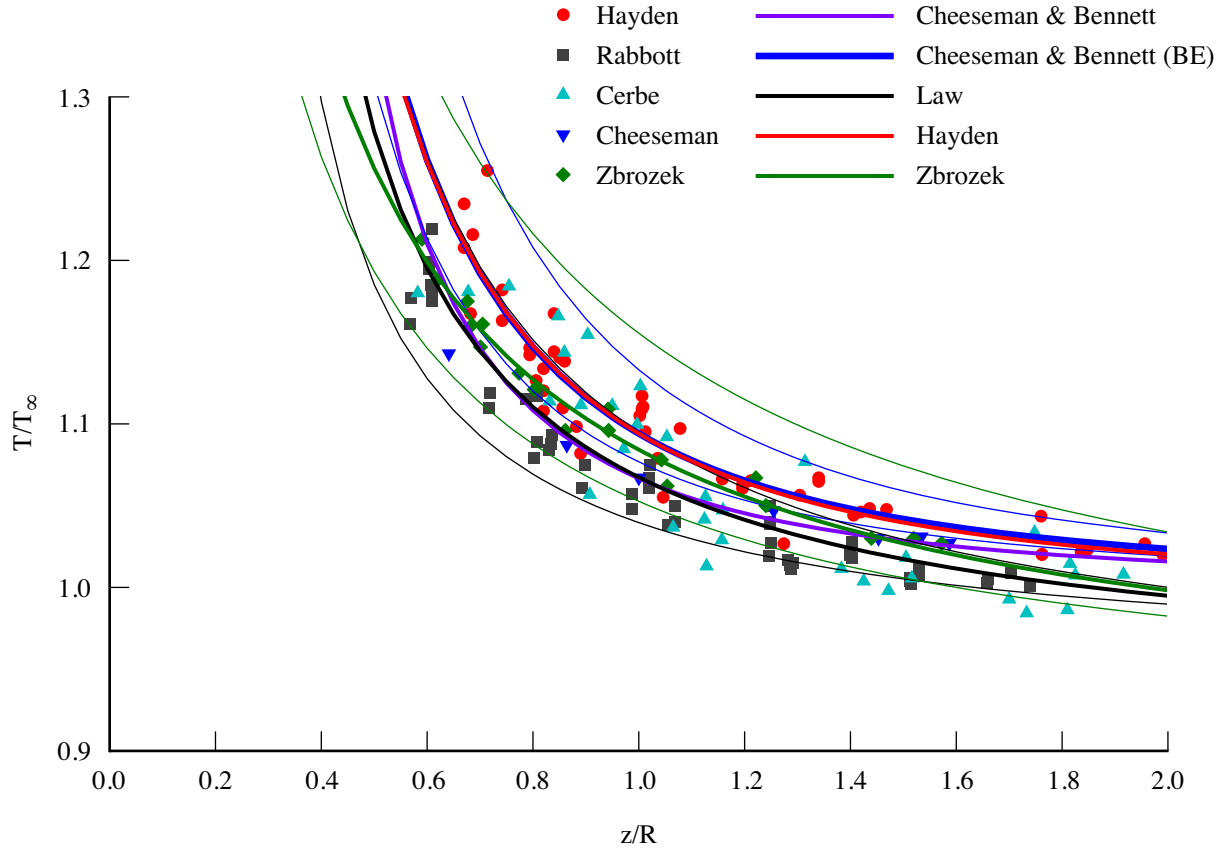
Figure 11-2. Ground effect models (hover).

hub moments, which influence the relationship between flapping and cyclic. The linear inflow variation caused by forward flight is $\Delta\lambda_f = \lambda_i(\kappa_x r \cos\psi + \kappa_y r \sin\psi)$, where λ_i is the mean inflow. Typically κ_x is positive, and roughly 1 at high speed; and κ_y is smaller in magnitude and negative. Both κ_x and κ_y must be zero in hover. Based on references 5–8, the following models are considered:

$$\text{Coleman and Feingold:} \quad \kappa_{x0} = f_x \frac{15\pi}{32} \tan\chi/2 = f_x \frac{15\pi}{32} \frac{\mu}{\sqrt{\mu^2 + \lambda^2} + |\lambda|}$$

$$\kappa_{y0} = -f_y 2\mu$$

$$\text{White and Blake:} \quad \kappa_{x0} = f_x \sqrt{2} \sin\chi = f_x \sqrt{2} \frac{\mu}{\sqrt{\mu^2 + \lambda^2}}$$

$$\kappa_{y0} = -2f_y \mu$$

where $\tan\chi = |\lambda|/\mu$ is the wake angle. Extending these results to include sideward velocity gives $\kappa_x = (\kappa_{x0}\mu_x + \kappa_{y0}\mu_y)/\mu$ and $\kappa_y = (-\kappa_{x0}\mu_y + \kappa_{y0}\mu_x)/\mu$. For flexibility, the empirical factors f_x and f_y have been introduced (values of 1.0 give the baseline model). There is also an inflow variation produced by any net aerodynamic moment on the rotor disk, which can be evaluated using a differential form of momentum theory:

$$\Delta\lambda_m = \frac{f_m}{\sqrt{\mu^2 + \lambda^2}} \left(-2C_{My} r \cos\psi + 2C_{Mx} r \sin\psi\right) = \lambda_{xm} r \cos\psi + \lambda_{ym} r \sin\psi$$

including empirical factor f_m. Note that the denominator of the hub moment term is zero for a hovering rotor at zero thrust; so this inflow contribution should not be used for cases of low speed and low thrust.

11-4.2   Rotor Forces

When direct control of the rotor thrust magnitude is used, the rotor collective pitch angle $\theta_{0.75}$ must be calculated from the thrust C_T/σ. If the commanded variable is the collective pitch angle, then it is necessary to calculate the rotor thrust, resulting in more computation, particularly since all quantities depending on the thrust (inflow, induced power factor, mean drag coefficient) are also unknown. There may be flight states where the commanded thrust can not be produced by the rotor, even with stall neglected in the section aerodynamics. This condition will manifest as an inability to solve for the collective pitch given the thrust. In this circumstance the trim method can be changed so the required or specified thrust is an achievable value, or commanded collective pitch control can be used.

Cyclic control consists of tip-path plane command, requiring calculation of the rotor cyclic pitch angles from the flapping; or no-feathering plane command, requiring calculation of the tip-path plane tilt from the cyclic control angles. The longitudinal tip-path plane tilt is β_c (positive forward) and the lateral tilt is β_s (positive toward retreating side). The longitudinal cyclic pitch angle is θ_s (positive aft), and the lateral cyclic pitch angle is θ_c (positive toward retreating side). Tip-path plane command is appropriate for main rotors. For rotors with no cyclic pitch, no-feathering plane command must be used.

The forces acting on the hub are the thrust T, drag H, and side force Y (positive in z, x, y-axis directions, respectively). The aerodynamic analysis is conducted for a clockwise rotating rotor, with appropriate sign changes for lateral velocity, flapping, and force. The analysis is conducted in dimensionless form, based on the actual radius and rotational speed of the flight state. The inplane hub forces are produced by tilt of the thrust vector with the tip-path plane, plus forces in the tip-path plane, and profile terms (produced by the blade drag coefficient). The orientation of the tip-path axes relative to the shaft axes is then $C^{PS} = X_{r\beta_s} Y_{-\beta_c}$. Then

$$\begin{pmatrix} C_H \\ C_Y \\ C_T \end{pmatrix} = C^{SP} \begin{pmatrix} 0 \\ 0 \\ C_T/C_{33}^{SP} \end{pmatrix} + \begin{pmatrix} C_{H\mathrm{tpp}} \\ rC_{Y\mathrm{tpp}} \\ 0 \end{pmatrix} + \begin{pmatrix} C_{Ho} \\ rC_{Yo} \\ 0 \end{pmatrix}$$

The inplane forces relative to the tip-path plane can be neglected, or calculated by blade element theory. Note that with thrust and tip-path plane command and $C_{H\mathrm{tpp}}$ and $C_{Y\mathrm{tpp}}$ neglected, it is not necessary to solve for the rotor collective and cyclic pitch angles. In general the inplane forces relative to the tip-path plane are not zero, and may be significant, as for a rotor with large flap stiffness. Figures 11-3a and b show respectively the tip-path plane tilt and thrust vector tilt with cyclic pitch control (no-feathering plane tilt), as functions of flap stiffness (frequency), for several rotor thrust values. The difference between tip-path plane tilt (figure 11-3a) and thrust vector tilt (figure 11-3b) is caused by tilt of the thrust vector relative to the tip-path plane.

The profile inplane forces can be obtained from simplified equations, or calculated by blade element theory. The simplified method uses:

$$\begin{pmatrix} C_{Ho} \\ C_{Yo} \end{pmatrix} = \frac{\sigma}{8} c_{d\mathrm{mean}} F_H \begin{pmatrix} \mu_x/\mu \\ -\mu_y/\mu \end{pmatrix}$$

where the mean drag coefficient $c_{d\mathrm{mean}}$ is from the profile power calculation. The function F_H accounts for the increase of the blade section velocity with rotor edgewise and axial speed: $C_{Ho} =$
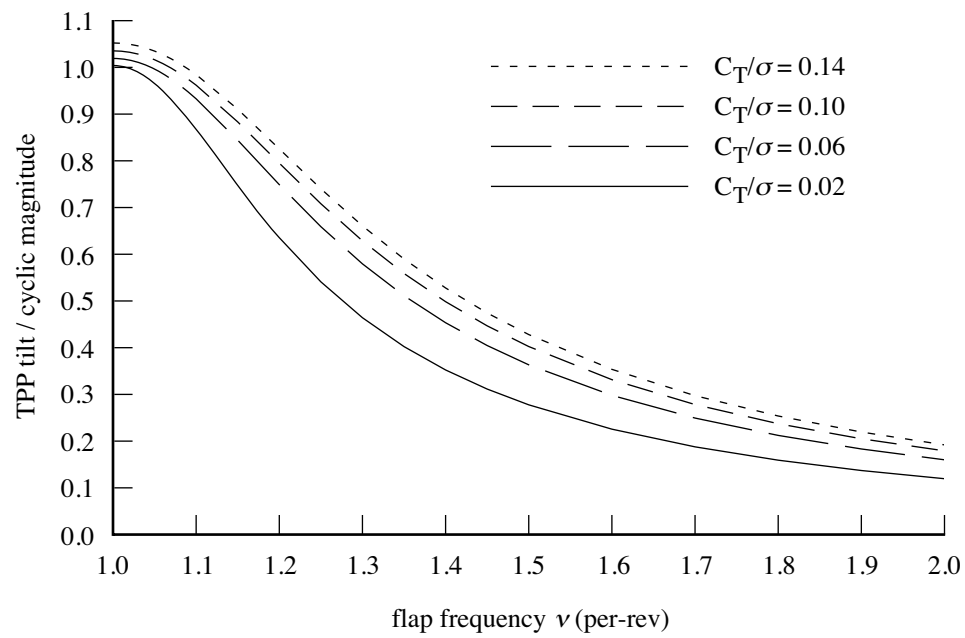
Figure 11-3a. Tip-path plane tilt with cyclic pitch.
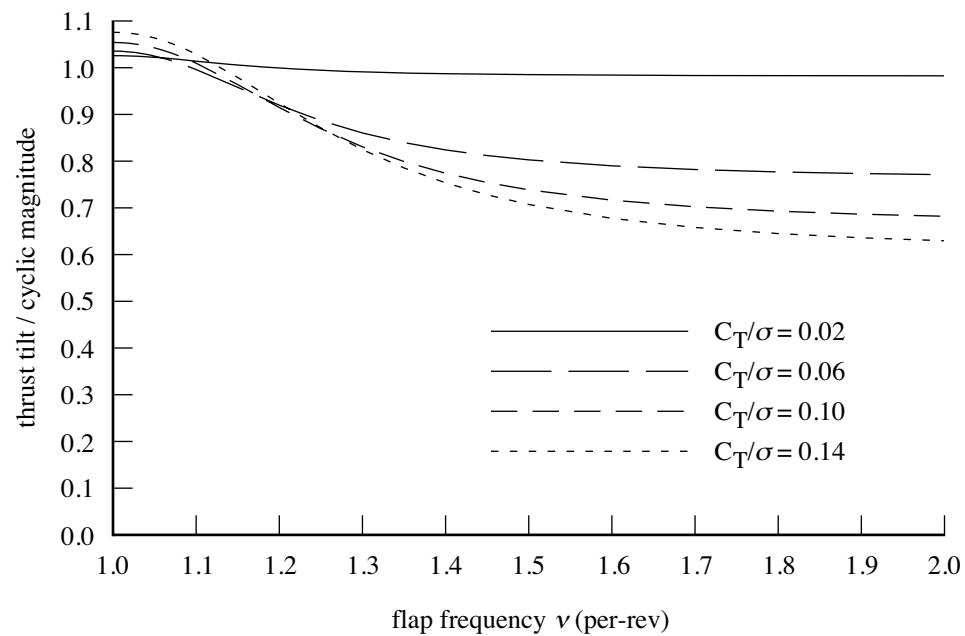
Figure 11-3b. Thrust vector tilt with cyclic pitch.

$\int \frac{1}{2}\sigma c_d U(r\sin\psi + \mu)dr = \int \frac{1}{2}\sigma c_d(u_T^2 + u_R^2 + u_P^2)^{1/2}(r\sin\psi + \mu)dr$; so (from Ref. 9)

$$F_H = 4\frac{1}{2\pi}\int_0^{2\pi}\int_0^1 \left((r + \mu\sin\psi)^2 + (\mu\cos\psi)^2 + \mu_z^2\right)^{1/2}(r\sin\psi + \mu)\,dr\,d\psi$$

$$\cong \sqrt{1+V^2}\left(3\mu + \frac{1}{4}\mu^3\frac{V^2-1}{(1+V^2)^2}\right) + \left(\mu\mu_z^2 + \frac{3}{4}\mu^3\right)\ln\left[\frac{\sqrt{1+V^2}+1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$.

11-4.3   Blade Element Theory

Blade element theory is the basis for the solution for the collective and cyclic pitch angles (or thrust and flap angles) and evaluation of the rotor inplane hub forces. The section aerodynamics are described by lift varying linearly with angle-of-attack, $c_\ell = c_{\ell\alpha}\alpha$ (no stall), and a constant mean drag coefficient $c_{d\text{mean}}$ (from the profile power calculation). The analysis is conducted in dimensionless form (based on density ρ, rotor rotational speed Ω, and blade radius R of the flight state). So in the following σ, ν, and γ are for the actual R and Ω; and $a = 5.7$ is the lift-curve slope used in the Lock number γ. The blade section aerodynamic environment is described by the three components of velocity, from which the yaw and inflow angles are obtained, and then the angle-of-attack:

$$u_T = r + \mu_x\sin\psi + \mu_y\cos\psi$$
$$u_R = \mu_x\cos\psi - \mu_y\sin\psi$$
$$u_P = \lambda + r(\dot\beta + \dot\alpha_x\sin\psi - \dot\alpha_y\cos\psi) + u_R\beta$$

$$U^2 = u_T^2 + u_P^2$$
$$\cos\Lambda = U/\sqrt{u_T^2 + u_P^2 + u_R^2}$$
$$\phi = \tan^{-1}u_P/u_T$$
$$\alpha = \theta - \phi$$

In reverse flow ($|\alpha| > 90$), $\alpha \leftarrow \alpha - 180\,\text{sign}\alpha$, and then $c_\ell = c_{\ell\alpha}\alpha$ still (airfoil tables are not used). The blade pitch consists of collective, cyclic, twist, and pitch-flap coupling terms. The flap motion is rigid rotation about a hinge with no offset, and only coning and once-per-revolution terms are considered:

$$\theta = \theta_{0.75} + \theta_{tw} + \theta_c\cos\psi + \theta_s\sin\psi - K_P\beta$$
$$\beta = \beta_0 + \beta_c\cos\psi + \beta_s\sin\psi$$

where $K_P = \tan\delta_3$. The twist is measured relative to $0.75R$; $\theta_{tw} = \theta_L(r - 0.75)$ for linear twist. The mean inflow is $\lambda_0 = \kappa\lambda_i$, using the induced velocity factor κ from the induced power model. The inflow includes gradients caused by edgewise flight and hub moments:

$$\lambda = \mu_z + \lambda_0(1 + \kappa_x r\cos\psi + \kappa_y r\sin\psi) + \Delta\lambda_m$$
$$= \mu_z + \lambda_0(1 + \kappa_x r\cos\psi + \kappa_y r\sin\psi) + \frac{f_m}{\sqrt{\mu^2 + \lambda^2}}\left(-2C_{My}r\cos\psi + 2C_{Mx}r\sin\psi\right)$$

From the hub moments

$$\begin{pmatrix}-C_{My}\\ C_{Mx}\end{pmatrix} = \frac{\sigma a}{2}\frac{\nu^2 - 1}{\gamma}\begin{pmatrix}\beta_c\\ \beta_s\end{pmatrix}$$

the inflow gradient is

$$\Delta\lambda_m = \left(\frac{f_m}{\sqrt{\mu^2 + \lambda^2}}\frac{\sigma a}{8}\right)\frac{\nu^2 - 1}{\gamma/8}\left(r\beta_c\cos\psi + r\beta_s\sin\psi\right) = K_m\frac{\nu^2 - 1}{\gamma/8}\left(r\beta_c\cos\psi + r\beta_s\sin\psi\right)$$

The constant K_m is associated with a lift-deficiency function:

$$C = \frac{1}{1 + K_m} = \frac{1}{1 + f_m \sigma a/(8\sqrt{\mu^2 + \lambda^2})}$$

The blade chord is $c(r) = c_{\text{ref}}\hat{c}(r)$, where c_{ref} is the thrust-weighted chord (chord at $0.75R$ for linear taper). Yawed flow effects increase the section drag coefficient, hence $c_d = c_{d\text{mean}}/\cos\Lambda$. The section forces in velocity axes and shaft axes are

$$L = \frac{1}{2}\rho U^2 cc_\ell \qquad\qquad F_z = L\cos\phi - D\sin\phi = \frac{1}{2}\rho Uc(c_\ell u_T - c_d u_P)$$

$$D = \frac{1}{2}\rho U^2 cc_d \qquad\qquad F_x = L\sin\phi + D\cos\phi = \frac{1}{2}\rho Uc(c_\ell u_P + c_d u_T)$$

$$R = \frac{1}{2}\rho U^2 cc_r = D\tan\Lambda \qquad\qquad F_r = -\beta F_z + R = -\beta F_z + \frac{1}{2}\rho Ucc_d u_R$$

These equations for the section environment and section forces are applicable to high inflow (large μ_z), sideward flight (μ_y), and reverse flow ($u_T < 0$). The total forces on the rotor hub are

$$T = N\int F_z\, dr$$

$$H = N\int F_x \sin\psi + F_r \cos\psi\, dr$$

$$Y = N\int -F_x \cos\psi + F_r \sin\psi\, dr$$

with an average over the rotor azimuth implied, along with the integration over the radius. Lift forces are integrated from the root cutout r_{root} to the tip loss factor B. Drag forces are integrated from the root cutout to the tip.

In coefficient form (forces divided by $\rho A V_{\text{tip}}^2$) the rotor thrust and inplane forces are:

$$C_T = \sigma\int \widehat{F}_z\, dr \qquad\qquad \widehat{F}_z = \frac{1}{2}\hat{c}U(c_\ell u_T - c_d u_P)$$

$$C_H = \sigma\int \widehat{F}_x \sin\psi + \widehat{F}_r \cos\psi\, dr \qquad\qquad \widehat{F}_x = \frac{1}{2}\hat{c}U(c_\ell u_P + c_d u_T)$$

$$C_Y = \sigma\int -\widehat{F}_x \cos\psi + \widehat{F}_r \sin\psi\, dr \qquad\qquad \widehat{F}_r = -\beta\widehat{F}_z + \frac{1}{2}\hat{c}Uc_d u_R$$

(and the sign of C_Y is changed for a clockwise rotating rotor). The terms $\Delta\widehat{F}_x \cong \widehat{F}_z\dot{\beta}$ and $\Delta\widehat{F}_r = -\widehat{F}_z\beta$ produce tilt of the thrust vector with the tip-path plane ($C_H = -C_T\beta_c$ and $C_Y = -C_T\beta_s$), which are accounted for directly. The section drag coefficient c_d produces the profile inplane forces. The approximation $u_P \cong \mu_z$ is consistent with the simplified method (using the function F_H), hence

$$\widehat{F}_{xo} = \frac{1}{2}\hat{c}U_0 c_d u_T \qquad C_{Ho} = \sigma\int \widehat{F}_{xo}\sin\psi + \widehat{F}_{ro}\cos\psi\, dr = \frac{\sigma}{2}\int \hat{c}U_0 c_d(r\sin\psi + \mu_x)\, dr$$

$$\widehat{F}_{ro} = \frac{1}{2}\hat{c}U_0 c_d u_R \qquad C_{Yo} = \sigma\int -\widehat{F}_{xo}\cos\psi + \widehat{F}_{ro}\sin\psi\, dr = -\frac{\sigma}{2}\int \hat{c}U_0 c_d(r\cos\psi + \mu_y)\, dr$$

where $U_0^2 = u_T^2 + \mu_z^2$, and $c_d = c_{d\text{mean}}/\cos\Lambda$. Using blade element theory to evaluate C_{Ho} and C_{Yo} accounts for the planform ($\hat{c}$) and root cutout. Using the function F_H implies a rectangular blade and no

root cutout (plus at most a 1% error approximating the exact integration). The remaining terms in the section forces produce the inplane loads relative to the tip-path plane:

$$\widehat{F}_{xi} = \widehat{F}_x - \widehat{F}_z\beta - \widehat{F}_{xo} = \frac{1}{2}\hat{c}Uc_\ell(u_P - u_T\dot{\beta}) + \frac{1}{2}\hat{c}Uc_d((1 - U_0/U)u_T + u_P\dot{\beta})$$

$$\widehat{F}_{ri} = \widehat{F}_r + \widehat{F}_z\beta - \widehat{F}_{ro} = \frac{1}{2}\hat{c}Uc_d(1 - U_0/U)u_R$$

$$C_{H\text{tpp}} = \sigma \int \; \widehat{F}_{xi}\sin\psi + \widehat{F}_{ri}\cos\psi \, dr$$

$$C_{Y\text{tpp}} = \sigma \int \; -\widehat{F}_{xi}\cos\psi + \widehat{F}_{ri}\sin\psi \, dr$$

(including small profile terms from $U_0 \neq U$).

Evaluating these inplane forces requires the collective and cyclic pitch angles and the flapping motion. The thrust equation must be solved for the rotor collective pitch or the rotor thrust. The relationship between cyclic pitch and flapping is defined by the rotor flap dynamics. The flap motion is rigid rotation about a central hinge, with a flap frequency $\nu > 1$ for articulated or hingeless rotors. The flapping equation of motion is

$$\ddot{\beta} + \nu^2\beta + 2\dot{\alpha}_y\sin\psi + 2\dot{\alpha}_x\cos\psi = \frac{\gamma}{a}\int \widehat{F}_z r \, dr + (\nu^2 - 1)\beta_p$$

including precone angle β_p; the Lock number $\gamma = \rho a c_{\text{ref}} R^4/I_b$. This equation is solved for the mean (coning) and 1/rev (tip-path plane tilt) flap motion:

$$\nu^2\beta_0 = \frac{\gamma}{a}\int \widehat{F}_z r \, dr + (\nu^2 - 1)\beta_p$$

$$(\nu^2 - 1)\begin{pmatrix} \beta_c \\ \beta_s \end{pmatrix} = \frac{\gamma}{a}\int \widehat{F}_z r \, dr \begin{pmatrix} 2\cos\psi \\ 2\sin\psi \end{pmatrix} + \begin{pmatrix} 2\dot{\alpha}_x \\ 2\dot{\alpha}_y \end{pmatrix}$$

with an average over the rotor azimuth implied. The solution for the coning is largely decoupled by introducing the thrust:

$$\nu_0^2\beta_0 = \frac{\gamma}{8}\frac{6C_T}{\sigma a} + (\nu_0^2 - 1)\beta_p + \frac{\gamma}{a}\int \widehat{F}_z(r - 3/4)\, dr$$

A separate flap frequency ν_0 is used for coning, in order to model teetering and gimballed rotors. For an articulated rotor, $\beta_p = 0$ should be used.

The thrust and flapping equations of motion that must be solved are:

$$E_t = \frac{6}{a}\int \widehat{F}_z \, dr - \frac{6C_T}{\sigma a}$$

$$\begin{pmatrix} E_c \\ E_s \end{pmatrix} = \frac{8}{a}\int \widehat{F}_z r \, dr \begin{pmatrix} 2\cos\psi \\ 2\sin\psi \end{pmatrix} - \frac{\nu^2 - 1}{\gamma/8}\begin{pmatrix} \beta_c \\ \beta_s \end{pmatrix} + \frac{16}{\gamma}\begin{pmatrix} \dot{\alpha}_x \\ \dot{\alpha}_y \end{pmatrix}$$

The solution v such that $E(v) = 0$ is required. The variables are $v = (\theta_{0.75}\, \theta_c\, \theta_s)^T$ for thrust and tip-path plane command; $v = (\theta_{0.75}\, \beta_c\, \beta_s)^T$ for thrust and no-feathering plane command; $v = (C_T/\sigma\, \theta_c\, \theta_s)^T$ for collective pitch and tip-path plane command; $v = (C_T/\sigma\, \beta_c\, \beta_s)^T$ for collective pitch and no-feathering plane command. Note that since $c_\ell = c_{\ell\alpha}\alpha$ is used (no stall), these equations are linear in θ. However, if $\partial T/\partial\theta_{0.75}$ is small, the solution may not produce a reasonable collective for commanded thrust. A

Newton–Raphson solution method is used: from $E(v_{n+1}) \cong E(v_n) + (dE/dv)(v_{n+1} - v_n) = 0$, the iterative solution is

$$v_{n+1} = v_n - C\,E(v_n)$$

where $C = f(dE/dv)^{-1}$, including the relaxation factor f. The derivative matrix dE/dv is obtained by numerical perturbation. Convergence of the Newton–Raphson iteration is tested in terms of $|E| < \epsilon$ for each equation, where ϵ is an input tolerance.

11–5 Power

The rotor power consists of induced, profile, interferenc, and parasite terms: $P = P_i + P_o + P_t + P_p$. The parasite power (including climb/descent power for the aircraft) is obtained from the wind axis drag force: $P_p = -XV = (v^F)^T F^F$.

The interference power can be produced by interactions from the wing. The component of the wing interference velocity v_{ind}^F parallel to the rotor force vector F^F (velocity roughly normal to the rotor disk) produces a power change $P_t \cong v_{\text{ind}}^T F$. The component of the interference velocity perpendicular to the rotor force vector (velocity roughly in the plane of the rotor disk) produces interference through the swirl, hence $P_t \propto (V/\Omega R)|v_{\text{ind}}||F|$. Thus the interference power is calculated from

$$P_t = -K_{\text{int}n}v_{\text{ind}}^T F + K_{\text{int}p}\frac{V}{\Omega R}\sqrt{(|v_{\text{ind}}||F|)^2 - (v_{\text{ind}}^T F)^2}$$

Separate interference factors K_{int} are used for the two terms. $K_{\text{int}p}$ is negative for favorable interference.

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. The empirical factor κ accounts for the effects of nonuniform inflow, non-ideal span loading, tip losses, swirl, blockage, and other phenomena that increase the induced power losses ($\kappa > 1$). For a ducted fan, $f_D = f_W/2$ is introduced. The induced power at zero thrust is zero in this model (or accounted for as a profile power increment). If κ is deduced from an independent calculation of induced power, nonzero P_i at low thrust will be reflected in large κ values.

The profile power is calculated from a mean blade drag coefficient: $P_o = \rho A(\Omega R)^3 C_{Po}$, $C_{Po} = (\sigma/8)c_{d\text{mean}}F_P$. The function $F_P(\mu, \mu_z)$ accounts for the increase of the blade section velocity with rotor edgewise and axial speed: $C_{Po} = \int \frac{1}{2}\sigma c_d U^3 dr = \int \frac{1}{2}\sigma c_d(u_T^2 + u_R^2 + u_P^2)^{3/2}dr$; so (from Ref. 9)

$$F_P = 4\frac{1}{2\pi}\int_0^{2\pi}\int_0^1 \left((r + \mu\sin\psi)^2 + (\mu\cos\psi)^2 + \mu_z^2\right)^{3/2} dr\,d\psi$$

$$\cong \sqrt{1 + V^2}\left(1 + \frac{5}{2}V^2 + \frac{3}{8}\mu^2\frac{4 + 7V^2 + 4V^4}{(1 + V^2)^2} - \frac{9}{16}\frac{\mu^4}{1 + V^2}\right)$$

$$+ \left(\frac{3}{2}\mu_z^4 + \frac{3}{2}\mu_z^2\mu^2 + \frac{9}{16}\mu^4\right)\ln\left[\frac{\sqrt{1 + V^2} + 1}{V}\right]$$

with $V^2 = \mu^2 + \mu_z^2$. This expression is exact when $\mu = 0$, and $f_P \sim 4V^3$ for large V.

Two performance methods are implemented, the energy method and the table method. The induced power factor and mean blade drag coefficient are obtained from equations with the energy method, or from tables with the table method. Optionally κ and $c_{d\text{mean}}$ can be specified for each flight state, superseding the performance method values.

11-5.1 Energy Performance Method

11-5.1.1 Induced Power

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. Reference values of κ are specified for hover, axial cruise (propeller), and edgewise cruise (helicopter): κ_{hover}, κ_{prop}, κ_{edge}. Two models are implemented: constant model and standard model. The constant model uses $\kappa = \kappa_{\text{hover}}$ if $\mu = \mu_z = 0$; or $\kappa = \kappa_{\text{prop}}$ if $|\mu| < 0.1|\mu_z|$; or $\kappa = \kappa_{\text{edge}}$ otherwise.

The standard model calculates an axial flow factor κ_{axial} from κ_{hover}, κ_{climb}, and κ_{prop}. Let $\Delta = C_T/\sigma - (C_T/\sigma)_{\text{ind}}$. For hover and low speed axial climb, including a variation with thrust, the inflow factor is

$$\kappa_h = \kappa_{\text{hover}} + k_{h1}\Delta_h + k_{h2}|\Delta_h|^{X_{h2}} + (\kappa_{\text{climb}} - \kappa_{\text{hover}})\frac{2}{\pi}\tan^{-1}\left[((|\mu_z|/\lambda_h)/M_{\text{axial}})^{X_{\text{axial}}}\right]$$

where $|\mu_z|/\lambda_h = M_{\text{axial}}$ is the midpoint of the transition between hover and climb, and X_{axial} is large for a fast transition. Figure 11-4 illustrates κ in hover (with a minimum value). Figure 11-5 shows the behavior of this function for a helicopter in climb ($X_{\text{axial}} = 0.65$). A polynomial describes the variation with axial velocity, scaled so $\kappa = \kappa_h$ at $\mu_z = 0$ and $\kappa = \kappa_p$ at $\mu_z = \mu_{z\text{prop}}$. Including variations with thrust and shaft angle:

$$\kappa_p = \kappa_{\text{prop}} + k_{p1}\Delta_p + k_{p2}|\Delta_p|^{X_{p2}} + k_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}}$$

$$\kappa_{\text{axial}} = \kappa_h + k_{a1}\mu_z + S(k_{a2}\mu_z^2 + k_{a3}|\mu_z|^{X_a})$$

where $S = (\kappa_p - (\kappa_h + k_{a1}\mu_{z\text{prop}}))/(k_{a2}\mu_{z\text{prop}}^2 + k_{a3}\mu_{z\text{prop}}^{X_a})$; $S = 0$ if $k_{a2} = k_{a3} = 0$ (not scaled); $\kappa_{\text{axial}} = \kappa_h$ if $\mu_{z\text{prop}} = 0$. A polynomial describes the variation with edgewise advance ratio, scaled so $\kappa = \kappa_{\text{axial}}$ at $\mu = 0$ and $\kappa = f_\alpha f_{\text{off}}\kappa_{\text{edge}}$ at $\mu = \mu_{\text{edge}}$. Thus the induced power factor is

$$\kappa = \kappa_{\text{axial}} + k_{e1}\mu + S(k_{e2}\mu^2 + k_{e3}\mu^{X_e})$$

where $S = (f_\alpha f_{\text{off}}\kappa_{\text{edge}} - (\kappa_{\text{axial}} + k_{e1}\mu_{\text{edge}}))/(k_{e2}\mu_{\text{edge}}^2 + k_{e3}\mu_{\text{edge}}^{X_e})$; $S = 0$ if $k_{e2} = k_{e3} = 0$ (not scaled); $\kappa = \kappa_{\text{axial}}$ if $\mu_{\text{edge}} = 0$. The function $f_\alpha = 1 - k_{e\alpha}\mu_z$ accounts for the influence of angle-of-attack (μ_z/μ) or rotor drag (C_X). The function $f_{\text{off}} = 1 - k_{o1}(1 - e^{-k_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$. Figure 11-6 illustrates κ in edgewise flight. Minimum and maximum values of the induced power factor, κ_{min} and κ_{max}, are also specified.

11-5.1.2 Profile Power

The profile power is calculated from a mean blade drag coefficient: $C_{Po} = (\sigma/8)c_{d\text{mean}}F_P$. Since the blade mean lift coefficient is $c_\ell \cong 6C_T/\sigma$, the drag coefficient is estimated as a function of blade loading C_T/σ (using thrust-weighted solidity). With separate estimates of the basic, stall, and compressibility drag, the mean drag coefficient is:

$$c_{d\text{mean}} = \chi S\left(c_{d\text{basic}} + c_{d\text{stall}} + c_{d\text{comp}}\right)$$

where χ is a technology factor. The factor $S = (Re_{\text{ref}}/Re)^{0.2}$ accounts for Reynolds number effects on the drag coefficient; Re is based on the thrust-weighted chord, $0.75V_{\text{tip}}$, and the flight state; and Re_{ref} corresponds to the input c_d information. The following models are implemented for the basic drag:

a) Array model: The basic drag $c_{d\text{basic}}$ is input as a function of C_T/σ; the array is linearly interpolated.
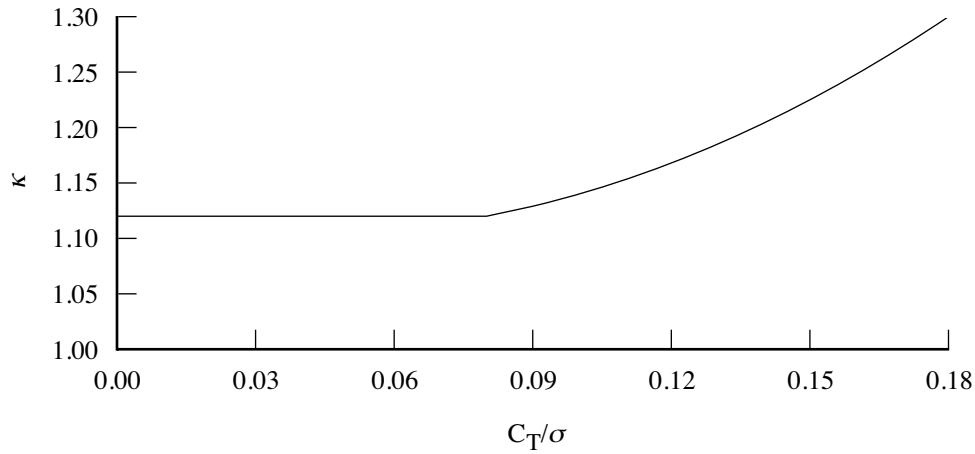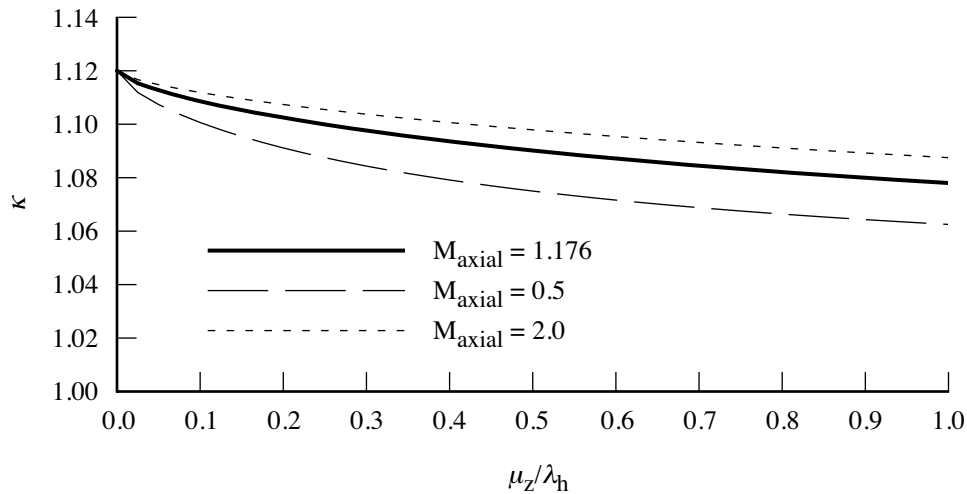
Figure 11-4. Induced power factor for rotor in hover.

Figure 11-5. Induced power factor for rotor in axial flight.

b) Equation model: The basic drag $c_{d\text{basic}}$ is a quadratic function of C_T/σ, plus an additional term allowing faster growth at high (sub-stall) angles of attack. Let $\Delta = |C_T/\sigma - (C_T/\sigma)_{D\text{min}}|$, where $(C_T/\sigma)_{D\text{min}}$ corresponds to the minimum drag and $\Delta_{\text{sep}} = |C_T/\sigma| - (C_T/\sigma)_{\text{sep}}$. Values of the basic drag equation are specified for helicopter (hover and edgewise) and propeller (axial climb and cruise) operation:

$$c_{dh} = d_{0\text{hel}} + d_{1\text{hel}}\Delta + d_{2\text{hel}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}}$$

$$c_{dp} = d_{0\text{prop}} + d_{1\text{prop}}\Delta + d_{2\text{prop}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}} + d_{p\alpha}\mu_z^2|\mu|^{X_{p\alpha}}$$

The separation term is present only if $\Delta_{\text{sep}} > 0$. The helicopter and propeller values are interpolated as
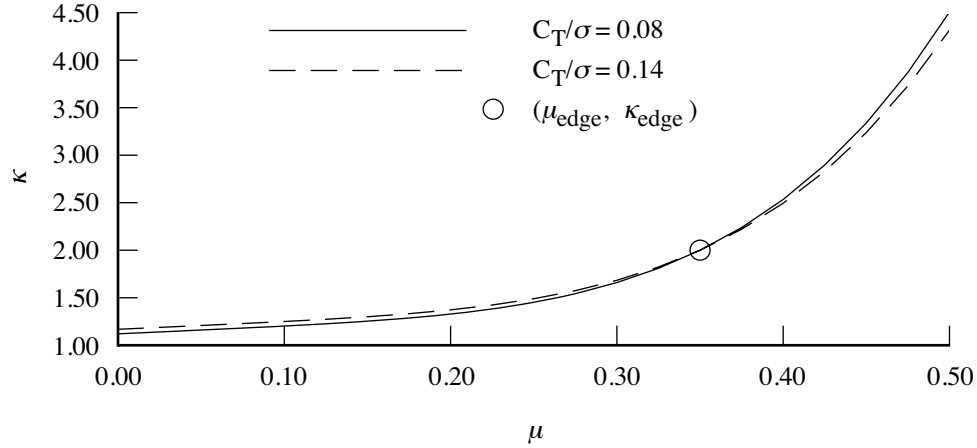
Figure 11-6. Induced power factor for rotor in edgewise flight.

a function of μ_z:

$$c_{d\text{basic}} = c_{dh} + (c_{dp} - c_{dh})\frac{2}{\pi}\tan^{-1}(|\mu_z|/\lambda_h)$$

so $|\mu_z|/\lambda_h = 1$ is the midpoint of the transition.

The stall drag increment represents the rise of profile power caused by the occurrence of significant stall on the rotor disk. Let $\Delta_s = |C_T/\sigma| - (f_s/f_\alpha f_{\text{off}})(C_T/\sigma)_s$ (f_s is an input factor). The function $f_\alpha = 1 - d_{s\alpha}\mu_z$ accounts for the influence of angle-of-attack (μ_z/μ) or rotor drag (C_X). The function $f_{\text{off}} = 1 - d_{o1}(1 - e^{-d_{o2}o_x})$ accounts for the influence of lift offset, $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$. Then $c_{d\text{stall}} = d_{s1}\Delta_s^{X_{s1}} + d_{s2}\Delta_s^{X_{s2}}$ (zero if $\Delta_s \leq 0$). The blade loading at which the stall affects the entire rotor power, $(C_T/\sigma)_s$, is an input function of the velocity ratio $V = \sqrt{\mu^2 + \mu_z^2}$.

The compressibility drag increment depends on the advancing tip Mach number M_{at}, and the tip airfoil thickness-to-chord ratio τ. The following models are implemented:

a) Drag divergence model: Let $\Delta M = M_{at} - M_{dd}$, where M_{dd} is the drag divergence Mach number of the tip section. Then the compressibility increment in the mean drag coefficient is

$$c_{d\text{comp}} = d_{m1}\Delta M + d_{m2}\Delta M^{X_m}$$

(ref. 10). M_{dd} is a function of the advancing tip lift coefficient, $c_{\ell(1,90)}$. The advancing tip lift is estimated from $\alpha_{(1,90)} = (\theta_{.75} + 0.25\theta_L + \theta_s - (\lambda - \beta_c)/(1 + \mu)) \cong 1.6(1 - 2.97\mu + 2.21\mu^2)(6C_T/\sigma a) + 0.25\theta_{tw}$ (zero above $\mu = 0.6$). Then the Korn expression (ref. 11) gives M_{dd} for small lift coefficient:

$$M_{dd} = \kappa_A - \kappa|c_\ell| - \tau = M_{dd0} - \kappa|c_\ell|$$

where M_{dd0} is the drag divergence Mach number at zero lift, and typically $\kappa = 0.16$.

b) Similarity model: From transonic small-disturbance theory (refs. 12–13), the scaled wave drag must be a function only of $K_1 = (M_{at}^2 - 1)/[M_{at}^2\tau(1 + \gamma)]^{2/3}$. An approximation for the wave drag increment is

$$\Delta c_d = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}}D(K_1) = \frac{\tau^{5/3}}{[M_{at}^2(1 + \gamma)]^{1/3}}1.774(K_1 + 1.674)^{5/2}$$
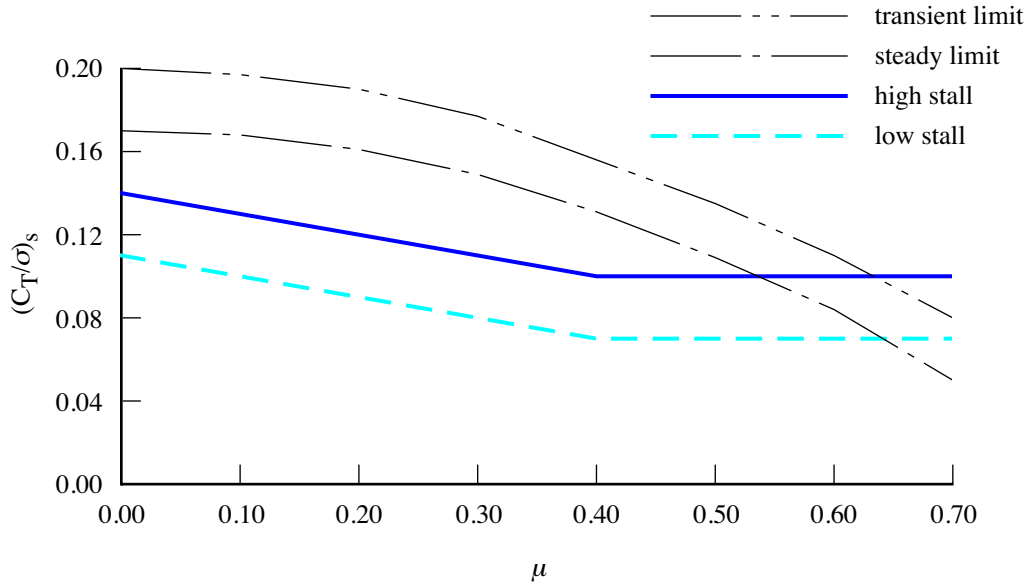
Figure 11-7. Stall function.

(constant for $K_1 > -0.2$). Integration of Δc_d over the rotor disk gives the compressibility increment in the profile power. Following Harris, the resulting compressibility increment in the mean drag coefficient is approximately:

$$c_{d\text{comp}} = 1.52 f(K_1 + 1)^2 [(1 + \mu)\tau]^{5/2}(1 + \gamma)^{1/2}$$

including the input correction factor f; $c_{d\text{comp}}$ is zero for $K_1 < -1$, and constant for $K_1 > -0.2$.

Figure 11-7 shows typical stall functions $(C_T/\sigma)_s$ for two rotors with different airfoils, and for reference typical helicopter rotor steady and transient load limits. Figure 11-8 illustrates the mean drag coefficient in hover, showing c_{dh} without and with the separation term, and the total for the high stall and low stall cases. Figure 11-9 illustrates the mean drag coefficient in forward flight, showing the compressibility term $c_{d\text{comp}}$, and the growth in profile power with C_T/σ and μ as the stall drag increment increases.

11-5.1.3 Twin Rotors

For twin rotors, the induced power is determined by the induced velocity of the rotor system, not the individual rotors. The induced power is still obtained using $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$ for each rotor, but the ideal induced velocity is calculated for an equivalent thrust C_{Te} based on the thrust and geometry of both rotors. The profile power calculation is not changed for twin rotors.

In hover, the twin rotor induced velocity is $v_i = \kappa_{\text{twin}} \sqrt{T/2\rho A_p}$, from the total thrust T and the projected disk area $A_p = (2 - m)A$. The overlap fraction m is calculated from the rotor hub separation ℓ. A correction factor for the twin rotor ideal power is also included. For a coaxial rotor, typically $\kappa_{\text{twin}} \cong 0.90$. So the ideal inflow is calculated for $C_{Te} = (C_{T1} + C_{T2})/(2 - m)$.

In forward flight, the induced velocity of a coaxial rotor is $v_i = \kappa_{\text{twin}} T/(2\rho AV)$, from the total
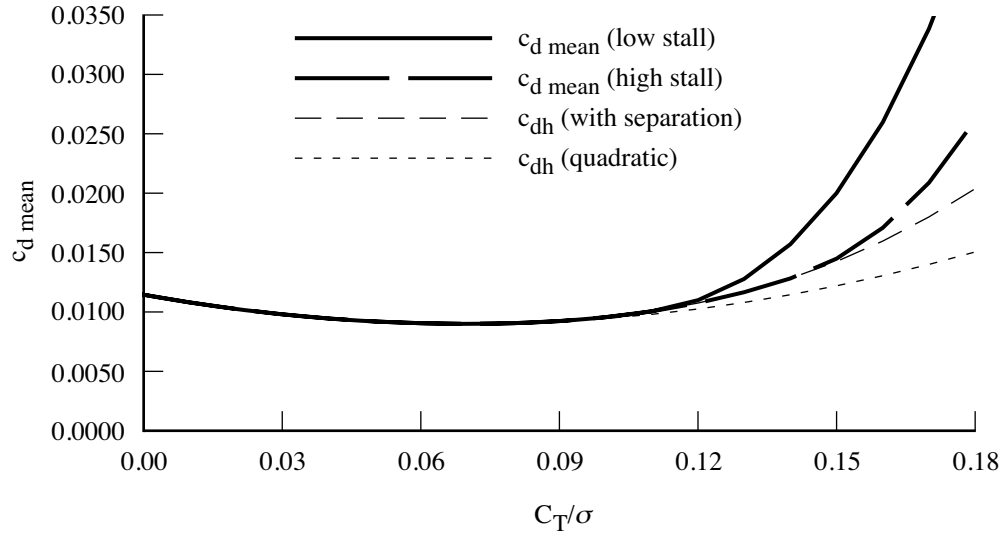
Figure 11-8. Mean drag coefficient for rotor in hover.
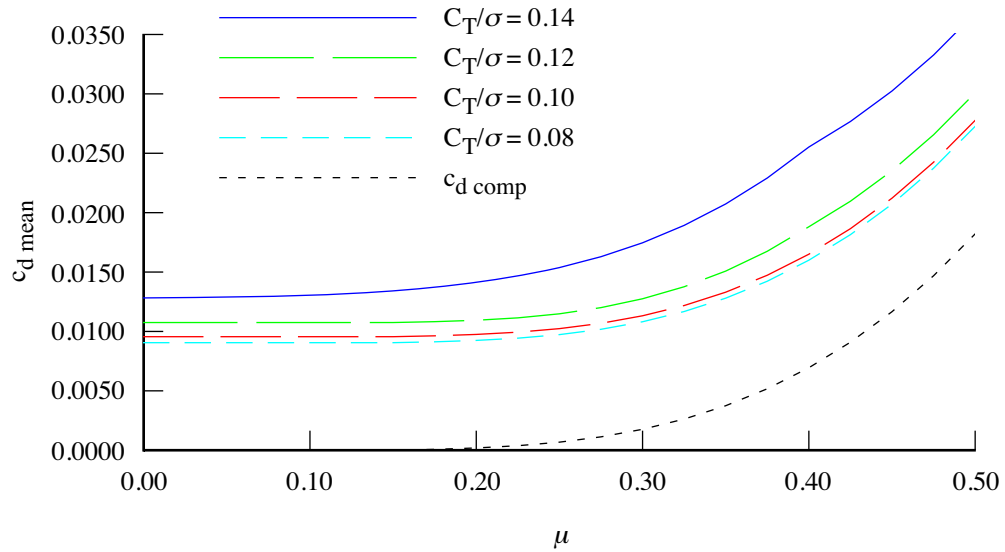
Figure 11-9a. Mean drag coefficient for rotor in forward flight, high stall.

thrust T and a span of $2R$. The correction factor for ideal induced power (biplane effect) is $\kappa_{\text{twin}} \cong 0.88$ to 0.81 for rotor separations of $0.06D$ to $0.12D$. The ideal inflow is thus calculated for $C_{Te} = C_{T1} + C_{T2}$. The induced velocity of side-by-side rotors is $v_i = \kappa_{\text{twin}} T/(2\rho A_e V)$, from the total thrust T and a span of $2R + \ell$, hence $A_e = A(1 + \ell/2R)^2$. The ideal inflow is thus calculated for $C_{Te} = (C_{T1} + C_{T2})/(1 + \ell/2R)^2$. The induced velocity of tandem rotors is $v_F = \kappa_{\text{twin}}(T_F/(2\rho AV) + x_R T_R/(2\rho AV))$ for the front rotor and $v_R = \kappa_{\text{twin}}(T_R/(2\rho AV) + x_F T_F/(2\rho AV))$ for the rear rotor. For large separation, $x_R \cong 0$ and $x_F \cong 2$; for the coaxial limit $x_R = x_F = 1$ is appropriate. Here $x_R = m$ and $x_F = 2 - m$ is used.
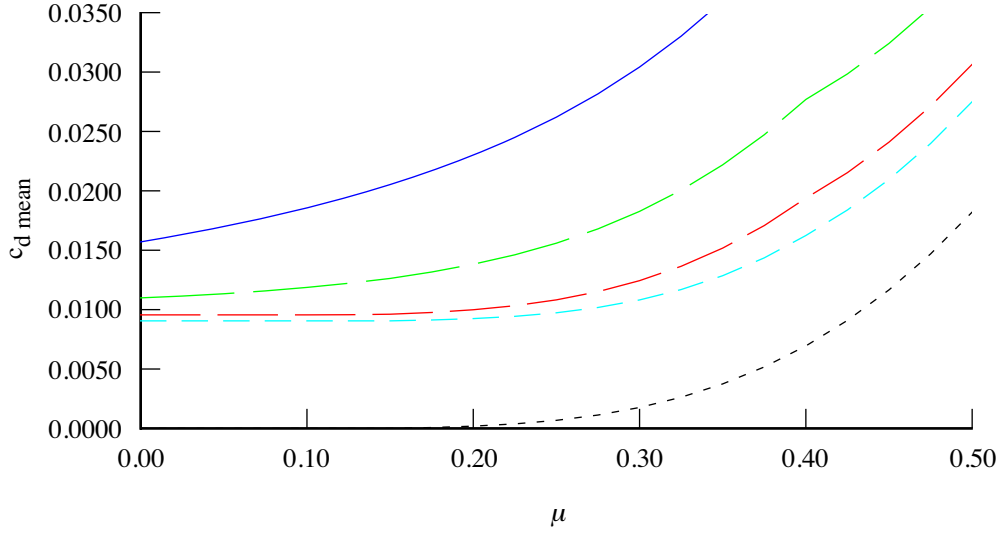
Figure 11-9b. Mean drag coefficient for rotor in forward flight, low stall.

To summarize, the model for twin rotor ideal induced velocity uses $C_{Te} = x_1 C_{T1} + x_2 C_{T2}$ and the correction factor κ_{twin}. In hover, $x_h = 1/(2-m)$; in forward flight of coaxial and tandem rotors, $x_f = 1$ for this rotor and $x_f = m$ or $x_f = 2-m$ for the other rotor; in forward flight of side-by-side rotors, $x_f = 1/(1+\ell/2R)^2$ ($x = 1/2$ if there is no overlap, $\ell/2R > 1$). The transition between hover and forward flight is accomplished using

$$x = \frac{x_f \mu^2 + x_h C \lambda_h^2}{\mu^2 + C \lambda_h^2}$$

with typically $C = 1$ to 4. This transition is applied to x for both rotors, and to κ_{twin}.

With a coaxial rotor in hover, the lower rotor acts in the contracted wake of the upper rotor. Momentum theory gives the ideal induced power for coaxial rotors with large vertical separation (ref. 14): $P_u = T_u v_u$, $v_u^2 = T_u/2\rho A$ for the upper rotor; and $P_\ell = (\bar{a} s/\sqrt{\tau}) T_\ell v_\ell$, $v_\ell^2 = T_\ell/2\rho A$ for the lower rotor. Here $\tau = T_\ell/T_u$; $\bar{a}$ is the average of the disk loading weighted by the induced velocity, hence a measure of nonuniform loading on the lower rotor ($\bar{a} = 1.05$ to 1.10 typically); and the momentum theory solution is

$$\frac{\bar{a} s}{\sqrt{\tau}} = \frac{1}{2\tau^{3/2}} \left(\sqrt{1 + 4(1+\tau)^2 \bar{a} \tau} - 1 \right)$$

The optimum solution for equal power of the upper and lower rotors is $\bar{a} s \tau = 1$, giving $\tau = T_\ell/T_u \cong 2/3$. Hence for the coaxial rotor in hover the ideal induced velocity is calculated from $C_{Te} = C_{Tu}$ for the upper rotor and from $C_{Te} = (\bar{a} s/\sqrt{\tau})^2 C_{T\ell}$ for the lower rotor, with $\kappa_{\text{twin}} = 1$. Thus $x_h = 1/(2-m) = 1/2$ and the input hover κ_{twin} is not used, unless the coaxial rotor is modeled as a tandem rotor with zero longitudinal separation.

11-5.2   Table Performance Method

The induced power is calculated from the ideal power: $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$. The induced power factor κ is obtained from an input table (linearly interpolated) that can be a function of edgewise advance ratio μ or axial velocity ratio μ_z, and of blade loading C_T/σ.

The profile power is calculated from a mean blade drag coefficient: $P_o = \rho A(\Omega R)^3 C_{Po} = \rho A(\Omega R)^3 \frac{\sigma}{8} c_d F_P$. The mean drag coefficient c_d, or alternatively $c_d F_P = 8C_{Po}/\sigma$, is obtained from an input table (linearly interpolated) that can be a function of edgewise advance ratio μ and blade loading C_T/σ.

11–6 Performance Indices

Several performance indices are calculated for each rotor. The induced power factor is $\kappa = P_i/P_{\text{ideal}}$. The rotor mean drag coefficient is $c_d = (8C_{Po}/\sigma)/F_P$, using the function $F(\mu, \mu_z)$ given above. The rotor effective lift-to-drag ratio is a measure of the induced and profile power: $L/D_e = VL/(P_i + P_o)$. The hover figure of merit is $M = Tf_D v/P$. The propeller propulsive efficiency is $\eta = TV/P$. These two indices can be combined as a momentum efficiency: $\eta_{\text{mom}} = T(V+w/2)/P$, where $w/2 = f_W v/2 = f_D v$.

11–7 Interference

The rotor can produce aerodynamic interference velocities at the other components (fuselage, wings, tails). The induced velocity at the rotor disk is κv_i, acting opposite the thrust (z-axis of tip-path plane axes). So $v_{\text{ind}}^P = -k^P \kappa v_i$, and $v_{\text{ind}}^F = C^{FP} v_{\text{ind}}^P$. The total velocity of the rotor disk relative to the air consists of the aircraft velocity and the induced velocity from this rotor: $v_{\text{total}}^F = v^F - v_{\text{ind}}^F$. The direction of the wake axis is thus $e_w^P = -C^{PF} v_{\text{total}}^F/|v_{\text{total}}^F|$ (for zero total velocity, $e_w^P = -k^P$ is used). The angle of the wake axis from the thrust axis is $\chi = \cos^{-1} |(k^P)^T e_w^P|$.

The interference velocity v_{int}^F at each component is proportional to the induced velocity v_{ind}^F (hence is in the same direction), with factors accounting for the stage of wake development and the position of the component relative to the rotor wake. The far wake velocity is $w = f_W v_i$, and the contracted wake area is $A_c = \pi R_c^2 = A/f_A$. The solution for the ideal inflow gives f_W and f_A. For an open rotor, $f_W = 2$. For a ducted rotor, the inflow and wake depend on the wake area ratio f_A, or on the ratio of the rotor thrust to total thrust: $f_T = T_{\text{rotor}}/T$. The corresponding velocity and area ratios at an arbitrary point on the wake axis are f_w and f_a, related by

$$f_a = \sqrt{\frac{\mu^2 + (\mu_z + f_w \lambda_i)^2}{(f_{Vx}\mu)^2 + (f_{Vz}\mu_z + \lambda_i)^2}}$$

Vortex theory for hover gives the variation of the induced velocity with distance z below the rotor disk:

$$v = v(0)\left(1 + \frac{z/R}{\sqrt{1 + (z/R)^2}}\right)$$

With this equation the velocity varies from zero far above the disk to $v = 2v(0)$ far below the disk. To use this expression in edgewise flow and for ducted rotors, the distance z/R is replaced by ζ_w/tR, where ζ_w is the distance along the wake axis, and the parameter t is introduced to adjust the rate of change (t small for faster transition to far wake limit). Hence the velocity inside the wake is $f_w v_i$, where

$$f_w = f_W f_z = \begin{cases} 1 + \dfrac{\zeta_w/tR}{\sqrt{1 + (\zeta_w/tR)^2}} & \zeta_w < 0 \\[3mm] 1 + (f_W - 1)\dfrac{\zeta_w/tR}{\sqrt{1 + (\zeta_w/tR)^2}} & \zeta_w > 0 \end{cases}$$

and the contracted radius is $R_c = R/\sqrt{f_a}$.

The wake is a skewed cylinder, starting at the rotor disk and with the axis oriented by e_w^P. The interference velocity is required at the position z_B^F on a component. Whether this point is inside or outside the wake cylinder is determined by finding its distance from the wake axis, in a plane parallel to the rotor disk. The position relative to the rotor hub is $\xi_B^P = C^{PF}(z_B^F - z_{\text{hub}}^F)$; the corresponding point on the wake axis is $\xi_A^P = e_w^P \zeta_w$. Requiring ξ_B^P and ξ_A^P have the same z value in the tip-path plane axes gives

$$\zeta_w = \frac{(k^P)^T C^{PF}(z_B^F - z_{\text{hub}}^F)}{(k^P)^T e_w^P}$$

from which f_z, f_w, f_a, and R_c are evaluated. The distance r from the wake axis is then

$$r^2 = \left((i^P)^T(\xi_B^P - \xi_A^P)\right)^2 + \left((j^P)^T(\xi_B^P - \xi_A^P)\right)^2$$

The transition from full velocity inside the wake to zero velocity outside the wake is accomplished in the distance sR_c, using

$$f_r = \begin{cases} 1 & r \leq R_c \\ 1 - (r - R_c)/(sR_c) & \\ 0 & r \geq (1+s)R_c \end{cases}$$

($s = 0$ for an abrupt transition, s large for always in wake).

The interference velocity at the component (at z_B^F) is calculated from the induced velocity v_{ind}^F, the factors $f_W f_z$ accounting for axial development of the wake velocity, the factor f_r accounting for immersion in the wake, and an input empirical factor K_{int}:

$$v_{\text{int}}^F = K_{\text{int}} \, f_W f_z f_r f_t \, v_{\text{ind}}^F$$

An additional factor f_t for twin rotors is included. Optionally the development along the wake axis can be a step function ($f_W f_z = 0$, 1, f_W above the rotor, on the rotor disk, and below the rotor disk, respectively); nominal ($t = 1$); or use an input rate parameter t. Optionally the wake immersion can use the contracted radius R_c or the uncontracted radius R; can be a step function ($s = 0$, so $f_r = 1$ and 0 inside and outside the wake boundary); can be always immersed ($s = \infty$ so $f_r = 1$ always); or can use an input transition distance s. Optionally the interference factor K_{int} can be reduced from an input value at low speed to zero at high speed, with linear variation over a specified speed range.

To account for the extent of the wing or tail area immersed in the rotor wake, the interference velocity is calculated at several points along the span and averaged. The increment in position is $\Delta z_B^F = C^{FB}(0 \, \Delta y \, 0)^T$, $\Delta y = (b/2)(-1 + (2i-1)/N)$ for $i = 1$ to N; where b is the wing span. The average interference is calculated separately for each wing panel (left and right), by interpolating the interference velocity at N points along the wing to $N/2$ points along the panel span.

For twin main rotors (tandem, side-by-side, or coaxial), the performance may be calculated for the rotor system, but the interference velocity is still calculated separately for each rotor, based on its disk loading. At the component, the velocities from all rotors are summed, and the total used to calculate the angle-of-attack and dynamic pressure. This sum must give the interference velocity of the twin rotor system, which requires the correction factor f_t. Consider differential momentum theory to estimate the induced velocity of twin rotors in hover. For the first rotor, the thrust and area in the non-overlap region are $(1-m)T_1$ and $(1-m)A$, hence the induced velocity is $v_1 = \kappa\sqrt{T_1/2\rho A}$; similarly $v_2 = \kappa\sqrt{T_2/2\rho A}$. In the overlap region the thrust and area are $mT_1 + mT_2$ and mA, hence the induced

velocity is $v_m = \kappa\sqrt{(T_1 + T_2)/2\rho A}$. So for equal thrust, the velocity in the overlap region (everywhere for the coaxial configuration) is $\sqrt{2}$ larger. The factor K_T is introduced to adjust the overlap velocity: $v_m = \kappa(K_T/\sqrt{2})\sqrt{(T_1 + T_2)/2\rho A}$. The interference velocities are calculated separately for the two rotors, with the correction factor f_t: $v_{\text{int1}} = f_t\kappa\sqrt{T_1/2\rho A}$ and $v_{\text{int2}} = f_t\kappa\sqrt{T_2/2\rho A}$. The sum $v_{\text{int1}} + v_{\text{int2}}$ must take the required value. Below the non-overlap region, the component is in the wake of only one of the rotors, so the interference velocity from the other rotor is zero, and thus $f_t = 1$. Below the overlap region, the component is in the wake of both rotors, and the sum of the interference velocities equals v_m if

$$f_{th} = \frac{K_T/\sqrt{2}}{\sqrt{\tau_1} + \sqrt{\tau_2}}$$

where $\tau_n = T_n/(T_1 + T_2)$ is the thrust ratio. For equal thrusts, $f_{th} = K_T/2$; or $f_{th} = 1/\sqrt{2}$ for the nominal velocity. The expression $f_t = f_{th}\cos^2\chi + \sin^2\chi$ gives the required correction factor, with $f_t = 1$ in edgewise flight. Optionally the correction for twin rotors can be omitted ($f_t = 1$); nominal ($K_T = \sqrt{2}$); or use an input velocity factor in overlap region (K_T).

11–8   Drag

The rotor component includes drag forces acting on the hub and spinner (at z_{hub}^F) and on the pylon (at z_{pylon}^F). The component drag contributions must be consistent. In particular, a rotor with a spinner (such as on a tiltrotor aircraft) would likely not have hub drag. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The body axes for the drag analysis are rotated about the y-axis relative to the rotor shaft axes: $C^{BF} = C^{BS}C^{SF}$, where $C^{BS} = Y_{-\theta_{\text{ref}}}$. The pitch angle θ_{ref} can be input, or the rotation appropriate for a helicopter rotor or a propeller can be specified.

> a) Consider a helicopter rotor, with the shaft axes oriented z-axis up and x-axis downstream. It is appropriate that the angle-of-attack is $\alpha = 0$ for forward flight and $\alpha = -90$ deg for hover, meaning that the body axes are oriented z-axis down and x-axis forward. Hence $\theta_{\text{ref}} = 180$ deg.
> b) Consider a propeller or tiltrotor, with the shaft axes oriented z-axis forward and x-axis up. It is appropriate that the angle-of-attack is $\alpha = 0$ in cruise and $\alpha = 90$ deg for helicopter mode (with a tilting pylon), meaning that the body axes are oriented z-axis down and x-axis forward. Hence $\theta_{\text{ref}} = 90$ deg.

The aerodynamic velocity relative to the air is calculated in component axes, v^B. The angle-of-attack α and dynamic pressure q are calculated from v^B. The reference areas for the drag coefficients are the rotor disk area $A = \pi R^2$ (for hub drag), pylon wetted area S_{pylon}, and spinner wetted area S_{spin}; these areas are input or calculated as described above.

The hub drag can be fixed, specified as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on the rotor disk area $A = \pi R^2$; or the drag can be estimated based on the gross weight, using a squared-cubed relationship or a square-root relationship. Based on historical data, the drag coefficient $C_D = 0.004$ for typical hubs, $C_D = 0.0024$ for current low drag hubs, and $C_D = 0.0015$ for faired hubs. For the squared-cubed relationship: $(D/q)_{\text{hub}} = k((W_{MTO}/N_{\text{rotor}})/1000)^{2/3}$ (W_{MTO}/N_{rotor} is the maximum takeoff gross weight per lifting rotor; units of k are ft²/k-lb²/³ or m²/Mg²/³). Based on

historical data, $k = 1.4$ for typical hubs, $k = 0.8$ for current low drag hubs, and $k = 0.5$ for faired hubs (English units). For the square-root relationship: $(D/q)_{\text{hub}} = k\sqrt{W_{MTO}/N_{\text{rotor}}}$ (W_{MTO}/N_{rotor} is the maximum takeoff gross weight per lifting rotor; units of k are $\text{ft}^2/\text{lb}^{1/2}$ or $\text{m}^2/\text{kg}^{1/2}$); based on historical data (ref. 15), $k = 0.074$ for single rotor helicopters, $k = 0.049$ for tandem rotor helicopters (probably a blade number effect), $k = 0.038$ for hingeless rotors, and $k = 0.027$ for faired hubs (English units). To handle multi-rotor aircraft, the scaling weight $w = W_{MTO}/N_{\text{rotor}}$ is calculated as for disk loading: $w = f_W W_{MTO}$ for main rotors or $w = fT$ for antitorque and auxiliary-thrust rotors.

The hub vertical drag can be fixed, specified as a drag area D/q; or the drag can be scaled, specified as a drag coefficient C_D based on the rotor disk area $A = \pi R^2$.

The pylon forward flight drag and vertical drag are specified as drag area or drag coefficient, based on the pylon wetted area. The spinner drag is specified as drag area or drag coefficient, based on the spinner wetted area.

The drag coefficient for the hub or pylon at angle-of-attack α is

$$C_D = C_{D0} + (C_{DV} - C_{D0})|\sin\alpha|^{X_d}$$

Optionally the variation can be quadratic ($X_d = 2$). For sideward flight, $C_{D\text{hub}} = C_{D0}$ for the hub and $C_{D\text{pylon}} = C_{DV}$ for the pylon. Then the total component drag force is

$$D = qAC_{D\text{hub}} + qS_{\text{pylon}}C_{D\text{pylon}} + qS_{\text{spin}}C_{D\text{spin}}$$

The force and moment produced by the drag are

$$F^F = \sum e_d D$$
$$M^F = \sum \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z_{cg}^F$ (separate locations are defined for the rotor hub and for the pylon), and e_d is the drag direction. The velocity relative to the air gives $e_d = -v^F/|v^F|$ (no interference).

11–9   Weights

The rotor configuration determines where the weights occur in the weight statement, as summarized in table 11-3. The rotor group consists of blade assembly, hub and hinge, fairing/spinner, blade fold structure, and inter-rotor shaft. The tail rotor (in empennage group) or the propeller/fan installation (in propulsion group) consists of the blade assembly, the hub and hinge, and the rotor/fan duct and rotor support.

There are separate weight models for main rotors, tail rotors, and auxiliary-thrust systems (propellers). The tail rotor model requires a torque calculated from the drive system rated power and main rotor rotational speed: $Q = P_{DS\text{limit}}/\Omega_{mr}$. The auxiliary-thrust model requires the design maximum thrust of the propeller.

Table 11-3. Principal configuration designation.

configuration	weight statement	weight model	performance model
main rotor	rotor group	rotor	rotor
tail rotor	empennage group	tail rotor	rotor
propeller	propulsion group	rotor, aux thrust	rotor

The flap moment of inertia I_b and the Lock number $\gamma = \rho a c R^4 / I_b$ are required for the blade motion solution. Several options are implemented to calculate I_b. The Lock number can be specified, and then $I_b = \rho a c R^4 / \gamma$ used, independent of the blade weight; this is the only option for the tail rotor and auxiliary thrust weight models, which do not give separate blade and hub weight estimates. The moment of inertia I_b can be calculated from the blade weight and the weight distribution. The Lock number can be specified, hence $I_b = \rho a c R^4 / \gamma$, and then mass added to the blade to achieve this value. An autorotation index $AI = KE/P = \frac{1}{2} N I_b \Omega^2 / P$ can be specified, hence the required I_b, and then mass added to the blade to achieve this value. Reference 16 describes this and other autorotation indices; $AI = KE/P \geq 3$ sec gives good autorotation characteristics for small helicopters.

In order to increase the moment of inertia, a tip weight W_t can be added to each blade at radial station r_t. Thus the total blade weight is $W_b = \chi w_b + dW_b + (1+f)W_t N$ (lb or kg); where w_b is the blade weight estimate, χ the technology factor, dW_b a specified weight increment; and the factor f accounts for the blade weight increase required by the centrifugal force due to W_t. The mass per blade is $M_b = W_b / N$ (slug or kg), or M_{b0} without the tip weight. The blade moment of inertia is

$$I_b = R^2(r_2^2(M_{b0} + fM_t) + r_t^2 M_t) = I_{b0} + R^2(r_t^2 + fr_2^2)M_t$$

where r_2 is the radius of gyration of the distributed mass. Typically $r_2 \cong 0.6$; $r_2 = 1/\sqrt{3} = 0.577$ for uniform mass distribution. If the required moment of inertia I_b is greater than I_{b0}, the tip mass M_t is needed. Additional mass is required inboard to react the centrifugal force increase due to M_t. This additional mass is less effective than M_t at increasing I_b. Assume a fraction a of the blade mass reacts the centrifugal force F, so $\Delta M / M_{b0} = a \Delta F / F_0$. The reference values are $M_{b0} = R \int m \, dr$ and $F_0 = \Omega^2 R^2 \int rm \, dr = \Omega^2 R r_1 M_{b0}$, where $r_1 \cong \frac{1}{2}$. Then

$$\Delta M = a \frac{M_{b0}}{F_0} \Delta F = a \frac{1}{\Omega^2 R r_1} \left(\Omega^2 R r_1 \Delta M + \Omega^2 R r_t M_t \right) = a \left(\Delta M + \frac{r_t}{r_1} M_t \right) = \frac{a r_t / r_1}{1 - a} M_t = f M_t$$

With $a \cong \frac{1}{3}$, $f \cong 1$. The tip mass required to produce $\Delta I_b = I_b - I_{b0}$ is $M_t = \Delta I_b / (R^2(r_t^2 + fr_2^2))$, and the total blade weight increment is $\Delta W_b = (1 + f)M_t N$ (lb or kg).

11–10   References

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Force

The force component is an object that can generate a force acting on the aircraft, possibly used for lift, propulsion, or control. The amplitude of the force can be a fixed value, or it can be connected to an aircraft control for trim. The direction of the force can be fixed or connected to aircraft control.

12–1   Control and Loads

The control variables are the force amplitude A, and the force incidence and yaw angles. The force orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x, y, or z-axis), and then applying a yaw angle ψ, then an incidence or tilt angle i (table 12-1). The control variables can be connected to the aircraft controls c_{AC}:

$$A = A_0 + T_A c_{AC}$$
$$\psi = \psi_0 + T_\psi c_{AC}$$
$$i = i_0 + T_i c_{AC}$$

with A_0, ψ_0 and i_0 zero, constant, or a function of flight speed (piecewise linear input). The force axes are $C^{BF} = U_i V_\psi$, where U and V depend on the nominal direction, as described in table 12-1. The force direction is $e_f = C^{FB} e_{f0}$. The force acts at position z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = e_f A$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$.

Table 12-1. Force orientation.

nominal (F axes)		e_{f0}	incidence, + for force	yaw, + for force	$C^{BF} = U_i V_\psi$
x	forward	i	up	right	$Y_i Z_\psi$
$-x$	aft	$-i$	up	right	$Y_{-i} Z_{-\psi}$
y	right	j	aft	up	$Z_i X_{-\psi}$
$-y$	left	$-j$	aft	up	$Z_{-i} X_\psi$
z	down	k	aft	right	$Y_{-i} X_{-\psi}$
$-z$	up	$-k$	aft	right	$Y_i X_\psi$

12–2   Performance and Weights

The force generation requires a fuel flow that is calculated from an input thrust-specific fuel consumption: $\dot{w} = A(\text{sfc})$. Units of sfc are pound/hour/pound, or kilogram/hour/Newton.

The force component weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group. The force component weight is calculated from specific weight and the design maximum force $F_{\max}$, plus a fixed increment: $W = SF_{\max} + \Delta W$.

Wing

The aircraft can have one or more wings, or no wings.

13–1   Geometry

The wing is described by planform area S, span b, mean chord $c = S/b$, and aspect ratio $A\!R = b^2/S$. These parameters are for the entire wing. The geometry is specified in terms of two of the following parameters: S or wing loading W/S, b (perhaps calculated from other geometry), c, $A\!R = b^2/S$. With more than one wing, the wing loading is obtained from an input fraction of design gross weight, $W = f_W W_D$. Optionally the span can be calculated from a specified ratio to the span of another wing; or the span can be calculated from a specified ratio to the radius of a designated rotor, $b = 2fR$. Optionally the wing span can be calculated from an appropriate specification of all wing panel widths.

Optionally for the tiltrotor configuration, the wing span can be calculated from the fuselage and rotor geometry: $b = 2(fR + d_{\text{fus}}) + w_{\text{fus}}$, where R is the rotor radius (cruise value for variable-diameter rotor), d_{fus} the rotor-fuselage clearance, and w_{fus} the fuselage width. Note that the corresponding option for the rotor hub position is $y_{\text{hub}} = \pm(fR + d_{\text{fus}} + {}^1\!/2 w_{\text{fus}})$. Optionally the wing span can be calculated from the rotor hub position: $b = 2|y_{\text{hub}}|$ (regardless of how the rotor position is determined). As implemented, symmetry is not assumed; rather the radius or hub position of the outermost designated rotors is used.

The wing is at position z^F, where the aerodynamic forces act. The component axes are the aircraft body axes, $C^{BF} = I$.

The wing planform is defined in terms of one or more wing panels (figure 13-1). Symmetry of the wing is assumed. The number of panels is P, with the panel index $p = 1$ to P. The wing span station η is scaled with the semi-span: $y = \eta(b/2), \eta = 0$ to 1. Each panel is a trapezoid, with a straight aerodynamic center and linear taper. The aerodynamic center locus (in wing axes) is defined by sweep Λ_p; dihedral δ_p; and offsets (x_{Ip}, z_{Ip}) at the inboard edge relative to the aerodynamic center of the previous panel. The wing position z^F is the mean aerodynamic center. The offset $(\bar{x}_A, \bar{z}_A)$ of the mean aerodynamic center from the root chord aerodynamic center is calculated (so the wing planform can be drawn; typically the aerodynamic center is drawn as the quarter-chord). Outboard panel edges are at η_{Ep} (input or calculated). A panel is characterized by span b_p (each side), mean chord c_p, and area $S_p = 2b_p c_p$ (both sides). The taper is defined by inboard and outboard chord ratios, $\lambda = c/c_{\text{ref}}$ (where c_{ref} is a panel or wing reference chord, depending on the options for describing the geometry).

The span for each panel (if there are more than two panels) can be a fixed input; a fixed ratio of the wing span, $b_p = f_{bp}(b/2)$; or free. The panel outboard edge (except at the wing tip) can be at a fixed input position y_{Ep}; at a fixed station $\eta_{Ep}, y_p = \eta_{Ep}(b/2)$; calculated from the rotor radius, $y_p = fR$; calculated from the fuselage and rotor geometry, $y_p = fR + d_{\text{fus}} + {}^1\!/2 w_{\text{fus}}$ (for a designated rotor); calculated from
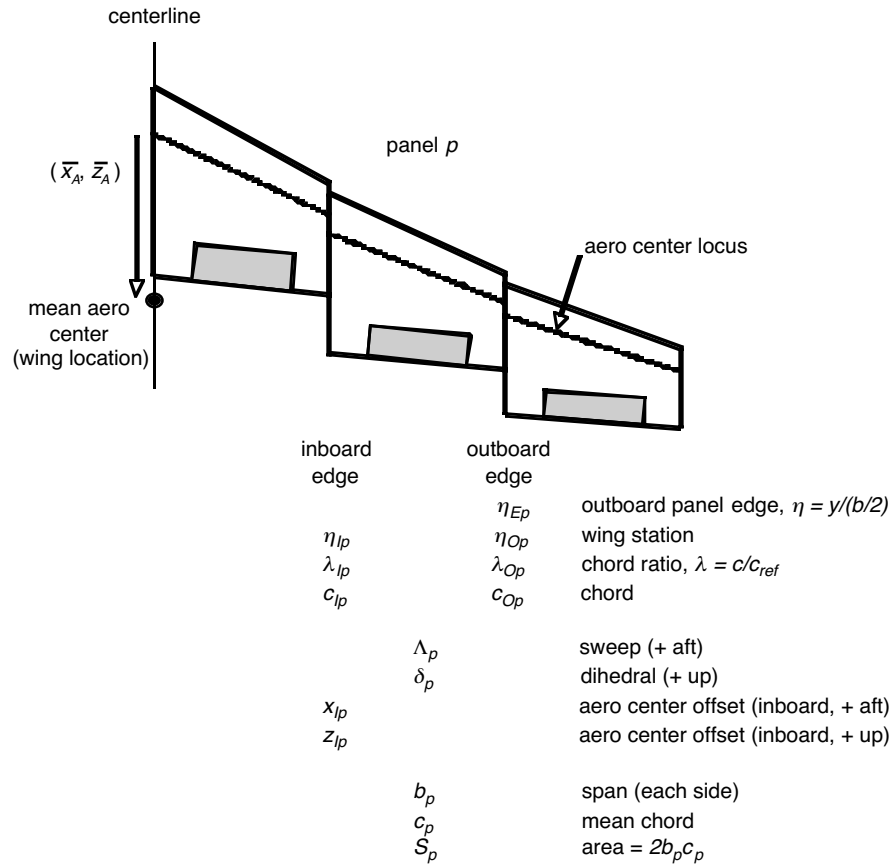
Figure 13-1. Wing geometry (symmetric, only right half-wing shown).

the hub position, $y_p = |y_{\text{hub}}|$ (for a designated rotor); or adjusted. An adjusted station is obtained from the last station and the span of this panel, $y_p = y_{p-1} + b_p$ or $y_p = y_{p-1} + f_{bp}(b/2)$; or from the next station and the span of the next panel, $y_p = y_{p+1} - b_{p+1}$ or $y_p = y_{p+1} - f_{b(p+1)}(b/2)$. The specification of panel spans and panel edges must be consistent, and sufficient to determine the wing geometry. Determining the panel edges requires the following steps.

a) Calculate the panel edges that are either at fixed values (input, or from width, or from hub position) or at fixed stations; root and tip edges are known.

b) Working from root to tip, calculate the adjusted panel edge y_p if panel span b_p or ratio f_{bp} is fixed, and if previous edge y_{p-1} is known.

c) Working from tip to root, calculate the adjusted panel edge y_p (if not yet known) if panel span b_{p+1} or ratio $f_{b(p+1)}$ is fixed, and if next edge y_{p+1} is known.

At the end of this process, all edges must be known and the positions y_p must be unique and sequential. If this geometry is being determined for a known span, then there must not be a fixed panel span or span ratio that has not been used. Alternatively, if the wing span is being calculated from the specification of all panel widths, then the process must leave one and only one fixed panel span or span ratio that has not been used. Since the wing span is to be calculated, each panel edge is known in the form $y_p = c_0 + c_1 b/2$.

Then the unused fixed panel span gives the equation $(c_0 + c_1 b/2)_O - (c_0 + c_1 b/2)_I = b_p$ (subscript O denotes outboard edge, subscript I denotes inboard edge), or the unused fixed panel span ratio gives the equation $(c_0 + c_1 b/2)_O - (c_0 + c_1 b/2)_I = f_p b/2$, which can be solved for the semispan $b/2$.

To complete the definition of the geometry, one of the following quantities is specified for each panel: panel area S_p; ratio of panel area to wing area, $f_s = S_p/S$; panel mean chord c_p; ratio of panel mean chord to wing mean chord, $f_c = c_p/c$; chord ratios $\lambda_I = c_I/c_{\rm ref}$ and $\lambda_O = c_O/c_{\rm ref}$ (taper); or free. The total wing area equals the sum of all panel areas:

$$S = \sum S_p + S \sum f_s + 2 \sum b_p c_p + 2c \sum b_p f_c + 2c_{\rm ref} \sum b_p \frac{1}{2}(\lambda_I + \lambda_O)$$

If there is one or more taper specification (and no free), then $c_{\rm ref}$ is calculated from this equation for S, and the mean chord is $c_p = \frac{1}{2}(c_I + c_O) = c_{\rm ref}\frac{1}{2}(\lambda_I + \lambda_O)$, $S_p = 2b_p c_p$. If there is one (and only one) free specification, then S_p is calculated from this equation for S, and the mean chord is $c_p = S_p/(2b_p)$, with $c_I = 2c_p/(1 + \lambda_O/\lambda_I)$, $c_O = 2c_p - c_I$.

Since the panels have linear taper ($c = c_{\rm ref}\lambda$), the mean aerodynamic chord is

$$S\bar{c}_A = \int_{-b/2}^{b/2} c^2\, dy = b \int_0^1 c_{\rm ref}^2 \lambda^2\, d\eta$$

$$= b \sum c_{\rm ref}^2 \frac{1}{3}(\lambda_I^2 + \lambda_I \lambda_O + \lambda_O^2)\, \Delta\eta_p = \sum \frac{1}{3}(c_I^2 + c_I c_O + c_O^2)\, 2b_p$$

$$S = \int_{-b/2}^{b/2} c\, dy = b \int_0^1 c_{\rm ref} \lambda\, d\eta$$

$$= b \sum c_{\rm ref} \frac{1}{2}(\lambda_I + \lambda_O)\, \Delta\eta_p = \sum \frac{1}{2}(c_I + c_O)\, 2b_p$$

These expressions are evaluated from panel c_I and c_O, as calculated using λ_I and λ_O, or using the ratio λ_O/λ_I ($c_{\rm ref}$ may not be the same for all panels).

The mean aerodynamic center is the point where there is zero moment due to lift: $\bar{x}_A C_L S = \bar{x}_A \int c_\ell c\, dy = \int x c_\ell c\, dy$, with $c c_\ell = \ell(y)$ the spanwise lift distribution. Thus

$$\int_0^1 \ell(\eta)(\bar{x}_A - x_{AC}(\eta))\, d\eta = 0$$

The locus of section aerodynamic centers x_{AC} is described by the panel sweep Λ_p and the offset x_{Ip} at the inboard end of the panel. These offsets can be a fixed input, a fraction of the root chord, or a fraction of the panel inboard chord. Assuming elliptical loading ($\ell = \sqrt{1 - \eta^2}$) gives

$$\frac{\pi}{4}\bar{x}_A = \int_0^1 \ell(\eta) x_{AC}\, d\eta = \sum \int \sqrt{1 - \eta^2}\left(\hat{x}_{Ip} + \frac{b}{2}\tan\Lambda_p\, \eta\right) d\eta$$

$$= \sum \left[\hat{x}_{Ip} \frac{1}{2}\left(\eta\sqrt{1 - \eta^2} + \sin^{-1}\eta\right) - \frac{b}{2}\tan\Lambda_p \frac{1}{3}(1 - \eta^2)^{3/2}\right]_{\eta_I}^{\eta_O}$$

where $\hat{x}_{Ip} = \sum_{q=2}^{p}\left(x_{Iq} + (b/2)\tan\Lambda_{q-1}(\eta_{O(q-1)} - \eta_{I(q-1)})\right) - (b/2)\tan\Lambda_p\eta_{Ip}$. The vertical position of the mean aerodynamic center is obtained in a similar fashion, from panel dihedral δ_p and offset z_{Ip} at the inboard edge of the panel. Assuming uniform loading ($\ell = 1$) gives

$$\bar{z}_A = \int_0^1 z_{AC}\, d\eta = \sum \int \left(\hat{z}_{Ip} + \frac{b}{2}\tan\delta_p\, \eta\right) d\eta = \sum \left[\hat{z}_{Ip}\eta + \frac{b}{2}\tan\delta_p \frac{1}{2}\eta^2\right]_{\eta_I}^{\eta_O}$$

Then $(\bar{x}_A, \bar{z}_A)$ is the offset of the mean aerodynamic center from the root chord aerodynamic center. Finally,

$$\Lambda = \tan^{-1}\left(\sum \frac{b_p}{b/2}\tan\Lambda_p\right)$$

$$\delta = \tan^{-1}\left(\sum \frac{b_p}{b/2}\tan\delta_p\right)$$

$$\lambda = \frac{2c}{c_{\text{root}}} - 1$$

are the wing overall sweep, dihedral, and taper.

The wing contribution to the aircraft operating length is $x_{\text{wing}} + (0.25c)\cos i$ (forward), $x_{\text{wing}} - (0.75c)\cos i$ (aft), and $y_{\text{wing}} \pm b/2$ (lateral).

13–2   Control and Loads

The control variables are flap δ_F, flaperon δ_f, aileron δ_a, and incidence i. The flaperon deflection can be specified as a fraction of flap deflection, or as an increment relative to the flap deflection, or the flaperon can be independent of the flap. The flaperon and aileron are the same surface, generating symmetric and antisymmetric loads, respectively, hence with different connections to pilot controls.

With more than one wing panel, each panel can have control variables: flap δ_{Fp}, flaperon δ_{fp}, aileron δ_{ap}, and incidence i_p. The outboard panel ($p \geq 2$) control or incidence can be specified independently, or in terms of the root panel ($p = 1$) control or incidence (either fraction or increment).

Each control is described by the ratio of the control surface chord to the wing panel chord, $\ell_f = c_f/c_p$; and by the ratio of the control surface span to wing panel span, $f_b = b_f/b_p$, such that the control surface area is obtained from the panel area by $S_f = \ell_f f_b S_p$.

13–3   Aerodynamics

The aerodynamic velocity of the wing relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack α_{wing} (hence C^{BA}) and dynamic pressure q are calculated from v^B. The reference area for the wing aerodynamic coefficients is the planform area, S. The wetted area contribution is twice the exposed area: $S_{\text{wet}} = 2(S - cw_{\text{fus}})$, where w_{fus} is the fuselage width.

The wing vertical drag can be fixed, specified as a drag area $(D/q)_V$; or the drag can be scaled, specified as a drag coefficient C_{DV} based on the wing area; or calculated from an airfoil section drag coefficient (for -90 deg angle-of-attack) and the wing area immersed in the rotor wake:

$$C_{DV} = \frac{1}{S}\,c_{d90}\left(S - S_{\text{center}} - f_{d90}b_F c_F(1 - \cos\delta_F) - f_{d90}b_f c_f(1 - \cos\delta_f)\right)$$

The term $S_{\text{center}} = c(w_{\text{fus}} + 2d_{\text{fus}})$ (where w_{fus} is the fuselage width and d_{fus} the rotor-fuselage clearance) is the area not immersed in the rotor wake, and is used only for tiltrotors. The last two terms account for the change in wing area due to flap and flaperon deflection, with an effectiveness factor f_{d90}.

From the control surface deflection and geometry, the lift coefficient, maximum lift angle, moment coefficient, and drag coefficient increments are evaluated: ΔC_{Lf}, $\Delta\alpha_{\text{max}f}$, ΔC_{Mf}, ΔC_{Df}. These increments are the sum of contributions from flap and flaperon deflection, hence weighted by the control surface area. The drag coefficient increment includes the contribution from aileron deflection.

13-3.1   Lift

The wing lift is defined in terms of lift curve slope $C_{L\alpha}$ and maximum lift coefficient $C_{L\max}$ (based on wing planform area). The three-dimensional lift curve slope is input directly or calculated from the two-dimensional lift curve slope:

$$C_{L\alpha} = \frac{c_{\ell\alpha}}{1 + c_{\ell\alpha}(1+\tau)/(\pi A\!R)}$$

where τ accounts for non-elliptical loading. The effective angle-of-attack is $\alpha_e = \alpha_{\text{wing}} + i - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. Let $\alpha_{\max} = C_{L\max}/C_{L\alpha}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection, $A_{\max} = \alpha_{\max} + \Delta\alpha_{\max f}$ and $A_{\min} = -\alpha_{\max} + \Delta\alpha_{\max f}$. Then

$$C_L = \begin{cases} C_{L\alpha}\alpha_e + \Delta C_{Lf} & A_{\min} \leq \alpha_e \leq A_{\max} \\[2mm] (C_{L\alpha}A_{\max} + \Delta C_{Lf})\left(\dfrac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\max}|}\right) & \alpha_e > A_{\max} \\[3mm] (C_{L\alpha}A_{\min} + \Delta C_{Lf})\left(\dfrac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\min}|}\right) & \alpha_e < A_{\min} \end{cases}$$

(for zero lift at 90 deg angle-of-attack). Note that $C_{L\alpha}A_{\max} + \Delta C_{Lf} = C_{L\alpha}\alpha_{\max} + \Delta C_{L\max f}$. In sideward flight, $C_L = 0$. Finally, $L = qSC_L$ is the lift force.

13-3.2   Pitch Moment

The wing pitch moment coefficient is $C_M = C_{Mac} + \Delta C_{Mf}$. Then $M = qScC_M$ is the pitch moment.

13-3.3   Roll Moment

The only wing roll moment considered is that produced by aileron control. Typically the flaperon and aileron are the same surface, but they are treated separately in this model. The aileron geometry is specified as for the flaperon and flap, hence includes both sides of the wing. The lift coefficient increment ΔC_{La} is evaluated as for the flaperon, so one-half of this lift acts up (on the right side) and one-half acts down. The roll moment is then $M_x = 2(\Delta L_a/2)y$, where y is the lateral position of the aileron aerodynamic center, measured from the wing centerline (defined as a fraction of the wing semi-span).

The roll moment coefficient is $C_\ell = -\frac{y}{b/2}\frac{1}{2}\Delta C_{La}$. Then $M_x = qSbC_\ell$ is the roll moment.

13-3.4   Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle-of-attack is $\alpha_e = \alpha_{\text{wing}} + i - \alpha_{D\min}$, where $\alpha_{D\min}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle-of-attack term and the control increment. If the angle-of-attack is greater than a separation angle $\alpha_s < \alpha_t$, there is an additional drag increase. Thus if $|\alpha_e| \leq \alpha_t$, the profile drag is

$$C_{Dp} = C_{D0}\left(1 + K_d|\alpha_e|^{X_d} + K_s(|\alpha_e| - \alpha_s)^{X_s}\right) + \Delta C_{Df}$$

where the separation (K_s) term is present only for $|\alpha_e| > \alpha_s$; and otherwise

$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d} + K_s(|\alpha_t| - \alpha_s)^{X_s}\right) + \Delta C_{Df}$$

$$C_{Dp} = C_{Dt} + (C_{DV} - C_{Dt})\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

Optionally there might be no angle-of-attack variation at low angles ($K_d = 0$ and/or $K_s = 0$), or quadratic variation ($X_d = 2$), or cubic variation for the separation term ($X_s = 3$). For sideward flight ($v_x^B = 0$) the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate the vertical coefficient: $C_D = C_{D0}\cos^2\phi_v + C_{DV}\sin^2\phi_v$. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!\!R}$$

Conventionally the Oswald efficiency e represents the wing parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). If C_{Dp} varies with angle-of-attack, then e is just the span efficiency factor for the induced power (and C_{L0} should be zero). The wing-body interference is specified as a drag area, or a drag coefficient based on the wing area. Then

$$D = qSC_D = qS\Big(C_{Dp} + C_{Di} + C_{Dwb}\Big)$$

is the drag force. The other forces and moments are zero.

13-3.5   Wing Panels

The wing panels can have separate controls, different incidence angles, and different interference from the rotors. Thus the lift, drag, and moment coefficients are evaluated separately for each panel, based on the panel area S_p and mean chord c_p. The coefficient increments due to control surface deflection are calculated using the ratio of the control surface area to panel area, $S_f/S_p = \ell_f f_b$. The lateral position of the aileron aerodynamic center is $\eta_a b_p$ from the panel inboard edge, so $y/(b/2) = \eta_{E(p-1)} + \eta_a b_p/(b/2)$ from the wing centerline. Then the total wing coefficients are:

$$C_L = \frac{1}{S}\sum S_p C_{Lp}$$
$$C_M = \frac{1}{Sc}\sum S_p c_p C_{Mp}$$
$$C_\ell = \frac{1}{Sb}\sum S_p b C_{\ell p}$$
$$C_{Dp} = \frac{1}{\langle qS\rangle}\sum q_p S_p C_{Dpp}$$

The sums are over all panels (left and right). The reference area $S = \sum S_p$ is used (accounting for possible absence of wing extensions). The dynamic pressure of each panel is used for the parasite drag, so $\langle qS\rangle = \sum q_p S_p$. The three-dimensional lift curve slope $C_{L\alpha}$ is calculated for the entire wing and used for each panel. The induced drag is calculated for the entire wing, from the total C_L. Since $C^{BF} = I$, the wing aerodynamic force is

$$F^F = C^{FB}C^{BA}F^A = C^{BA}qS\begin{pmatrix}-C_{Di}\\0\\-C_L\end{pmatrix} + \sum C_p^{BA}q_p S_p\begin{pmatrix}-C_{Dpp} - C_{Dwb}\\0\\0\end{pmatrix}$$

The effect of rotor and wing interference is represented directly in the calculation of the induced drag, so the lift does not get tilted by the interference; C^{BA} and q do not include the interference velocity. The interference does affect the magnitude and direction of the parasite drag; C_p^{BA} and q_p include the interference velocity at each panel.

13-3.6   Interference

With more than one wing, the interference velocity at other wings is proportional to the induced velocity of the wing producing the interference: $v_{\text{int}}^F = K_{\text{int}} v_{\text{ind}}^F$. The induced velocity is obtained

from the induced drag, assumed to act in the k^B direction: $\alpha_{\text{ind}} = v_{\text{ind}}/|v^B| = C_{Di}/C_L = C_L/(\pi e A\!R)$, $v_{\text{ind}}^F = C^{FB} k^B |v^B| \alpha_{\text{ind}}$. For tandem wings, typically $K_{\text{int}} = 2$ for the interference of the front wing on the aft wing, and $K_{\text{int}} = 0$ for the interference of the aft wing on the front wing. For biplane wings, the mutual interference is typically $K_{\text{int}} = 0.7$ (upper on lower, and lower on upper). The induced drag is then

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!R} + C_L \sum_{\text{other wings}} \alpha_{\text{int}}$$

$$\alpha_{\text{int}} = K_{\text{int}} \alpha_{\text{ind}} = K_{\text{int}} \left(\frac{C_L}{\pi e A\!R} \right)_{\text{other wing}}$$

The induced velocity from the rotors is included in the angle-of-attack of the wing. The rotor interference must also be accounted for in the wing induced power:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!R} + C_L \left[\sum_{\text{other wings}} K_{\text{int}} \alpha_{\text{ind}} + \sum_{\text{rotors}} C_{\text{int}} \alpha_{\text{ind}} \right]$$

The angle $\alpha_{\text{ind}} = v_{\text{ind}}/V$ is obtained from the rotor induced velocity λ_i. If the interference is wing-like, $v_{\text{ind}} = \Omega R \lambda_i$ (so $v_{\text{ind}} \propto L/\rho b^2 V \propto T/2\rho AV$). If the interference is propeller-like, $v_{\text{ind}} = V\lambda_i$ (so $v_{\text{ind}} \propto \Gamma/b \propto T/2\rho A\Omega R$).

The wing interference at the tail produces an angle-of-attack change $\epsilon = E(C_L/C_{L\alpha})$, where $E = d\epsilon/d\alpha$ is an input factor determined by the aircraft geometry. Then from the velocity v^B of the wing,

$$v_{\text{int}}^F = C^{FB} \begin{pmatrix} -\epsilon v_z^B \\ 0 \\ \epsilon v_x^B \end{pmatrix}$$

is the interference velocity at the tail.

The wing interference at the rotor can produce interference power. The induced velocity at the rotor disk is $v_{\text{int}}^F = K_{\text{int}} v_{\text{ind}}^F$, with $v_{\text{ind}}^F = C^{FB} k^B |v^B| \alpha_{\text{ind}}$ again. Separate interference factors K_{int} are used for the components of the interference velocity parallel to and perpendicular to the rotor force vector (roughly normal to and in the plane of the rotor disk).

13–4   Wing Extensions

The wing can have extensions, defined as wing portions of span b_X at each wing tip. For the tiltrotor configuration in particular, the wing weight depends on the distribution of wing area outboard (the extension) and inboard of the rotor and nacelle location. Wing extensions are defined as a set of wing panels at the tip. The extension span and area are the sum of the panel quantities, $b_X = \sum_{\text{ext}} b_p$ and $S_X = \sum_{\text{ext}} S_p$. The inboard span and area are then $b_I = b - 2b_X$, $S_I = S - S_X$. Optionally the wing extensions can be considered a kit, hence the extensions can be absent for designated flight conditions or missions. As a kit, the wing extension weight is considered fixed useful load. With wing extensions removed, the aerodynamic analysis considers only the remaining wing panels. The total wing coefficients are then based on the area without the extensions. For the induced drag and interference, the effective aspect ratio is then reduced by the factor $(b_I/b)^2$, since the lift and drag coefficients are still based on total wing area S.

13–5   Wing Kit

The wing can be a kit, the kit weight an input fraction of the total wing weight. The wing kit weight can be part of the wing group, or considered fixed useful load. With the kit removed, there are no aerodynamic loads or aerodynamic interference generated by the wing, and the wing kit weight is omitted.

13–6   Weights

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including compound helicopter).

The AFDD wing weight models are based on parameters for the basic wing plus the wing tip extensions (not the total wing and extensions). The tiltrotor wing model requires the weight on the wing tips (both sides), consisting of: rotor group, engine system, drive system (except drive shaft), engine section or nacelle group, air induction group, rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions.

Empennage

The aircraft can have one or more tail surfaces, or no tail surface. Each tail is designated as horizontal or vertical, affecting some parameter definitions.

14–1   Geometry

The tail is described by planform area S, span b, chord $c = S/b$, and aspect ratio $A\!R = b^2/S$. The tail volume can be referenced to rotor radius and disk area, $V = S\ell/RA$; to wing area and chord for horizontal tails, $V = S\ell/S_w c_w$; or to wing area and span for vertical tails, $V = S\ell/S_w b_w$. Here the tail length is $\ell = |x_{ht} - x_{cg}|$ or $\ell = |x_{vt} - x_{cg}|$ for horizontal tail or vertical tail, respectively. The geometry is specified in terms of S or V; and b, or $A\!R$, or c. The elevator or rudder is described by the ratio of control surface chord to tail chord, c_f/c; and the ratio of control surface span to tail span, b_f/b.

For a canted tail plane ($\phi > 0$), tail volumes can be specified for both primary behavior (horizontal or vertical as designated, using $S\cos^2\phi$) and secondary behavior (vertical or horizontal, using $S\sin^2\phi$).

The tail contribution to the aircraft operating length is $x_{\text{tail}} + 0.25c$ (forward), $x_{\text{tail}} - 0.75c$ (aft), and $y_{\text{tail}} \pm (b/2)C$ (lateral), where $C = \cos\phi$ for a horizontal tail and $C = \cos(\phi - 90)$ for a vertical tail.

14–2   Control and Loads

The tail is at position z^F, where the aerodynamic forces act. The scaled input for tail position can be referenced to the fuselage length, or to the rotor radius.

The horizontal tail can have a cant angle ϕ (positive tilt to left, becomes vertical tail for $\phi = 90$ deg). Thus the component axes are given by $C^{BF} = X_{-\phi}$. The control variables are elevator δ_e and incidence i.

The convention for nominal orientation of the vertical tail is positive lift to the left, so aircraft sideslip (positive to right) generates positive tail angle-of-attack and positive tail lift. The vertical tail can have a cant angle ϕ (positive tilt to right, becomes horizontal tail for $\phi = 90$), so the component axes are given by $C^{BF} = X_{-90+\phi}$. The control variables are rudder δ_r and incidence i.

14–3   Aerodynamics

The aerodynamic velocity of the tail relative to the air, including interference, is calculated in component axes, v^B. The angle-of-attack α_{tail} (hence C^{BA}) and dynamic pressure q are calculated from v^B. The reference area for the tail aerodynamic coefficients is the planform area, S. The wetted area contribution is $S_{\text{wet}} = 2S$. From the elevator or rudder deflection and geometry, the lift coefficient, maximum lift angle, and drag coefficient increments are evaluated: $\Delta C_{Lf}, \Delta\alpha_{\text{max}f}, \Delta C_{Df}$.

14-3.1   Lift

The tail lift is defined in terms of lift curve slope $C_{L\alpha}$ and maximum lift coefficient $C_{L\max}$ (based on tail planform area). The three-dimensional lift curve slope is input directly or calculated from the two-dimensional lift curve slope:

$$C_{L\alpha} = \frac{c_{\ell\alpha}}{1 + c_{\ell\alpha}(1+\tau)/(\pi A\!R)}$$

where τ accounts for non-elliptical loading. The effective angle-of-attack is $\alpha_e = \alpha_{\text{tail}} + i - \alpha_{zl}$, where α_{zl} is the angle of zero lift; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. Let $\alpha_{\max} = C_{L\max}/C_{L\alpha}$ be the angle-of-attack increment (above or below zero lift angle) for maximum lift. Including the change of maximum lift angle caused by control deflection, $A_{\max} = \alpha_{\max} + \Delta\alpha_{\max f}$ and $A_{\min} = -\alpha_{\max} + \Delta\alpha_{\max f}$. Then

$$C_L = \begin{cases} C_{L\alpha}\alpha_e + \Delta C_{Lf} & A_{\min} \leq \alpha_e \leq A_{\max} \\[2ex] (C_{L\alpha}A_{\max} + \Delta C_{Lf})\left(\dfrac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\max}|}\right) & \alpha_e > A_{\max} \\[2ex] (C_{L\alpha}A_{\min} + \Delta C_{Lf})\left(\dfrac{\pi/2 - |\alpha_e|}{\pi/2 - |A_{\min}|}\right) & \alpha_e < A_{\min} \end{cases}$$

(for zero lift at 90 deg angle-of-attack). Note that $C_{L\alpha}A_{\max} + \Delta C_{Lf} = C_{L\alpha}\alpha_{\max} + \Delta C_{L\max f}$. In sideward flight (defined by $(v_x^B)^2 + (v_z^B)^2 < (0.05|v^B|)^2$), $C_L = 0$. Finally, $L = qSC_L$ is the lift force.

14-3.2   Drag

The drag area or drag coefficient is defined for forward flight and vertical flight. The effective angle-of-attack is $\alpha_e = \alpha_{\text{tail}} + i - \alpha_{D\min}$, where $\alpha_{D\min}$ is the angle of minimum drag; in reverse flow ($|\alpha_e| > 90$), $\alpha_e \leftarrow \alpha_e - 180\,\text{sign}\alpha_e$. For angles of attack less than a transition angle α_t, the drag coefficient equals the forward flight (minimum) drag C_{D0}, plus an angle-of-attack term and the control increment. Thus if $|\alpha_e| \leq \alpha_t$, the profile drag is

$$C_{Dp} = C_{D0}\left(1 + K_d|\alpha_e|^{X_d}\right) + \Delta C_{Df}$$

and otherwise

$$C_{Dt} = C_{D0}\left(1 + K_d|\alpha_t|^{X_d}\right) + \Delta C_{Df}$$

$$C_{Dp} = C_{Dt} + (C_{DV} - C_{Dt})\sin\left(\frac{\pi}{2}\frac{|\alpha_e| - \alpha_t}{\pi/2 - \alpha_t}\right)$$

Optionally there might be no angle-of-attack variation at low angles ($K_d = 0$), or quadratic variation ($X_d = 2$). In sideward flight (defined by $(v_x^B)^2 + (v_z^B)^2 < (0.05|v^B|)^2$), the drag is obtained using $\phi_v = \tan^{-1}(-v_z^B/v_y^B)$ to interpolate the vertical coefficient: $C_{Dp} = C_{D0}\cos^2\phi_v + C_{DV}\sin^2\phi_v$. The induced drag is obtained from the lift coefficient, aspect ratio, and Oswald efficiency e:

$$C_{Di} = \frac{(C_L - C_{L0})^2}{\pi e A\!R}$$

Conventionally the Oswald efficiency e can represent the tail parasite drag variation with lift, as well as the induced drag (hence the use of C_{L0}). Then

$$D = qSC_D = qS\left(C_{Dp} + C_{Di}\right)$$

is the drag force. The other forces and moments are zero.

14–4   Weights

The empennage group consists of the horizontal tail, vertical tail, and tail rotor. The tail plane weight consists of the basic structure and fold structure. The tail weight (empennage group) model depends on the configuration: helicopters and compounds, or tiltrotors and tiltwings. Separate weight models are available for horizontal and vertical tails.

The AFDD tail weight model depends on the design dive speed at sea level (input or calculated). The calculated dive speed is $V_{\text{dive}} = 1.25 V_{\text{max}}$, from the maximum speed at the design gross weight and sea level standard conditions.

Fuel Tank

15–1   Fuel Capacity

The fuel tank capacity $W_{\text{fuel}-\text{cap}}$ (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required, $W_{\text{fuel}-\text{miss}}$ (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{\text{fuel}-\text{cap}} = \max(f_{\text{fuel}-\text{cap}}W_{\text{fuel}-\text{miss}}, W_{\text{fuel}-\text{miss}} + W_{\text{reserve}})$$

where $f_{\text{fuel}-\text{cap}} \geq 1$ is an input factor. Alternatively, the fuel tank capacity $W_{\text{fuel}-\text{cap}}$ can be input. The corresponding volumetric fuel tank capacity is $V_{\text{fuel}-\text{cap}} = W_{\text{fuel}-\text{cap}}/\rho_{\text{fuel}}$ (gallons or liters), where ρ_{fuel} is the fuel density (input as weight per volume).

For missions that are not used to size the fuel tank, the fuel weight may be fallout, or the fuel weight may be specified (with or without auxiliary tanks). The fuel weight for a flight condition or the start of a mission is specified as an increment d, plus a fraction f of the fuel tank capacity, plus auxiliary tanks:

$$W_{\text{fuel}} = \min(d_{\text{fuel}} + f_{\text{fuel}}W_{\text{fuel}-\text{cap}}, W_{\text{fuel}-\text{cap}}) + \sum N_{\text{auxtank}}W_{\text{aux}-\text{cap}}$$

where $W_{\text{aux}-\text{cap}}$ is the capacity of each auxiliary fuel tank. The fuel capacity of the wing can be estimated from

$$W_{\text{fuel}-\text{wing}} = \rho_{\text{fuel}} \sum f c_{tb} t_w b_w$$

where c_{tb} is the torque box chord, t_w the wing thickness, and b_w the wing span; and f is the input fraction of the wing torque box that is filled by primary fuel tanks, for each wing. This calculation is performed in order to judge whether fuel tanks outside the wing are needed.

15–2   Geometry

The fuel tank is at position z^F, where the inertial forces act.

15–3   Fuel Reserves

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments, for example 200 miles plus 20 minutes at V_{be}. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, so $W_{\text{fuel}} = (1 + f_{\text{res}})W_{\text{fuel}-\text{miss}}$. Fuel reserves can be an input fraction of the fuel capacity, so $W_{\text{fuel}} = W_{\text{miss}-\text{seg}} + f_{\text{res}}W_{\text{fuel}-\text{cap}}$. If more than one criterion for reserve fuel is specified, the maximum reserve is used.

```
if  W_fuel > W_fuel−max
    for designated auxiliary tank
        N_auxtank = N_auxtank + n
        ΔW_fuel = −n f_auxtank W_aux−cap
        ΔW_fuel−max = n W_aux−cap
        repeat if  W_fuel > W_fuel−max
    if  W_fuel ≤ W_fuel−max − W_aux−cap
        N_auxtank = N_auxtank − n
        ΔW_fuel−max = −n W_aux−cap
        W_fuel = n W_fuel−max  (capped)
else if  W_fuel < W_fuel−max
    for designated auxiliary tank
    (then for last nonzero  N_auxtank)
        N_auxtank = N_auxtank − n
        ΔW_fuel = n f_auxtank W_aux−cap
        ΔW_fuel−max = −n W_aux−cap
        repeat if  W_fuel < W_fuel−max
    undo last increment
        N_auxtank = N_auxtank + n
        ΔW_fuel = −n f_auxtank W_aux−cap
        ΔW_fuel−max = n W_aux−cap
```

Figure 15-1. Outline of N_{auxtank} calculation.

15–4   Auxiliary Fuel Tank

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank, $W_{\mathrm{aux-cap}}$, is an input parameter. The number of auxiliary fuel tanks on the aircraft, N_{auxtank} for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel.

Figure 15-1 describes the process for determining N_{auxtank} from the fuel weight W_{fuel} and the aircraft maximum fuel capacity $W_{\mathrm{fuel-max}} = W_{\mathrm{fuel-cap}} + \sum N_{\mathrm{auxtank}} W_{\mathrm{aux-cap}}$. The fuel weight adjustment ΔW_{fuel} is made if fuel weight is fallout from fixed gross weight and payload, accounting for the operating weight update when N_{auxtank} changes. If the auxiliary tank weight is greater than the increment in fuel weight needed, then the fallout fuel weight $W_{\mathrm{fuel}} = W_G - W_O - W_{\mathrm{pay}}$ can not be achieved; in such a case, the fuel weight is capped at the maximum fuel capacity and the payload weight adjusted instead. The tanks changed can be the first size, the first size already used, or a designated size. The tanks can be added or dropped in groups of n ($n = 2$ for pairs).

The weight and drag of N_{auxtank} tanks are included in the performance calculation. Optionally the number of auxiliary tanks required can be calculated at the beginning of designated mission segments (based on the aircraft fuel weight at that point), and tanks dropped if no longer needed. The weight of the auxiliary fuel tanks is an input fraction of the tank capacity: $W_{\mathrm{auxtank}} = \sum f_{\mathrm{auxtank}} N_{\mathrm{auxtank}} W_{\mathrm{aux-cap}}$.

15-4.1   Auxiliary Fuel Tank Drag

The auxiliary fuel tanks are located at position z^F. The drag area for one auxiliary tank is specified,

$(D/q)_{\text{auxtank}}$. The velocity relative to the air gives the drag direction $e_d = -v^F/|v^F|$ and dynamic pressure $q = \frac{1}{2}\rho|v^F|^2$ (no interference). Then

$$F^F = e_d\, q\, N_{\text{auxtank}}(D/q)_{\text{auxtank}}$$

is the total drag force, calculated for each auxiliary tank size.

15–5   Weights

The fuel system consists of the tanks (including support) and the plumbing.

The weight of the auxiliary fuel tanks is part of the fixed useful load; it is an input fraction of the tank capacity: $W_{\text{auxtank}} = \sum f_{\text{auxtank}} N_{\text{auxtank}} W_{\text{aux}-\text{cap}}$.

The AFDD weight model for the plumbing requires the fuel flow rate (for all engines), calculated for the takeoff rating and conditions.

Propulsion

The propulsion group is a set of components and engine groups, connected by a drive system. The engine model describes a particular engine, used in one or more engine group. The components (rotors) define the power required. The engine groups define the power available. Figure 16-1 illustrates the power flow.

16–1 Drive System

The drive system defines gear ratios for all the components it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed $V_{\text{tip-ref}}$ is defined for each drive system state. By convention, the "hover tip speed" refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant. By convention, the gear ratio of the primary rotor is $r = 1$. For dependent rotors, either the gear ratio is specified (for each drive system state) or a tip speed is specified and the gear ratio calculated ($r = \Omega_{\text{dep}}/\Omega_{\text{prim}}, \Omega = V_{\text{tip-ref}}/R$). For the engine group, either the gear ratio is specified (for each drive system state) or the gear ratio calculated from the specification engine turbine speed $\Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}}$ and the reference tip speed of the primary rotor ($r = \Omega_{\text{spec}}/\Omega_{\text{prim}}, \Omega_{\text{prim}} = V_{\text{tip-ref}}/R$). The latter option means the specification engine turbine speed N_{spec} corresponds to $V_{\text{tip-ref}}$ for all drive system states. To determine the gear ratios, the reference tip speed and radius are used, corresponding to hover.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius the rotational speed of the primary rotor is obtained ($\Omega_{\text{prim}} = V_{\text{tip}}/R$); from the gear ratios, the rotational speed of dependent rotors ($\Omega_{\text{dep}} = r\Omega_{\text{prim}}$) and the engine groups ($N = (60/2\pi)r_{\text{eng}}\Omega_{\text{prim}}$) are obtained; and from the rotor radius, the tip speed of the dependent rotor ($V_{\text{tip}} = \Omega_{\text{dep}}R$) is obtained. The flight state specification of the tip speed can be an input value; the reference tip speed; a function of flight speed or a conversion schedule; or one of several default values. These relationships between tip speed and rotational speed use the actual radius of the rotors in the flight state, which for a variable-diameter rotor may not be the same as the reference, hover radius.

A designated drive system state can have a variable speed (variable gear ratio) transmission, by introducing a factor f_{gear} on the gear ratio when the speeds of the dependent rotors and engines are evaluated. The factor f_{gear} is a component control, which can be connected to an aircraft control and thus set for each flight state.
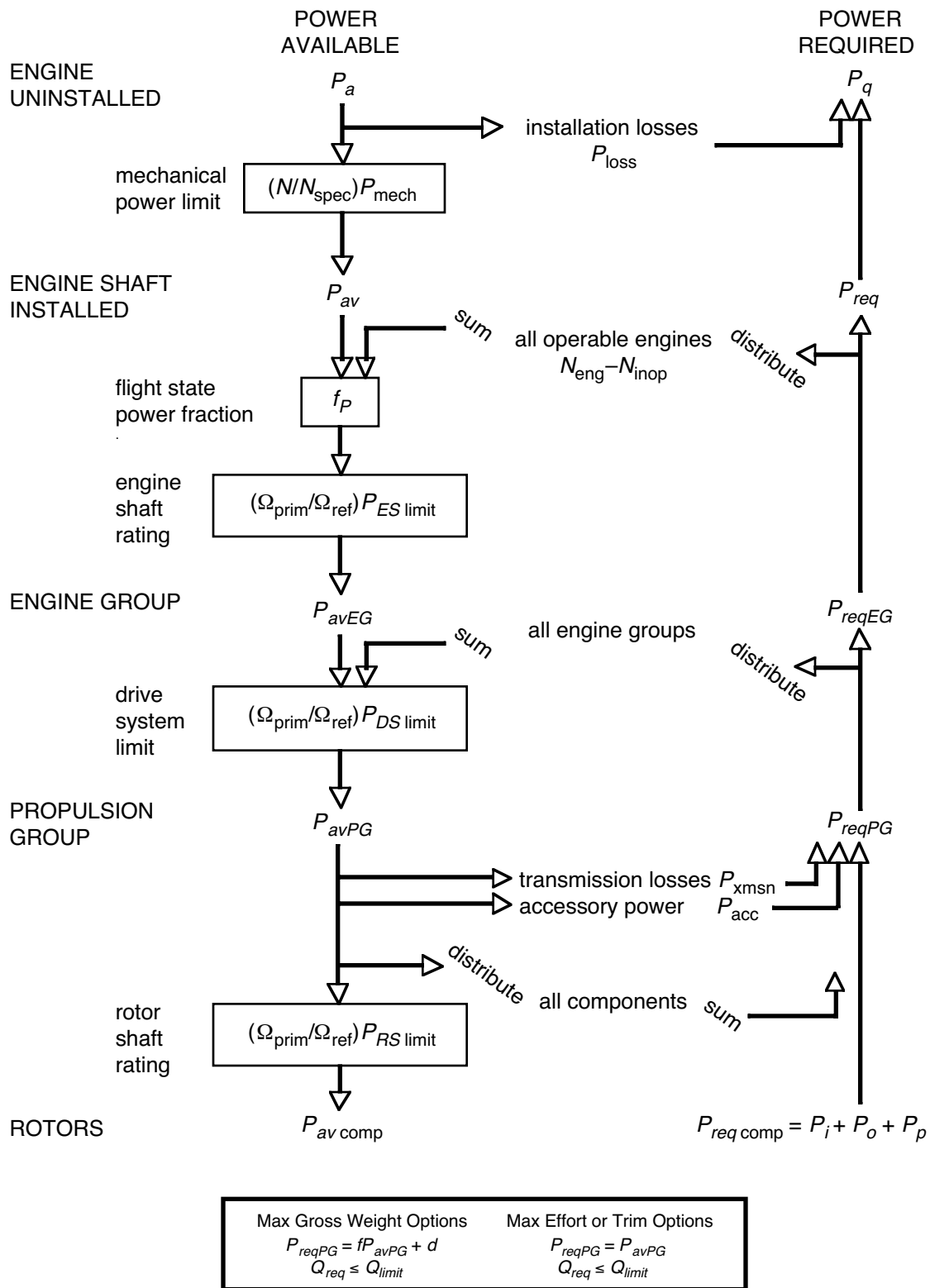
Figure 16-1. Power flow.

An optional conversion schedule is defined in terms of two speeds: hover and helicopter mode for speeds below $V_{C\text{hover}}$, cruise mode for speeds above $V_{C\text{cruise}}$, and conversion mode for speeds between $V_{C\text{hover}}$ and $V_{C\text{cruise}}$. The tip speed is $V_{\text{tip}-\text{hover}}$ in helicopter and conversion mode, and $V_{\text{tip}-\text{cruise}}$ in airplane mode. Drive system states are defined for helicopter, cruise, and conversion mode flight. The flight state specifies the nacelle tilt angle, tip speeds, control state, and drive system state, including the option to obtain any or all of these quantities from the conversion schedule.

Several default values of the tip speed are defined for use by the flight state, including cruise, maneuver, one-engine inoperative, drive system limit conditions, and a function of flight speed (piecewise linear input). Optionally these default values can be input as a fraction of the hover tip speed. Optionally the tip speed can be calculated from an input $C_T/\sigma = t_0 - \mu t_1$, so $V_{\text{tip}} = \sqrt{T/\rho A \sigma t_0 + (Vt_1/2t_0)^2} + (Vt_1/2t_0)$; or from $\mu = V/V_{\text{tip}}$, so $V_{\text{tip}} = V/\mu$; or from $M_{at} = M_{\text{tip}}\sqrt{(1+\mu)^2 + \mu_z^2}$, so $V_{\text{tip}} = \sqrt{(c_s M_{at})^2 - V_z^2} - V$.

The sizing task might change the hover tip speed (reference tip speed for drive system state #1), the reference tip speed of a dependent rotor, a rotor radius, or the specification engine turbine speed N_{spec}. In such cases the gear ratios and other parameters are recalculated. Note that it is not consistent to change the reference tip speed of a dependent rotor if the gear ratio is a fixed input.

16–2   Power Required

The component power required P_{comp} is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power:

$$P_{reqPG} = P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$$

The transmission losses are calculated as an input fraction ℓ_{xmsn} of the component power, plus windage loss:

$$P_{\text{xmsn}} = \ell_{\text{xmsn}} f_{\text{xmsm}} P_{\text{comp}} + P_{\text{windage}} (\Omega_{\text{prim}}/\Omega_{\text{ref}})$$

The factor f_{xmsm} can equal 1, or can include a function of the drive shaft rating (increasing the losses at low power):

$$f_{\text{xmsm}} P_{\text{comp}} = \begin{cases} \frac{1}{2} P_{X\text{limit}} & Q < \frac{1}{4} \\ \left(\frac{7}{3} - \frac{4}{3}Q\right) P_{\text{comp}} & \frac{1}{4} < Q < 1 \\ P_{\text{comp}} & 1 < Q \end{cases}$$

where $Q = P_{\text{comp}}/P_{X\text{limit}}$, $P_{X\text{limit}} = r P_{DS\text{limit}}$, and $r = N/N_{\text{spec}} = \Omega_{\text{prim}}/\Omega_{\text{ref}}$. Accessory losses are calculated as the sum of an input constant; terms that scale with air density and rotor speed; a fraction of power required (such as ECU losses); IRS fan loss (if IR suppression system is on); deice power loss (if deice system is on); and an increment specified for each flight state:

$$P_{\text{acc}} = P_{\text{acc}0} + P_{\text{acc}d}\sigma + P_{\text{acc}n}\sigma(\Omega_{\text{prim}}/\Omega_{\text{ref}}) + \ell_{\text{acc}}(P_{\text{comp}} + P_{\text{xmsn}}) + \ell_{\text{IRfan}}\sigma N_{\text{eng}} P_{\text{eng}} + P_{\text{acc}i} + dP_{\text{acc}}$$

where $\sigma = \rho/\rho_0$ is the density ratio.

The power required for the propulsion group must be distributed to the engine groups. With only one engine group, $P_{reqEG} = P_{reqPG}$. An engine group power can be fixed at a fraction f_P of the engine power available: $P_{reqEG} = f_P(N_{\text{eng}} - N_{\text{inop}})P_{\text{eng}}$ at the specified rating (each operating engine

at $f_P P_{\text{eng}}$). The power required for the remaining (perhaps all) engine groups is distributed proportional to the engine rated power:

$$P_{reqEG} = \left(P_{reqPG} - \sum_{\text{fixed}} P_{reqEG}\right) \frac{(N_{\text{eng}} - N_{\text{inop}})P_{\text{eng}}}{\sum_{\text{notfixed}}(N_{\text{eng}} - N_{\text{inop}})P_{\text{eng}}}$$

If the sum of the fixed P_{reqEG} exceeds the propulsion group power required, or if the power is fixed for all engine groups, then each is reduced by the ratio $P_{reqPG}/\sum_{\text{fixed}} P_{reqEG}$. The result is that the power is distributed proportional to f_P times the engine rated power. The fuel flow of the propulsion group is obtained from the sum over the engine groups: $\dot{w}_{reqPG} = \sum \dot{w}_{reqEG}$.

16–3 Geometry

The length of the drive system ℓ_{DS} can be input or calculated. The calculated length is the sum of the longitudinal, lateral, and vertical distances from the primary rotor hub to the other hub locations, for all rotors in the propulsion group: $\ell_{DS} = f \sum(|\Delta x| + |\Delta y| + |\Delta z|)$, where f is an input factor.

16–4 Drive System Rating

The drive system rating is defined as a power limit, $P_{DS\text{limit}}$. The rating is properly a torque limit, $Q_{DS\text{limit}} = P_{DS\text{limit}}/\Omega_{\text{ref}}$, but is expressed as a power limit for clarity. The drive system rating can be specified as follows:

a) Input $P_{DS\text{limit}}$.
b) From the engine takeoff power rating, $P_{DS\text{limit}} = f_{\text{limit}} \sum N_{\text{eng}} P_{\text{eng}}$ (summed over all engine groups).
c) From the power available at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}} P_{av}$ (largest of all conditions and segments).
d) From the power required at the transmission sizing conditions and missions, $P_{DS\text{limit}} = f_{\text{limit}}(\Omega_{\text{ref}}/\Omega_{\text{prim}}) \sum N_{\text{eng}} P_{req}$ (largest of all conditions and segments).

with f_{limit} an input factor. The drive system rating is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft ($P_{RS\text{limit}}$) and of each engine group ($P_{ES\text{limit}}$). The engine shaft rating is calculated as for the drive system rating, without the sum over engine groups. The rotor shaft rating is either input or calculated from the rotor power required at the transmission sizing flight conditions. The power limit is associated with a reference rotational speed, and when applied the limit is scaled with the rotational speed of the flight state. The rotation speed for the drive system rating $P_{DS\text{limit}}$ is the hover speed of the primary rotor of the propulsion group (for the first drive state). The rotation speed for the engine shaft rating $P_{ES\text{limit}}$ is the corresponding engine turbine speed. The rotation speed for the rotor shaft rating $P_{RS\text{limit}}$ is the corresponding speed of that rotor.

16–5 Weights

The drive system consists of gear boxes and rotor shafts, drive shafts, rotor brakes, clutches, and gas drive. The drive system weight depends on the rotor and engine rotational speeds, evaluated for the propulsion group primary rotor and a specified engine group, at a specified drive system state (gear ratio).

The AFDD drive system weight model depends on f_Q, the second (main or tail) rotor rated torque as a fraction of the total drive system rated torque; and on f_P, the second (main or tail) rotor rated power as a fraction of the total drive system rated power. These parameters are related by the rotational speeds of the two rotors: $f_P = f_Q \Omega_{\text{other}}/\Omega_{\text{main}}$. Typically $f_P = f_Q = 0.6$ for twin rotors (tandem, coaxial, and tiltrotor configurations). For the single-main-rotor and tail-rotor configuration, typically $f_Q = 0.03$ and $f_P = 0.15$ (0.18 with a two-bladed teetering main rotor).

Engine Group

The engine group consists of one or more engines of a specific type. For each engine type an engine model is defined. The Referred Parameter Turboshaft Engine Model (RPTEM) is implemented.

The engine size is described by the power P_{eng}, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines N_{eng} is specified for each engine group.

If the sizing task determines the engine power for a propulsion group, the power P_{eng} of at least one engine group is found (including the first engine group). The total power required is $P_{PG} = r \sum N_{\text{eng}} P_{\text{eng}}$, where $r = \max(P_{reqPG}/P_{avPG})$. The sized power is $P_{\text{sized}} = P_{PG} - \sum_{\text{fixed}} N_{\text{eng}} P_{\text{eng}}$. Then the sized engine power is $P_{\text{eng}} = f_n P_{\text{sized}}/N_{\text{eng}}$ for the n-th engine group (with $f_1 = \sum_{n \neq 1, \text{sized}} f_n$ for the first group).

17–1 Engine Performance

17-1.1 Power Available

Given the flight condition and engine rating, the power available P_a is calculated (from the specific power SP_a and mass flow $\dot{m}_a$). The flight condition information includes the altitude, temperature, flight speed, and primary rotor speed; a power fraction f_P; and the states of the engine, drive system, and IR suppressor. The engine turbine speed is $N = (60/2\pi) r_{\text{eng}} \Omega_{\text{prim}}$, where Ω_{prim} is the current rotor speed and r_{eng} is the gear ratio (depending on the drive system state, including a factor f_{gear} for a variable speed transmisson). If the reference primary rotor speed Ω_{prim} corresponds to the specification turbine speed N_{spec}, then $r_{\text{eng}} = \Omega_{\text{spec}}/\Omega_{\text{prim}}$; alternatively, the engine gear ratio can be a fixed input.

In the engine model, installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\text{loss}}$), and then the mechanical limit applied: $P_{av} = \min(P_{av}, r P_{\text{mech}R})$, $r = N/N_{\text{spec}}$. The mechanical limit is properly a torque limit, $Q_{\text{mech}} = P_{\text{mech}}/N_{\text{spec}}$, but is expressed as a power limit for clarity.

The engine model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines):

$$P_{avEG} = f_P (N_{\text{eng}} - N_{\text{inop}}) P_{av}$$

including a specified power fraction f_P. The propulsion group power available is obtained from the sum over the engine groups: $P_{avPG} = \sum P_{avEG}$.

The drive system rating at the flight condition is $r P_{DS\text{limit}}$, where $r = \Omega_{\text{prim}}/\Omega_{\text{ref}}$. Optionally this limit is applied to the propulsion group power: $P_{avPG} = \min(P_{avPG}, r P_{DS\text{limit}})$. Similarly the

engine shaft limit at the flight condition is optionally applied to the engine group power: $P_{avEG} = \min(P_{avEG}, rP_{ESlimit})$

17-1.2 Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q, flight condition, and engine rating. The flight condition includes the altitude, temperature, flight speed, and primary rotor speed; and engine, drive system, and IR suppressor states. The engine turbine speed is $N = r_{eng}\Omega_{prim}$.

The engine model deals with a single engine. The power required of a single engine is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):

$$P_{req} = P_{reqEG}/(N_{eng} - N_{inop})$$

In the engine model, installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{loss}$).

The engine model gives the performance of a single engine. The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

$$\dot{m}_{reqEG} = (N_{eng} - N_{inop})\dot{m}_{req}$$
$$\dot{w}_{reqEG} = (N_{eng} - N_{inop})\dot{w}_{req}K_{ffd}$$
$$F_{NEG} = (N_{eng} - N_{inop})F_N$$
$$D_{auxEG} = (N_{eng} - N_{inop})D_{aux}$$

The fuel flow has also been multiplied by a factor accounting for deterioration of the engine efficiency.

17-1.3 Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}: $P_{av} = P_a - P_{loss}$ and $P_{req} = P_q - P_{loss}$. The inlet ram recovery efficiency η_d is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{loss} = (\ell_{in}+\ell_{ex})P_a$ or $P_{loss} = (\ell_{in}+\ell_{ex})P_q$. Installation effects on the jet thrust are included in the engine model. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux}\dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux})\dot{m}_{aux}V = (1 - \eta_{aux})f_{aux}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

17–2   Control and Loads

The engine orientation is specified by selecting a nominal direction e_{f0} in body axes (positive or negative x, y, or z-axis; usually thrust forward, hence positive x-axis); then applying a yaw angle ψ; and then an incidence or tilt angle i (table 17-1). The yaw and incidence angles can be connected to the aircraft controls c_{AC}:

$$\psi = \psi_0 + T_\psi c_{AC}$$
$$i = i_0 + T_i c_{AC}$$

with ψ_0 and i_0 zero, constant, or a function of flight speed (piecewise linear input). Hence the incidence and yaw angles can be fixed orientation or can be control variables. Optionally the lateral position of the engine group can be set equal to that of a designed rotor (useful for tiltrotors when the rotor hub lateral position is calculated from the clearance or wing geometry).

The engine group produces a jet thrust F_N, acting in the direction of the engine; a momentum drag D_{aux}, acting in the wind direction; and a nacelle drag D_{nac}, acting in the wind direction. The engine group is at location z^F. The force and moment acting on the aircraft in body axes are thus:

$$F^F = e_f F_N + e_d D_{\text{aux}} + e_d D_{\text{nac}}$$
$$M^F = \widetilde{\Delta z^F} F^F$$

where $\Delta z^F = z^F - z^F_{cg}$, e_f is the engine thrust direction and e_d is the drag direction. The velocity relative to the air gives $e_d = -v^F/|v^F|$ (no interference). The engine axes are $C^{BF} = U_i V_\psi$, where U and V depend on the nominal direction, as described in table 17-1. The engine direction is $e_f = C^{FB} e_{f0}$.

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention $\alpha_{\text{tilt}} = 0$ for cruise and $\alpha_{\text{tilt}} = 90$ deg for helicopter mode. The engine incidence angle is then connected to α_{tilt} by defining the matrix T_i appropriately. If the engine nominal direction is defined for airplane mode ($+x$), then $i = \alpha_{\text{tilt}}$ should be used; if the engine nominal direction is defined for helicopter mode ($-z$), then $i = \alpha_{\text{tilt}} - 90$ should be used.

Table 17-1. Engine orientation.

nominal (F axes)		e_{f0}	incidence, + for force	yaw, + for force	$C^{BF} = U_i V_\psi$
x	forward	i	up	right	$Y_i Z_\psi$
$-x$	aft	$-i$	up	right	$Y_{-i} Z_{-\psi}$
y	right	j	aft	up	$Z_i X_{-\psi}$
$-y$	left	$-j$	aft	up	$Z_{-i} X_\psi$
z	down	k	aft	right	$Y_{-i} X_{-\psi}$
$-z$	up	$-k$	aft	right	$Y_i X_\psi$

17–3   Nacelle Drag

The engine group includes a nacelle, which contributes to the aircraft drag. The component drag contributions must be consistent. The pylon is the rotor support and the nacelle is the engine support. The drag model for a tiltrotor aircraft with tilting engines would use the pylon drag (and no nacelle drag), since the pylon is connected to the rotor shaft axes; with non-tilting engines it would use the nacelle drag as well.

The nacelle drag acts at the engine location z^F. The nacelle axes are the engine axes, hence C^{BF} is calculated as described above (see table 17-1). For the engine nominal direction forward ($+x$-axis), the nacelle z-axis is downward and the x-axis is forward; zero incidence angle corresponds to zero angle-of-attack; and 90 deg incidence angle corresponds to 90 deg angle-of-attack (vertical drag). The velocity, angle-of-attack, and dynamic pressure are calculated at the nacelle (without interference). The reference area for the nacelle drag coefficient is the nacelle wetted area. The wetted area per engine is

input, or calculated either from the engine system (engine, exhaust, and accessories) weight or from the engine system plus drive system weight:

$$S_{\text{wet}} = k\big(w/N_{\text{eng}}\big)^{2/3}$$

where $w = W_{ES}$ or $w = W_{ES} + W_{gbrs}/N_{EG}$, and the units of k are ft^2/lb$^{2/3}$ or m^2/kg$^{2/3}$. The reference area is then $S_{\text{nac}} = N_{\text{eng}}S_{\text{wet}}$. The nacelle area is included in the aircraft wetted area if the drag coefficient is nonzero. The drag area or drag coefficient is defined for forward flight and for vertical flight. The drag coefficient at angle-of-attack α is

$$C_D = C_{D0} + (C_{DV} - C_{D0})|\sin\alpha|^{X_d}$$

typically using $X_d = 2$. In sideward flight, $C_D = C_{D0}$ is used. The nacelle drag is $D_{\text{nac}} = qS_{\text{nac}}C_D$.

17–4   Engine Scaling

The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and N_{spec}). Figures 17-1 through 17-3 present historical data for engine specific fuel consumption, weight, and specific power; the variation at a given power reflects technology insertion with time.

The engine model includes reference values of the engine performance parameters: P_{0R}, SP_{0R}, $P_{\text{mech}R}$, sfc_{0C}, SF_{0C}, N_{spec}, and $N_{\text{opt}0C}$. Mass flow and fuel flow are obtained from $\dot{m}_{0R} = P_{0R}/SP_{0R}$ and $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The reference power at each engine rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. Similarly for specific power and mechanical limits: $r_{s0R} = SP_{0R}/SP_{0C}$ and $r_{m0R} = P_{\text{mech}R}/P_{0C}$. These ratios are kept fixed when the engine is scaled.

The engine size is specified as takeoff power $P_{to} = P_{\text{eng}}$: power at rating R, for SLS static conditions and specification turbine speed N_{spec}. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows. If P_{0C} is not equal to the reference value of the engine model, then the engine is scaled. To reflect advanced technology, the specific power, specific fuel consumption, and specification turbine speed can be specified: $SP_{0C} = SP_{\text{tech}}$, $\text{sfc}_{0C} = \text{sfc}_{\text{tech}}$, $N_{\text{spec}} = N_{\text{tech}}$. The default values are the reference values of the engine model.

The actual (perhaps scaled) values of the performance parameters are available for the engine group: P_{0R}, SP_{0R}, $P_{\text{mech}R}$, sfc_{0C}, F_{g0C}, N_{spec}, $N_{\text{opt}0C}$.

17–5   Weights

The component weight consists of engine system, engine section or nacelle group, and air induction group. The engine system consists of engine, exhaust system, and accessories. The engine section or nacelle group consists of engine support, engine cowling, and pylon support. These weights are for the engine group, consisting of N_{eng} engines. The engine system weight $W_{ES} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}}$ is used for the engine nacelle wetted area, the rotor pylon wetted area, and the rotor moving weight.
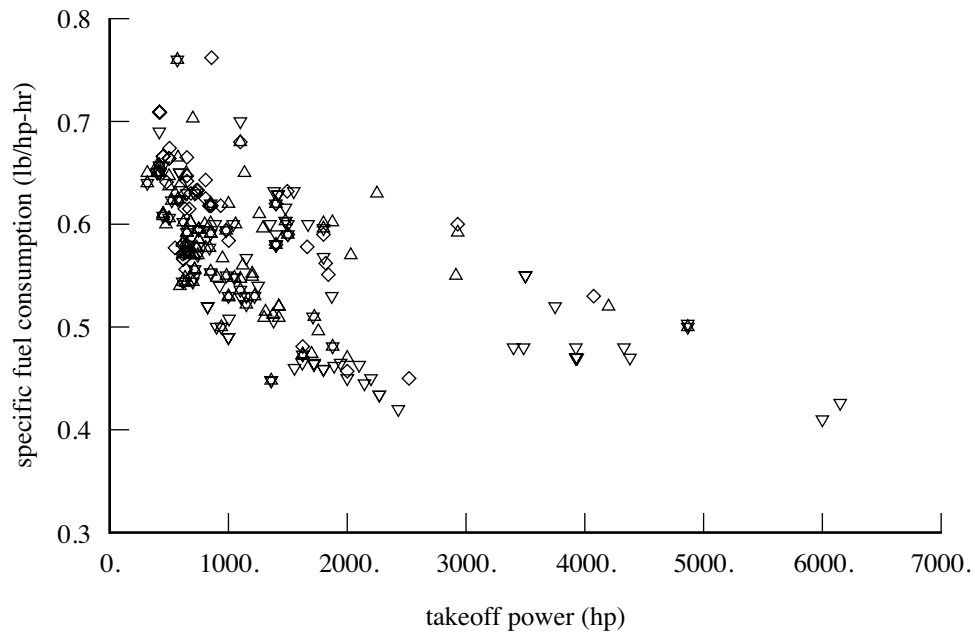
Figure 17-1. Historical data for turboshaft engine specific fuel consumption.
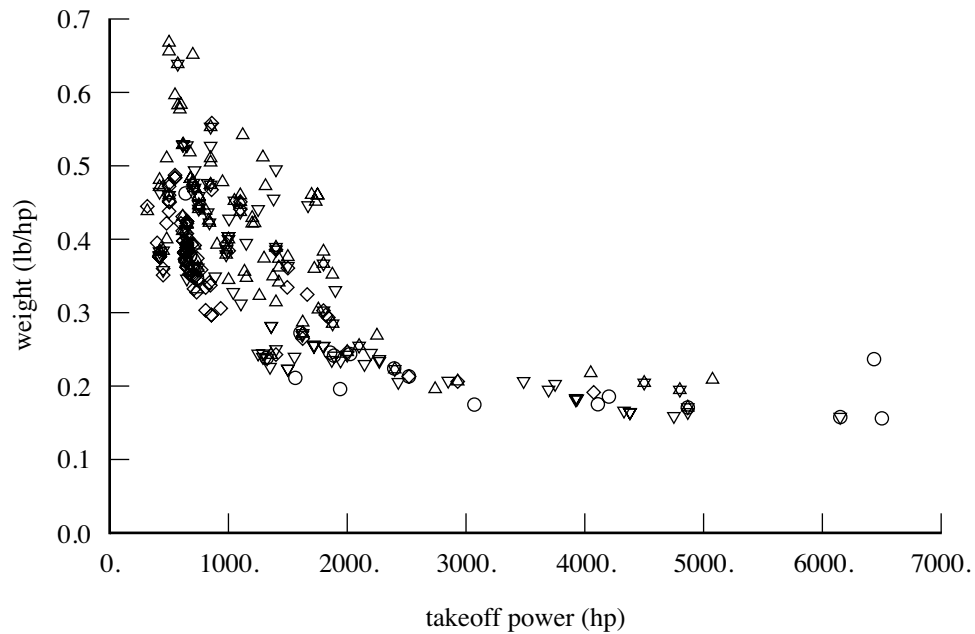

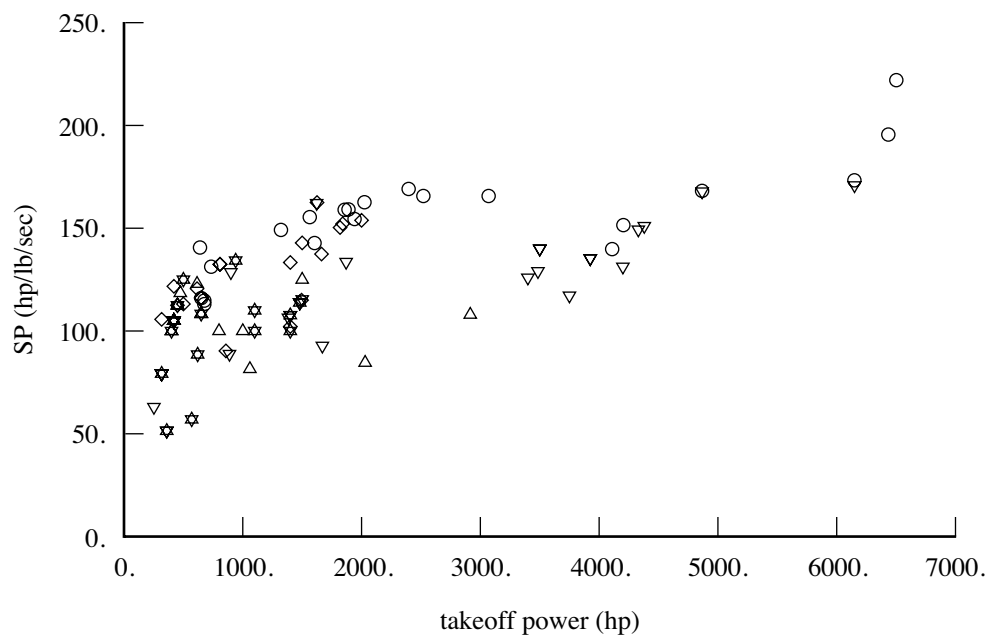Figure 17-2. Historical data for turboshaft engine weight.

Figure 17-3. Historical data for turboshaft engine specific power.

Referred Parameter Turboshaft Engine Model

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of engine performance data for existing or projected engines over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck. The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The use of referred parameters to curve-fit engine performance data was suggested by David Woodley from Boeing during the JVX program (1983). The RPTEM was developed and documented by Michael P. Scully and Henry Lee of ASRAO, U.S. Army Aeroflightdynamics Directorate (AFDD), with a subsequent implementation written by Sam Ferguson (1995).

18–1   Operating Environment

The operating condition and atmosphere give the standard conditions (temperature T_{std} and pressure p_{std}) for a specified pressure altitude; the sea-level standard conditions (temperature T_0 and pressure p_0); and the operating temperature T and pressure $p = p_{\mathrm{std}}$. Here the temperatures are °R or °K. The engine characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$.

The flight Mach number $M = V/c_s = V/c_{s0}\sqrt{\theta}$ is obtained from the aircraft speed V.

The inlet ram air temperature ratio and pressure ratio are obtained then from M and the inlet ram recovery efficiency η_d:

$$\theta_M = \left(1 + \frac{\gamma-1}{2}M^2\right) = \left(1 + 0.2M^2\right)$$

$$\delta_M = \left(1 + \frac{\gamma-1}{2}\eta_d M^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 0.2\eta_d M^2\right)^{3.5}$$

where the ratio of specific heats $\gamma = 1.4$.

18–2   Engine Ratings

The power available from a turboshaft engine depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine. Typical engine ratings are given in table 18-1. Engine power is generally specified in terms of SLS static MCP. Takeoff typically uses MRP. CRP or ERP are restricted to use in one-engine inoperative (OEI) emergencies.

Table 18-1. Typical engine ratings.

rating	description	time limit
MCP	maximum continuous power	∞
IRP	intermediate rated power	30 min
MRP	maximum rated power	10 min
CRP	contingency rated power	2.5 min
ERP	emergency rated power	1.0 min

18–3   Performance Characteristics

The engine performance is described by: the power available P_a, at each engine rating and the specification engine turbine speed N_{spec}; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce power required P_q at engine turbine speed N; and the gross jet thrust F at a given power required P_q. Then the specific power is $SP = P/\dot{m}$, and the specific fuel consumption is $\text{sfc} = \dot{w}/P$.

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). Referred or corrected engine parameters are used in the model:

power	$\dfrac{P}{\delta\sqrt{\theta}}$	fuel flow	$\dfrac{\dot{w}}{\delta\sqrt{\theta}}$
mass flow	$\dfrac{\dot{m}}{\delta/\sqrt{\theta}}$	thrust	$\dfrac{F}{\delta}$
specific power	$\dfrac{P/\dot{m}}{\theta}$	turbine speed	$\dfrac{N}{\sqrt{\theta}}$

For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: power P_{0R}, specific power SP_{0R}, and mechanical power limit $P_{\text{mech}R}$. The mass flow

is then $\dot{m}_{0R} = P_{0R}/SP_{0R}$. The gross jet thrust F_{g0C} is given at MCP. These characteristics are at the specification turbine speed N_{spec}.

The installed power required P_{req} and power available $P_{av} > P_{req}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust F_N, from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such as IR suppressor or cooling air) are treated as momentum drag D_{aux}.

The difference between net and gross jet thrust is the momentum drag: $F_n = F_g - \dot{m}_{req}V = \dot{m}_{req}(V_j - V)$, where V_j is the engine jet exhaust velocity. Note that traditional units for mass flow are pound/sec (pps), while this equation requires slug/sec ($\dot{m}_{req}/g$ replaces $\dot{m}_{req}$).

The uninstalled power required is P_q, the power available P_a, the gross jet thrust F_g, and net jet thrust F_n. The engine model calculates P_a as a function of flight condition and engine rating; or calculates engine mass flow, fuel flow, and jet thrust at P_q.

18–4   Installation

The difference between installed and uninstalled power is the inlet and exhaust losses P_{loss}:

$$P_{av} = P_a - P_{\text{loss}}$$
$$P_{req} = P_q - P_{\text{loss}}$$

The inlet ram recovery efficiency η_d (through δ_M) is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available or power required: $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_a$ or $P_{\text{loss}} = (\ell_{in} + \ell_{ex})P_q$. So

$$P_{av} = P_a(1 - \ell_{in} - \ell_{ex})$$
$$P_q = P_{req}/(1 - \ell_{in} - \ell_{ex})$$

The engine model gives uninstalled power and the gross thrust F_g for a nominal exhaust nozzle area. The gross jet thrust F_G and exhaust power loss (ℓ_{ex}) are both functions of the exhaust nozzle area. Smaller exhaust nozzle areas increase exhaust losses and increase gross thrust. Thus the ratio of installed to uninstalled thrust is approximated by a function of the exhaust power loss:

$$F_G/F_g = K_{fgr} = K_{fgr0} + K_{fgr1}\ell_{ex} + K_{fgr2}\ell_{ex}^2 + K_{fgr3}\ell_{ex}^3$$

so the net installed jet thrust is

$$F_N = K_{fgr}F_g - \dot{m}_{req}V$$

The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{\text{aux}} = f_{\text{aux}}\dot{m}_{req}$:

$$D_{\text{aux}} = (1 - \eta_{\text{aux}})\dot{m}_{\text{aux}}V = (1 - \eta_{\text{aux}})f_{\text{aux}}\dot{m}_{req}V$$

where η_{aux} is the ram recovery efficiency. Exhaust losses (ℓ_{ex}) and auxiliary air flow parameters (η_{aux}, f_{aux}) are defined for IR suppressor on and off. Inlet particle separator loss is added to the inlet losses (ℓ_{in}).

18–5   Power Turbine Speed

The shaft power available is a function of the gas power available P_G and the power turbine efficiency η_t: $P_a = \eta_t P_G$. Generally the power turbine speed N has a significant effect on η_t, but almost no effect

on P_G. The model used for the efficiency variation is $\eta_t \cong 1 - |(N/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}$, where N_{opt} is the speed for peak efficiency, hence

$$\frac{P(N)}{P(N_{\mathrm{spec}})} = \frac{\eta_t(N)}{\eta_t(N_{\mathrm{spec}})} = \frac{1 - |(N/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}{1 - |(N_{\mathrm{spec}}/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}$$

Two approximations for the optimum turbine speed are used. The first is a linear expression in $p = P/(P_{0R}\delta\sqrt{\theta})$:

$$N_{\mathrm{opt}} = N_{\mathrm{spec}}\sqrt{\theta}\left(K_{NoptA}\sqrt{\theta_M} + \frac{K_{NoptB}}{\delta_M}p\right)$$

and the second is a cubic function of $p = P/(P_{0C}\delta\sqrt{\theta})$:

$$N_{\mathrm{opt}} = N_{\mathrm{opt0C}}\sqrt{\theta}\left(K_{Nopt0} + K_{Nopt1}p + K_{Nopt2}p^2 + K_{Nopt3}p^3\right)[\theta_M]^{X_{Nopt}}$$

The second expression is based on a larger data sample. For power available calculations, $P = P_a(N_{\mathrm{spec}})$; for power required calculations, $P = P_q(N)$.

18–6   Power Available

Given the flight condition and engine rating, the power available P_a is calculated as follows. The specific power and referred mass flow (at N_{spec}, relative to SP_0 and $\dot{m}_0$ for this rating) are approximated by functions of the ambient temperature ratio θ and inlet ram air ratios:

$$SP_a(N_{\mathrm{spec}}) = SP_0\,\theta\,K_{spa}\left[\delta_M\sqrt{\theta_M}\right]^{X_{spa}}$$

$$\dot{m}_a(N_{\mathrm{spec}}) = \dot{m}_0\left(\delta/\sqrt{\theta}\right)e^{K_{mfa}}\left[\delta_M\sqrt{\theta_M}\right]^{X_{mfa}}$$

where the static lapse rate (K_{spa}, K_{mfa}) and ram air exponents (X_{spa}, X_{mfa}) are piecewise linear functions of θ. The power available is then

$$P_a(N_{\mathrm{spec}}) = P_0\frac{SP_a(N_{\mathrm{spec}})}{SP_0}\frac{\dot{m}_a(N_{\mathrm{spec}})}{\dot{m}_0} = P_0\left(\delta\sqrt{\theta}\right)K_{spa}e^{K_{mfa}}\left[\delta_M\sqrt{\theta_M}\right]^{X_{spa}+X_{mfa}}$$

This expression for $\dot{m}_a$ is used only to calculate P_a; elsewhere the $\dot{m}_q$ expression below (for performance at power required) is used to obtain the mass flow at a power P_q. Finally

$$P_a(N) = P_a(N_{\mathrm{spec}})\frac{1 - |(N/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}{1 - |(N_{\mathrm{spec}}/N_{\mathrm{opt}}) - 1|^{X_{N\eta}}}$$

is the power available at turbine speed N. Installation losses P_{loss} are subtracted from P_a ($P_{av} = P_a - P_{\mathrm{loss}}$), and then the mechanical limit is applied: $P_{av} = \min(P_{av}, rP_{\mathrm{mechR}})$, $r = N/N_{\mathrm{spec}}$. The mechanical limit is properly a torque limit, $Q_{\mathrm{mech}} = P_{\mathrm{mech}}/N_{\mathrm{spec}}$, but is expressed as a power limit for clarity.

18-6.1   Piecewise Linear Parameters

Several parameters K are input as piecewise linear functions of the temperature ratio θ. One format of the input is a set of I regions, with the function in the i-th region given by $K = K_{0i} + K_{1i}\theta$. The break point between the i and $i - 1$ regions is

$$\theta_{bi} = -\frac{K_{0i} - K_{0(i-1)}}{K_{1i} - K_{1(i-1)}}$$

$$K_{bi} = K_{0i} + K_{1i}\theta_b$$

for $i = 2$ to I. Another format is a set of $I + 1$ break points, with the values at the i-th point (θ_{bi}, K_{bi}). Then the coefficients between i and $i + 1$ are

$$K_{0i} = \frac{K_{bi}\theta_{b(i+1)} - K_{b(i+1)}\theta_{bi}}{\theta_{b(i+1)} - \theta_{bi}}$$

$$K_{1i} = \frac{-K_{bi} + K_{b(i+1)}}{\theta_{b(i+1)} - \theta_{bi}}$$

for $i = 1$ to I. The interpolation is performed using $K = K_{0i} + K_{1i}\theta$ for θ in the range θ_{bi} to $\theta_{b(i+1)}$. Outside the defined regions, the linear expression is continued; hence θ_{bi} is used only for $i = 2$ to I.

18–7 Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required P_q (which might equal the power available), flight condition, and engine rating. Installation losses P_{loss} are added to P_{req} ($P_q = P_{req} + P_{\text{loss}}$). The referred quantities (relative to SLS static MCP quantities) are approximated by cubic functions of $q = P_q(N_{\text{spec}})/(P_{0C}\delta\sqrt{\theta})$:

$$\dot{w}_{req} = \dot{w}_{0C} \left(\delta\sqrt{\theta}\right) \left(K_{ffq0} + K_{ffq1}q + K_{ffq2}q^2 + K_{ffq3}q^3\right) [\theta_M]^{-X_{ffq}}$$

$$\dot{m}_{req} = \dot{m}_{0C} \left(\delta/\sqrt{\theta}\right) \left(K_{mfq0} + K_{mfq1}q + K_{mfq2}q^2 + K_{mfq3}q^3\right) [\theta_M]^{X_{mfq}}$$

$$F_g = F_{g0C} \left(\delta\right) \left(K_{fgq0} + K_{fgq1}q + K_{fgq2}q^2 + K_{fgq3}q^3\right) [\theta_M]^{X_{fgq}}$$

at N_{spec}, with $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The mass flow and fuel flow are primarily functions of the gas power P_G, and are assumed to be independent of η_t, hence independent of turbine speed. However, these equations are functions of $P_q(N_{\text{spec}})$, obtained from $P_q(N)$ using

$$P_q(N_{\text{spec}}) = P_q(N) \frac{1 - |(N_{\text{spec}}/N_{\text{opt}}) - 1|^{X_{N\eta}}}{1 - |(N/N_{\text{opt}}) - 1|^{X_{N\eta}}}$$

Then the installed net jet thrust F_N and momentum drag D_{aux} are calculated.

18–8 Scaling

The parameters of the engine model can be defined for a specific engine, but it is also necessary to scale the parameters as part of the aircraft sizing task, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and N_{spec}).

The engine model includes reference values of the engine performance parameters: P_{0R}, SP_{0R}, $P_{\text{mech}R}$, sfc_{0C}, SF_{0C}, N_{spec}, $N_{\text{opt}0C}$. Mass flow and fuel flow are obtained from $\dot{m}_{0R} = P_{0R}/SP_{0R}$ and $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The reference power at each engine rating R defines a ratio to MCP: $r_{p0R} = P_{0R}/P_{0C}$. Similarly for specific power and mechanical limits: $r_{s0R} = SP_{0R}/SP_{0C}$, $r_{m0R} = P_{\text{mech}R}/P_{0C}$. These ratios are kept fixed when the engine is scaled.

The engine size is specified as takeoff power $P_{to} = P_{\text{eng}}$, which is the power at rating R, for SLS static conditions and specification turbine speed N_{spec}. Hence the MCP is $P_{0C} = P_{to}/r_{p0R}$, and the power at all other ratings follows. If P_{0C} is not equal to the reference value of the engine model, then the engine is scaled. To reflect advanced technology, the specific power, specific fuel consumption, and

specification turbine speed can be specified: $SP_{0C} = SP_{\text{tech}}, \text{sfc}_{0C} = \text{sfc}_{\text{tech}}, N_{\text{spec}} = N_{\text{tech}}$. The default values are the reference values of the engine model. In the following paragraph, the subscript "tech" refers to these quantities; the subscript "ref" means the engine model reference values.

The engine technology parameters SP_{0C} and sfc_{0C} are assumed to vary linearly with mass flow $\dot{m}_{0C}$ up to a limit $\dot{m}_{\text{lim}}$, and constant thereafter at SP_{lim} and sfc_{lim}. The mass flow at the technology condition is $\dot{m}_{\text{tech}} = P_{\text{ref}}/SP_{\text{tech}}$, with the technology values SP_{tech} and sfc_{tech}. The intercept values are projected from the technology values: $K_{sp0} = SP_{\text{tech}} - K_{sp1}\dot{m}_{\text{tech}}, K_{sp1} = (SP_{\text{lim}} - SP_{\text{tech}})/(\dot{m}_{\text{lim}} - \dot{m}_{\text{tech}})$; and similarly for sfc. Then for $\dot{m}_{0C} < \dot{m}_{\text{lim}}$

$$SP_{0C} = K_{sp0} + K_{sp1}\dot{m}_{0C}$$

$$\text{sfc}_{0C} = K_{sfc0} + K_{sfc1}\dot{m}_{0C}$$

and for $\dot{m}_{0C} \geq \dot{m}_{\text{lim}}$

$$SP_{0C} = SP_{\text{lim}}$$

$$\text{sfc}_{0C} = \text{sfc}_{\text{lim}}$$

These equations are used if $\dot{m}_{\text{lim}} > \dot{m}_{\text{tech}}$. Otherwise the model sets $K_{sp1} = K_{sfc1} = 0$, so there is no variation with scale: $SP_{0C} = SP_{\text{tech}}$ and $\text{sfc}_{0C} = \text{sfc}_{\text{tech}}$. Usually the effect of size gives $K_{sp2} \geq 0$ and $K_{sfc2} \leq 0$. The power at the limit is $P_{\text{lim}} = SP_{\text{lim}}\dot{m}_{\text{lim}}$. Using $\dot{m}_{0C} = P_{0C}/SP_{0C}$, the specific power equation can be solved for the mass flow given the power:

$$\dot{m}_{0C} = \begin{cases} P_{0C}/K_{sp0} & K_{sp1} = 0 \\ P_{0C}/SP_{\text{lim}} & P_{0C} \geq P_{\text{lim}} \\ \sqrt{(\frac{K_0}{2K_1})^2 + \frac{P_{0C}}{K_1}} - \frac{K_0}{2K_1} & \text{otherwise} \end{cases}$$

From this mass flow, SP_{0C} and sfc_{0C} are calculated, hence the fuel flow $\dot{w}_{0C} = \text{sfc}_{0C}P_{0C}$. The specific thrust available at MCP is assumed to be constant, and the specification power turbine speed decreases with the mass flow:

$$F_{g0C} = SF_{0C}\dot{m}_{0C}$$

$$N_{\text{spec}} = \left(N_{\text{spec}} - K_{Ns2}/\sqrt{\dot{m}_{0C}}\right)_{\text{tech}} + K_{Ns2}/\sqrt{\dot{m}_{0C}} = K_{Ns1} + K_{Ns2}/\sqrt{\dot{m}_{0C}}$$

$$N_{\text{opt0C}} = \frac{(N_{\text{opt0C}})_{\text{ref}}}{N_{\text{tech}}} N_{\text{spec}} = K_{No}N_{\text{spec}}$$

Then the power and specific power at all ratings R are obtained from the ratios: $P_{0R} = r_{p0R}P_{0C}$, $SP_{0R} = r_{s0R}SP_{0C}, P_{\text{mech}R} = r_{m0R}P_{0C}$.

18–9   Engine Speed

The model as described in the previous sections may not adequately account for variation of engine performance with engine speed, so it is also possible to define the parameters corresponding to a set of engine speed ratios $r = N/N_{\text{spec}}$. Then the engine performance and power available quantities are linearly interpolated to obtain the values at the required engine speed N. If this option is used, then the correction based on $P(N)/P(N_{\text{spec}}) = \eta_t(N)/\eta_t(N_{\text{spec}})$ is not applied.

18–10   Weight

The engine weight can be a fixed input value, calculated as a function of power, or scaled with engine mass flow. As a function of power, the weight of one engine is:

$$W_{\text{one eng}} = K_{0\text{eng}} + K_{1\text{eng}}P + K_{2\text{eng}}P^{X_{\text{eng}}}$$

where P is the installed takeoff power (SLS static, specified rating) per engine. A constant weight per power W/P is given by using only $K_{1\text{eng}}$. Alternatively, the specific weight $SW = P/W$ can be scaled with the mass flow $\dot{m}_{0C}$. The scaling is determined from the specific weight SW_{ref} at the mass flow $\dot{m}_{\text{tech}}$, and the limit SW_{lim} at $\dot{m}_{\text{lim}}$. Then

$$SW = \begin{cases} K_{sw0} + K_{sw1}\dot{m}_{0C} & \dot{m}_{0C} < \dot{m}_{\text{lim}} \\ SW_{\text{lim}} & \dot{m}_{0C} \geq \dot{m}_{\text{lim}} \end{cases}$$

(if $\dot{m}_{\text{lim}} < \dot{m}_{\text{tech}}$, then $K_{sw1} = 0$, so $SW = SW_{\text{ref}}$) and $W_{\text{one eng}} = P/SW$.

18–11   Units

In this engine model, only the reference values and scaling constants are dimensional. Conventional English units and SI units are shown in table 18-2. Units of specific power and specific fuel consumption follow from these conventions.

Table 18-2. Conventional units.

	power P	mass flow $\dot{m}$	fuel flow $\dot{w}$	force F	turbine speed N
English:	horsepower	pound/sec	pound/hour	pound	rpm
SI:	kiloWatt	kilogram/sec	kilogram/hour	Newton	rpm

18–12   Typical Parameters

Typical values of the principal parameters describing the engine and its performance are given in table 18-3 for several generic engine sizes. These values represent good current technology. Advanced technology can be introduced by reducing the specific fuel consumption and weight, and increasing the specific power. Typical ratios of the power, specific power, and mass flow to the values at MCP are given in table 18-4 for several ratings. Figure 18-1 shows typical performance characteristics: fuel flow $\dot{w}$, mass flow $\dot{m}$, and net jet thrust F_g variation with power P and speed (referred quantities, normalized, at N_{spec}). Figure 18-2 shows typical variation of the power available with engine turbine speed. Figures 18-3 to 18-8 show typical power available characteristics: specific power SP, mass flow $\dot{m}$, and power P variation with temperature ratio θ, for static and 200 knots conditions and several engine ratings (referred quantities, normalized, at N_{spec}).

Table 18-3. Typical engine performance parameters.

power (MCP)	P_{0C}	500	1000	2000	4000	8000	16000	hp
specific power	SP_{0C}	116	125	134	143	153	164	hp/lb/sec
mechanical limit	P_{mech}	750	1500	3000	6000	12000	24000	hp
specific fuel cons.	sfc_{0C}	0.54	0.48	0.44	0.40	0.38	0.35	lb/hp-hr
specific jet thrust	SF_{0C}	5.9	7.3	8.9	11.0	13.6	16.7	lb/lb/sec
gross jet thrust	F_{g0C}	25	58	134	308	707	1625	lb
mass flow	$\dot{m}_{0C}$	4.3	8.0	15.0	27.9	52.1	97.3	lb/sec
turbine speed	N_{spec}	35600	26100	19100	14000	10200	7500	rpm
weight	W/P_{to}	0.34	0.23	0.18	0.16	0.16	0.15	lb/hp

Table 18-4. Typical parameter ratios for various ratings (percent).

		IRP	MRP	CRP	P_{mech}
power	P_0/P_{0C}	120	127	133	150
specific power	SP_0/SP_{0C}	117	123	128	
mass flow	$\dot{m}_0/\dot{m}_{0C}$	102.6	103.2	103.9	
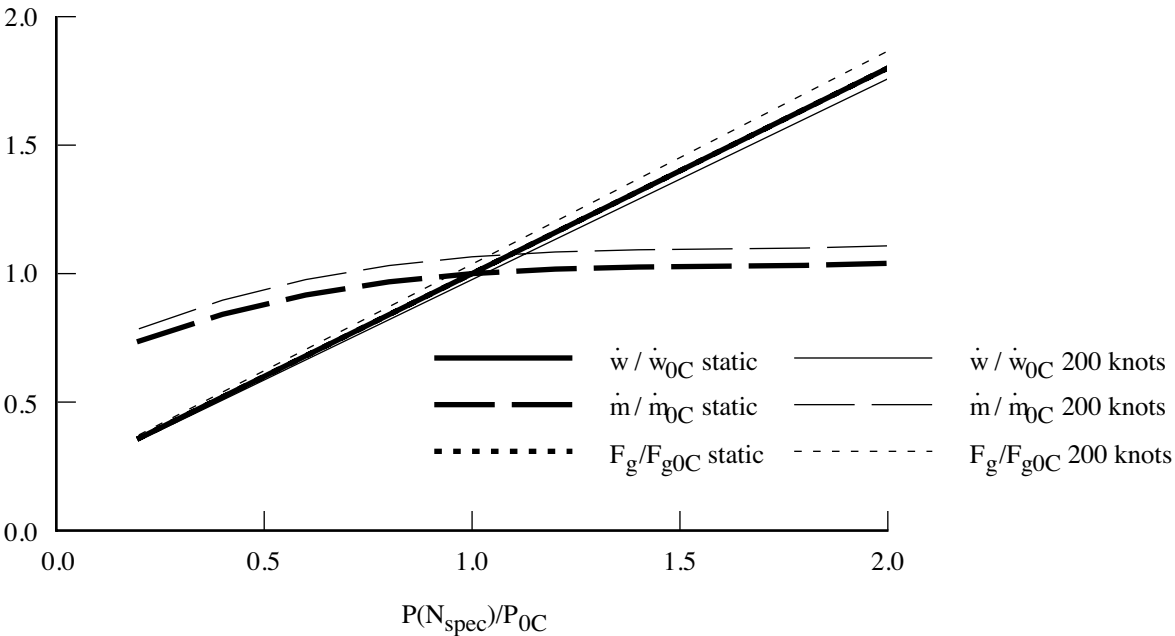
Figure 18-1. Fuel flow, mass flow, and net jet thrust variation with power.
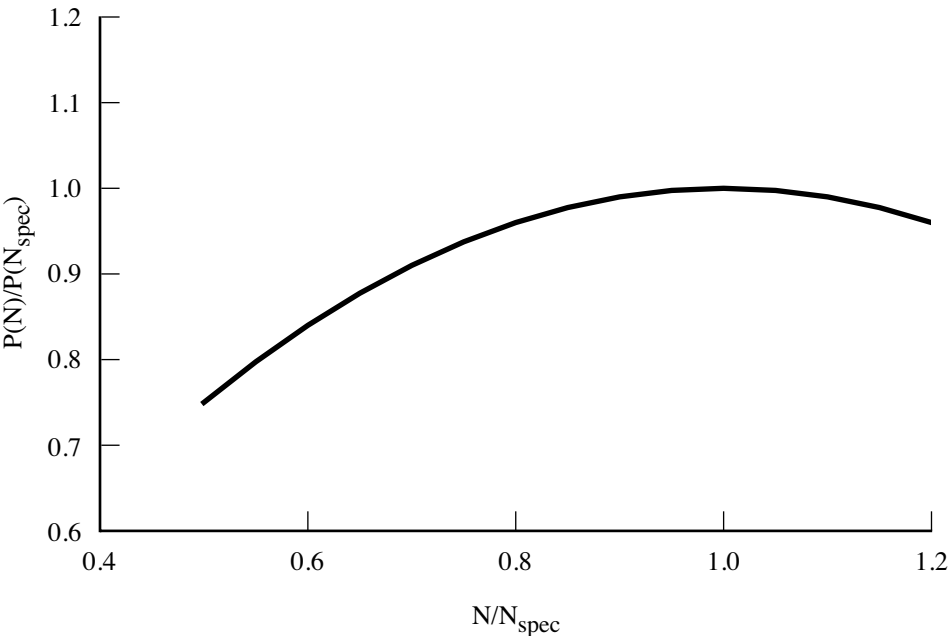
Figure 18-2. Power variation with turbine speed.
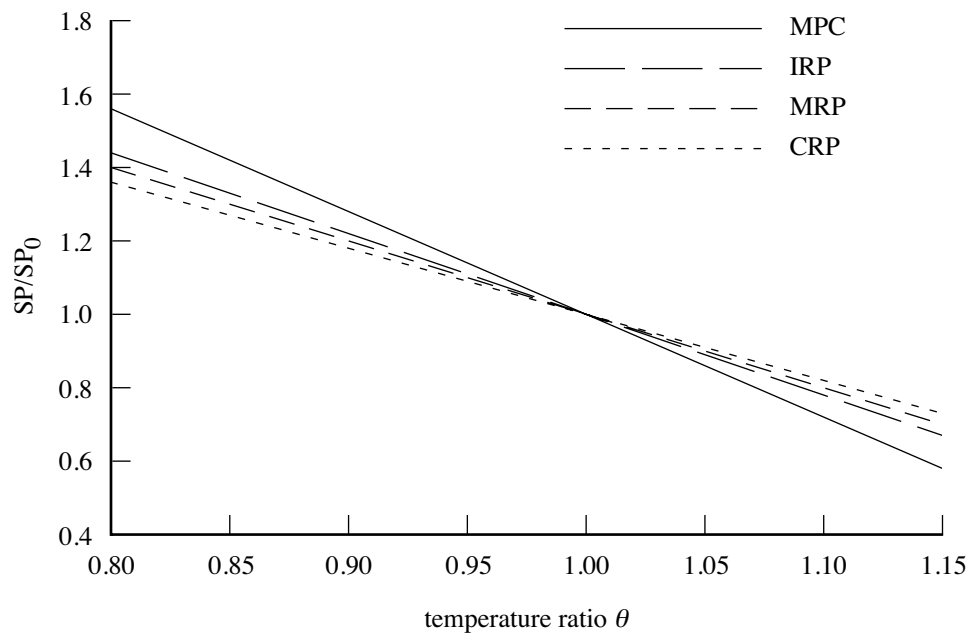
Figure 18-3. Specific power variation with temperature ratio, static.
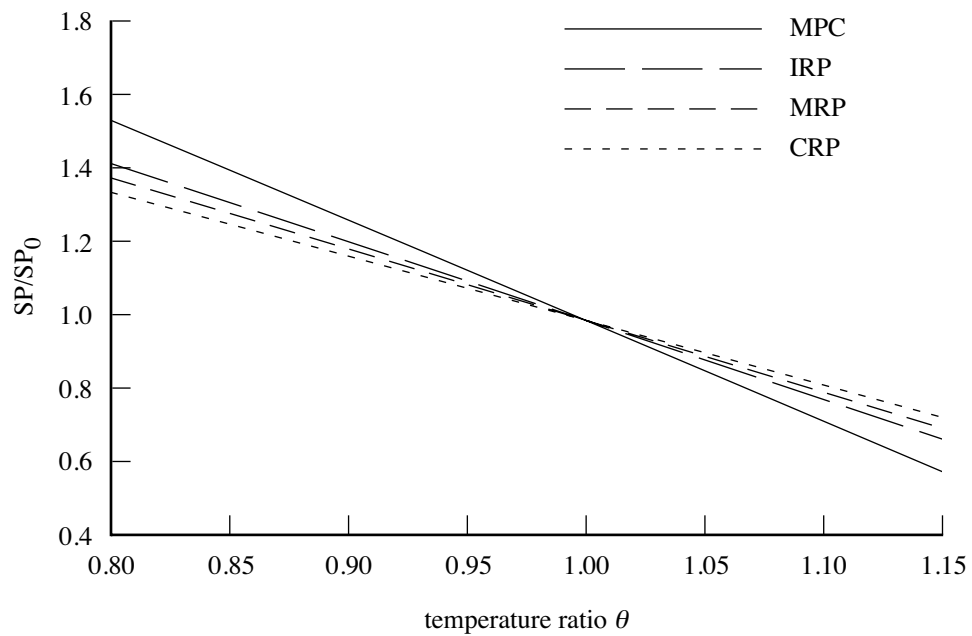
Figure 18-4. Specific power variation with temperature ratio, 200 knots.
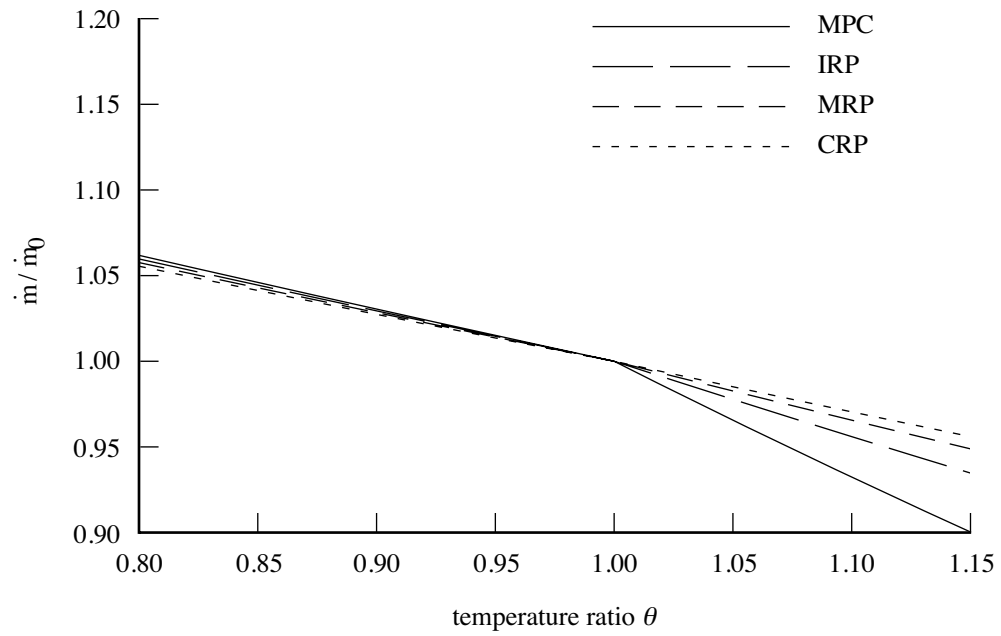
Figure 18-5. Mass flow variation with temperature ratio, static.
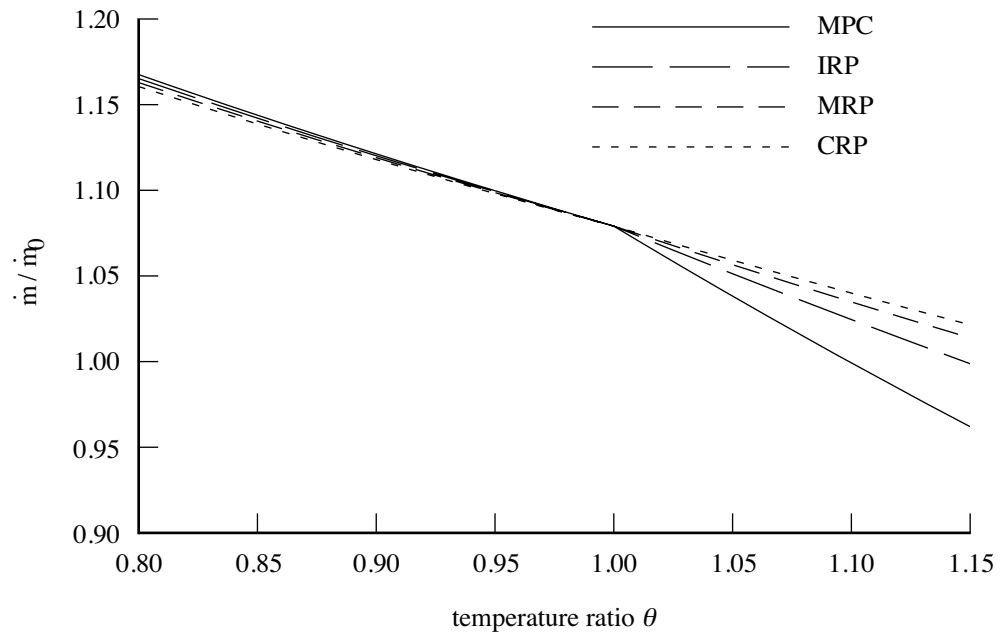
Figure 18-6. Mass flow variation with temperature ratio, 200 knots.
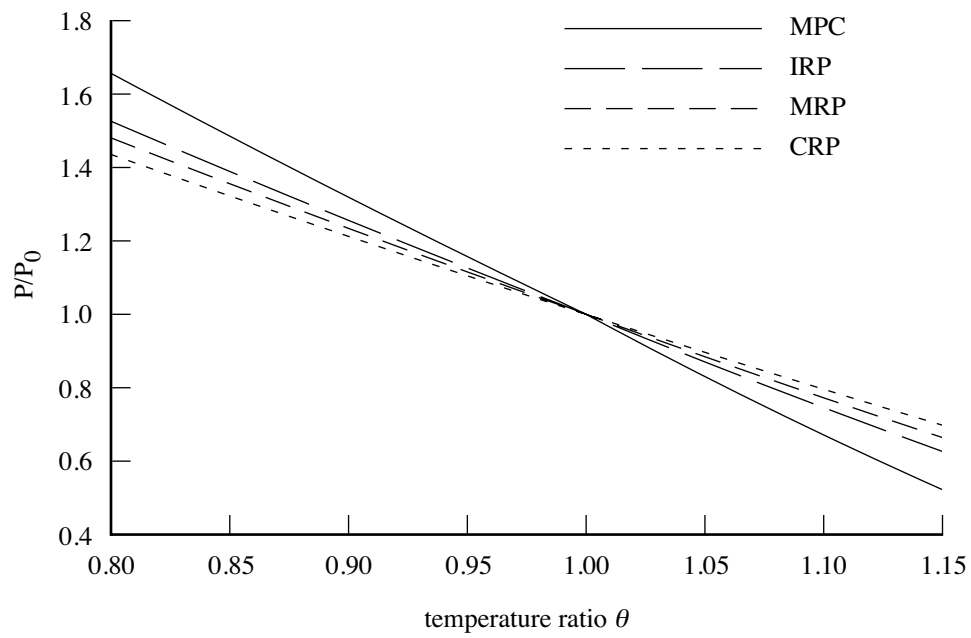
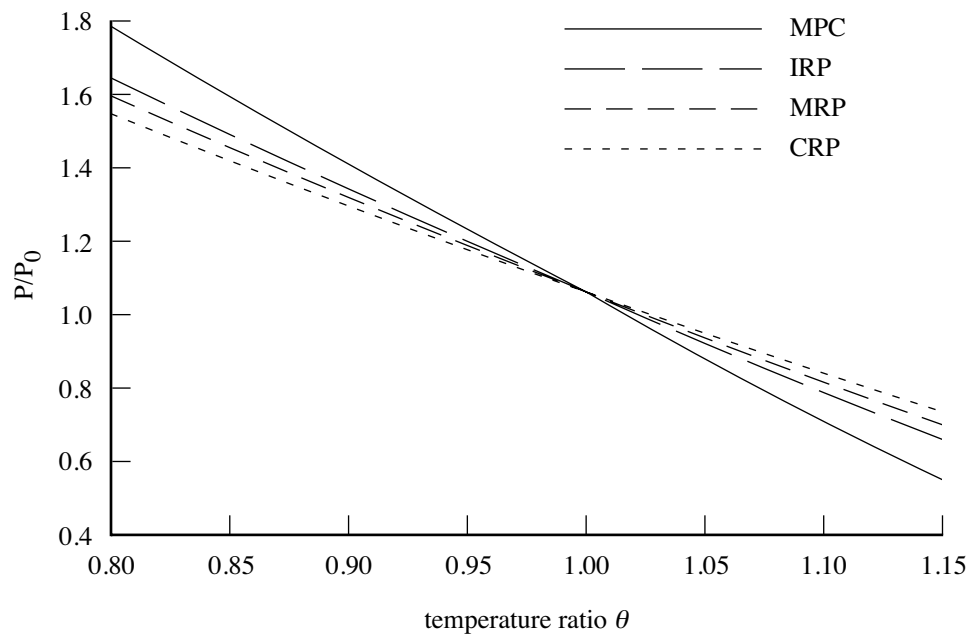Figure 18-7. Power variation with temperature ratio, static.

Figure 18-8. Power variation with temperature ratio, 200 knots.

AFDD Weight Models

This chapter presents the rotorcraft weight models developed by the U.S. Army Aeroflightdynamics Directorate (AFDD). For some weight groups several models are available, designated AFDDnn. The weights are estimated from parametric equations, based on the weights of existing turbine-powered helicopters and tiltrotors. The figures of this chapter compare the weights calculated from these equations with the actual weights. The results of these equations are the weight in pounds, and the units of the parameters are noted in the tables. Technology factors χ are included in the weight equations. The input usually includes a weight increment dW that can be added to the results of the weight model. Thus typically a component or element weight is obtained from $W = \chi W_{\mathrm{model}} + dW$. Weight of individual elements in a group can be fixed by using dW and setting the corresponding technology factor $\chi = 0$. With $\chi \neq 0$, the increment dW can account for something not included in the parametric model.

The weight models are implemented as part of the aircraft components. The weights are entered into the weight statement data structure (extended RP8A format) for each component, reflected in the organization of this chapter.

19–1   Wing Group

The wing group consists of: basic structure (primary structure, consisting of torque box and spars, plus extensions); fairings (leading edge and trailing edge); fittings (non-structural); fold/tilt structure; and control surfaces (flaps, ailerons, flaperons, and spoilers). There are separate models for a tiltrotor or tiltwing configuration and for other configurations (including compound helicopter).

19-1.1   Tiltrotor or Tiltwing Wing

Wing weight equations for a tiltrotor or tiltwing aircraft are based on methodology developed by Chappell and Peyran (refs. 1 and 2). The wing is sized primarily to meet torsional stiffness requirements. The primary structure weight is calculated from torque box and spar weights:

$$W_{\mathrm{box}} = A_{tb}\rho_{tb}b_w/e_{tb}$$
$$W_{\mathrm{spar}} = C_t A_{sp}\rho_{sp}b_w/e_{sp}$$
$$w_{\mathrm{prim}} = (W_{\mathrm{box}} + W_{\mathrm{spar}})f_{\mathrm{units}}$$
$$W_{\mathrm{prim}} = \chi_{\mathrm{prim}}w_{\mathrm{prim}}$$

A consistent mass-length-time system is used in the equations for W_{box} and W_{spar}, which therefore have units of slug or kg. The primary structure weight W_{prim} however has units of lb or kg, hence a conversion factor $f_{\mathrm{units}} = g$ is required for English units. The wing fairing (leading edge and trailing edge), control

surface (flaps, ailerons, flaperons, and spoilers), fittings (non-structural), and fold/tilt weights are:

$$w_{\text{fair}} = S_{\text{fair}} U_{\text{fair}} \qquad\qquad\qquad W_{\text{fair}} = \chi_{\text{fair}} w_{\text{fair}}$$

$$w_{\text{flap}} = S_{\text{flap}} U_{\text{flap}} \qquad\qquad\qquad W_{\text{flap}} = \chi_{\text{flap}} w_{\text{flap}}$$

$$w_{\text{fit}} = f_{\text{fit}} T_{\text{cap}} \qquad\qquad\qquad W_{\text{fit}} = \chi_{\text{fit}} w_{\text{fit}}$$

$$w_{\text{fold}} = f_{\text{fold}} \left(W_{\text{prim}} + W_{\text{fair}} + W_{\text{flap}} + W_{\text{fit}} + W_{\text{tip}} \right) \qquad W_{\text{fold}} = \chi_{\text{fold}} w_{\text{fold}}$$

The control surface area S_{flap} for a tiltrotor wing is the sum of the flap and flaperon areas. The fairing area is

$$S_{\text{fair}} = (b_w - w_{\text{attach}}) \, c_w \, (1 - w_{tb}) - S_{\text{flap}}$$

The wing extension weight is:

$$w_{\text{ext}} = S_{\text{ext}} U_{\text{ext}} \qquad W_{\text{ext}} = \chi_{\text{ext}} w_{\text{ext}}$$

$$w_{\text{efold}} = f_{\text{efold}} W_{\text{ext}} \qquad W_{\text{efold}} = \chi_{\text{efold}} w_{\text{efold}}$$

and these terms are added to W_{prim} and W_{fold}. The tiltrotor wing weight (and wing folding weight in fuselage group) depends on W_{tip}, the weight on the wing tips; which is the sum of rotor group, engine section or nacelle group, air induction group, engine system, drive system (except drive shaft), rotary wing and conversion flight controls, hydraulic group, trapped fluids, and wing extensions. The weight on wing tip is used as the fraction $f_{\text{tip}} = W_{\text{tip}}/W_{SD}$; the mass on the wing tip is M_{tip} (slug or kg).

To estimate the wing weights, the required stiffness is scaled with input frequencies (per rev) of the wing primary bending and torsion modes. First the torque box is sized to meet the torsional stiffness (frequency) requirement. Next spar cap area is added as required to meet the chord and beam bending frequency requirements. Finally spar cap area is added if necessary for a jump takeoff condition. Wing section form factors, relating typical airfoil and torque box geometry to ideal shapes, are input or calculated from the thickness-to-chord ratio and the torque box chord to wing chord ratio:

$$
\begin{aligned}
F_B &= 0.073 \sin(2\pi(\tau_w - 0.151)/0.1365) + 0.14598\tau_w \\
&\quad + 0.610 \sin(2\pi(w_{tb} + 0.080)/2.1560) - (0.4126 - 1.6309\tau_w)(w_{tb} - 0.131) + 0.0081 \\
F_C &= 0.640424 w_{tb}^2 - 0.89717 w_{tb} + 0.4615\tau_w + 0.655317 \\
F_T &= ((0.27 - \tau_w)/0.12)0.12739 \left(-0.96 + \sqrt{3.32 + 94.6788 w_{tb} - (w_{tb}/0.08344)^2} \right) \\
&\quad - 2.7545 w_{tb}^2 + 5.1799 w_{tb} - 0.2683 \\
F_{VH} &= 0.25 \sin(5.236 w_{tb}) + 0.325
\end{aligned}
$$

for beam bending, chord bending, torsion, and spar cap vertical/horizontal bending. The ideal shape for torsional stiffness is a tube of radius t_w, so the torsional stiffness $J = F_T A_{tb} t_w^2/4$. The ideal shape for chord bending is two caps c_{tb} apart, so $I_{Ctb} = F_C A_{tb} c_{tb}^2/4$. The ideal shape for beam bending is two caps t_w apart, so $I_{Bsp} = F_{VH} A_{sp} t_w^2/4$ and $I_{Btb} = F_B A_{tb} t_w^2/4$. The torque box cross-sectional area is obtained from the wing torsion frequency;

$$GJ = (\omega_T \Omega)^2 \frac{1}{2}(b_w - w_{\text{attach}})\frac{1}{2} M_{\text{tip}} r_{\text{pylon}}^2$$

$$A_{tb} = 4GJ/(G_{tb} F_T t_w^2)$$

It is assumed that between the points of attachment to the fuselage, the torque box is very stiff in torsion, but does deflect in bending. So the effective length is $\frac{1}{2}(b_w - w_{\text{attach}})$ for torsion, and $\frac{1}{2}b_w$ for bending.

The spar cap cross-sectional area (in addition to torque box material) is obtained from beam and chord bending frequencies:

$$EI_C = (\omega_C \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{\text{tip}} f_{\text{mode}}$$

$$EI_B = (\omega_B \Omega)^2 \frac{1}{24} b_w^3 \frac{1}{2} M_{\text{tip}} f_{\text{mode}}$$

$$EI_{Ctb} = E_{tb} F_C A_{tb} c_{tb}^2 / 4$$
$$EI_{Csp} = EI_C - EI_{Ctb}$$
$$A_{Csp} = EI_{Csp} / (E_{sp} c_{tb}^2 / 4)$$

$$EI_{Btb} = E_{tb} F_B A_{tb} t_w^2 / 4$$
$$EI_{VH} = E_{sp} F_{VH} A_{Csp} t_w^2 / 4$$
$$EI_{Bsp} = EI_B - EI_{Btb} - EI_{VH}$$
$$A_{Bsp} = EI_{Bsp} / (E_{sp} t_w^2 / 4)$$
$$EI_{sp} = EI_{VH} + EI_{Bsp}$$

$$A_{sp} = A_{Csp} + A_{Bsp}$$

where EI_{Csp} and EI_{Bsp} are replaced by zero if negative (no additional spar material required). The factor $f_{\text{mode}} = 1 - f_{\text{tip}}$ is a mode shape correction for fuselage motion. Next the primary structure, fairing, flap, and fitting weights are calculated as before; and the sum $W_{\text{wing}} = W_{\text{prim}} + W_{\text{fair}} + W_{\text{flap}} + W_{\text{fit}}$. Additional spar cap material for a jump takeoff condition is obtained from the ultimate applied bending moment at the wing root:

$$M_U = T_{\text{cap}} \ell_w \left(0.75(1 - f_{\text{tip}}) - 0.375(\ell_w / b_w)(W_{\text{wing}} / W_{SD}) \right)$$

where T_{cap} is the maximum thrust capability of one rotor, equal to the greater of $n_{\text{jump}} W_{SD} / N_{\text{rotor}}$ or $(C_T / \sigma) \rho A_b V_{\text{tip}}^2$ (from an input C_T / σ at the jump takeoff condition, SLS and hover rotor speed). The bending moment capacity of the wing is

$$M_{tb} = 2 EI_{Btb} \epsilon_U / t_w$$
$$M_{sp} = 2 C_m EI_{sp} \epsilon_U / t_w$$

Then the additional cross-section area is obtained from the moment deficit:

$$\Delta M = M_U - (M_{tb} + M_{sp})$$
$$\Delta A_{sp} = 2 \Delta M / (\epsilon_U E_{sp} t_w)$$
$$\Delta W_{\text{spar}} = C_j \Delta A_{sp} \rho_{sp} b_w / e_{sp}$$

where ΔM is replaced by zero if negative (no additional spar material required). If ΔW_{spar} is positive, it is added to W_{spar}, and the primary structure, fairing, flap, and fitting weights are recalculated. Parameters are defined in table 19-1, including units as used in these equations. Here a consistent mass-length-time system is used, producing W_{box} and W_{spar} in slug or kg. Typically the input uses conventional English units for density (lb/in^3) and modulus (lb/in^2).

Table 19-1. Parameters for tiltrotor wing weight.

parameter	definition	units
$W_{\text{box}}, W_{\text{spar}}$	wing torque box and spar weights	slug or kg
W_{SD}	structural design gross weight	lb
$(C_T/\sigma)_{\text{jump}}$	rotor maximum thrust capability (jump takeoff)	
n_{jump}	load factor at W_{SD} (jump takeoff)	
N_{rotor}	number of rotors	
b_w	wing span (length of torque box)	ft or m
$\ell_w = b_w - w_{fus}$	wing length (span less fuselage width)	ft or m
c_w	wing chord	ft or m
w_{tb}	torque box chord to wing chord ratio	
$c_{tb} = w_{tb}c_w$	torque box chord	ft or m
τ_w	wing airfoil thickness-to-chord ratio	
$t_w = \tau_w c_w$	wing thickness	ft or m
r_{pylon}	pylon radius of gyration (pitch inertia = $r_{\text{pylon}}^2 M_{\text{tip}}$)	ft or m
Ω	rotor speed for wing weight design condition	rad/sec
ω_T	wing torsion mode frequency (fraction rotor speed)	per rev
ω_B	wing beam bending mode frequency (fraction rotor speed)	per rev
ω_C	wing chord bending mode frequency (fraction rotor speed)	per rev
ρ_{tb}	density of torque box material	slug/ft^3 or kg/m^3
ρ_{sp}	density of spar cap material	slug/ft^3 or kg/m^3
G_{tb}	torque box shear modulus	lb/ft^2 or N/m^2
E_{tb}	torque box modulus	lb/ft^2 or N/m^2
E_{sp}	spar modulus	lb/ft^2 or N/m^2
ϵ_U	ultimate strain allowable (minimum of spar and torque box)	
C_t	weight correction for spar taper (equivalent stiffness)	
C_j	weight correction for spar taper (equivalent strength)	
C_m	strength correction for spar taper (equivalent stiffness)	
e_{tb}	structural efficiency factor, torque box	
e_{sp}	structural efficiency factor, spars	
U_{fair}	unit weight of leading and trailing edge fairings	lb/ft^2 or kg/m^2
U_{flap}	unit weight of control surfaces	lb/ft^2 or kg/m^2
S_{fair}	area of leading and trailing edge fairings	ft^2 or m^2
S_{flap}	area of control surfaces	ft^2 or m^2
f_{fit}	wing fittings and brackets (fraction maximum thrust of one rotor)	
f_{fold}	wing fold/tilt (fraction total weight excluding fold, including weight on tips)	
w_{attach}	width of wing structural attachments to body	ft or m
U_{ext}	unit weight of wing extension	lb/ft^2 or kg/m^2
S_{ext}	area of wing extensions (span b_{ext} times mean chord c_{ext})	ft^2 or m^2
f_{efold}	wing extension fold/tilt (fraction extension weight)	

19-1.2   Aircraft Wing

There are two models intended for the wing of a compound helicopter: area method and parametric method. For the area method (based on weight per unit area), the total wing weight excluding folding is:

$$w_{\text{wing}} = S_w U_w$$

$$f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}}$$

Typically $U_w = 5$ to 9 lb/ft^2. For the parametric method (AFDD93), the total wing weight including folding is:

$$w_{\text{wing}} = 5.66411 f_{LG\text{loc}} \left(\frac{W_{SD}}{1000 \cos \Lambda_w} \right)^{0.847} n_z^{0.39579} S_w^{0.21754} A_w^{0.50016}$$

$$((1 + \lambda_w)/\tau_w)^{0.09359} (1 - b_{\text{fold}})^{-0.14356}$$

$$f_{\text{prim}} = 1 - f_{\text{fair}} - f_{\text{flap}} - f_{\text{fit}} - f_{\text{fold}}$$

where $f_{LG\text{loc}} = 1.7247$ if the landing gear is on the wing, and 1.0 otherwise. Based on 25 fixed-wing aircraft, the average error of the aircraft wing equation is 3.4% (figure 19-1). Then the primary structure, secondary structure, and control surface weights are:

$$W_{\text{prim}} = \chi_{\text{prim}} f_{\text{prim}} w_{\text{wing}}$$

$$W_{\text{fair}} = \chi_{\text{fair}} f_{\text{fair}} w_{\text{wing}}$$

$$W_{\text{flap}} = \chi_{\text{flap}} f_{\text{flap}} w_{\text{wing}}$$

$$W_{\text{fit}} = \chi_{\text{fit}} f_{\text{fit}} w_{\text{wing}}$$

$$W_{\text{fold}} = \chi_{\text{fold}} f_{\text{fold}} w_{\text{wing}}$$

The wing extension weight is:

$$w_{\text{ext}} = S_{\text{ext}} U_{\text{ext}} \qquad W_{\text{ext}} = \chi_{\text{ext}} w_{\text{ext}}$$

$$w_{\text{efold}} = f_{\text{efold}} W_{\text{ext}} \qquad W_{\text{efold}} = \chi_{\text{efold}} w_{\text{efold}}$$

and these terms are added to W_{prim} and W_{fold}. Parameters are defined in table 19-2, including units as used in these equations.

Table 19-2. Parameters for aircraft wing weight.

parameter	definition	units
S_w	wing planform area (theoretical)	ft^2
U_w	unit weight of wing planform	lb/ft^2
W_{SD}	structural design gross weight	lb
n_z	design ultimate flight load factor at W_{SD}	g
Λ_w	wing sweep angle	deg
A_w	wing aspect ratio	
λ_w	wing taper ratio (tip chord/root chord)	
τ_w	wing airfoil thickness-to-chord ratio	
b_{fold}	fraction wing span that folds (0 to 1)	
f_{fair}	fairings (fraction total wing weight)	
f_{flap}	control surfaces (fraction total wing weight)	
f_{fit}	fittings (fraction total wing weight)	
f_{fold}	fold/tilt (fraction total wing weight)	
U_{ext}	unit weight of wing extension	lb/ft^2
S_{ext}	area of wing extensions (span b_{ext} times mean chord c_{ext})	ft^2
f_{efold}	wing extension fold/tilt (fraction extension weight)	

19–2   Rotor Group

The rotor group consists of: blades, hub and hinge, spinner, and blade fold structure. The blade and hub-hinge weights for the AFDD82 model are:

$$w_{\text{blade}} = 0.02606 N_{\text{rotor}} N_{\text{blade}}^{0.6592} R^{1.3371} c^{0.9959} V_{\text{tip}}^{0.6682} \nu_{\text{blade}}^{2.5279} \qquad\qquad W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}}$$

$$w_{\text{hub}} = 0.003722 N_{\text{rotor}} N_{\text{blade}}^{0.2807} R^{1.5377} V_{\text{tip}}^{0.4290} \nu_{\text{hub}}^{2.1414} (W_{\text{blade}}/N_{\text{rotor}})^{0.5505} \qquad W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}}$$

Based on 37 aircraft, the average error of the blade equation is 7.7% (figure 19-2). Based on 35 aircraft, the average error of the hub equation is 10.2% (figure 19-3). The blade and hub-hinge weights for the AFDD00 model are:

$$w_{\text{blade}} = 0.0024419 f_{\text{tilt}} N_{\text{rotor}} N_{\text{blade}}^{0.53479} R^{1.74231} c^{0.77291} V_{\text{tip}}^{0.87562} \nu_{\text{blade}}^{2.51048} \qquad\qquad W_{\text{blade}} = \chi_{\text{blade}} w_{\text{blade}}$$

$$w_{\text{hub}} = 0.0061182 N_{\text{rotor}} N_{\text{blade}}^{0.20373} R^{0.60406} V_{\text{tip}}^{0.52803} \nu_{\text{hub}}^{1.00218} (W_{\text{blade}}/N_{\text{rotor}})^{0.87127} \qquad W_{\text{hub}} = \chi_{\text{hub}} w_{\text{hub}}$$

where $f_{\text{tilt}} = 1.17940$ for tilting rotors; 1.0 otherwise. Based on 51 aircraft, the average error of the blade equation is 7.9% (figure 19-4) and the average error of the hub equation is 12.2% (figure 19-5). This equation for hub weight was developed using the actual blade weight. Using the blade weight equation instead gives

$$w_{\text{hub}} = 0.18370 N_{\text{rotor}} N_{\text{blade}}^{0.16383} R^{0.19937} V_{\text{tip}}^{0.06171} \nu_{\text{hub}}^{0.46203} (w_{\text{blade}}/N_{\text{rotor}})^{1.02958}$$

$$= 0.00037547 f_{\text{tilt}}^{1.02958} N_{\text{rotor}} N_{\text{blade}}^{0.71443} R^{1.99321} c^{0.79577} V_{\text{tip}}^{0.96323} \nu_{\text{hub}}^{0.46203} \nu_{\text{blade}}^{2.58473}$$

which really just adds chord and f_{tilt} to the regression parameters. Based on 51 aircraft, the average error of this equation is 9.2% (figure 19-5). For teetering and gimballed rotors, the flap frequency ν should be the coning frequency. Thus using the blade weight equation results in a lower average error, and best represents legacy rotor systems. The hub weight equation using the actual blade weight is best for advanced technology rotors with blades lighter than trend.

The fairing/spinner and blade fold weights are:

$$W_{\text{spin}} = \chi_{\text{spin}} 7.386 N_{\text{rotor}} D_{\text{spin}}^2$$

$$W_{\text{fold}} = \chi_{\text{fold}} f_{\text{fold}} W_{\text{blade}}$$

The blade weight is for all blades of the rotors. If the weight is evaluated separately for each rotor, then $N_{\text{rotor}} = 1$ should be used in the equations. Typically $f_{\text{fold}} = 0.04$ for manual fold and $f_{\text{fold}} = 0.28$ for automatic fold. Parameters are defined in table 19-3, including units as used in these equations.

Table 19-3. Parameters for rotor weight.

parameter	definition	units
N_{rotor}	number of rotors	
N_{blade}	number of blades per rotor	
R	rotor radius in hover	ft
c	rotor mean geometric blade chord	ft
V_{tip}	rotor hover tip velocity	ft/sec
$\nu_{\text{blade}}, \nu_{\text{hub}}$	flap natural frequency (for weight estimate)	per rev
D_{spin}	spinner diameter	ft
f_{fold}	blade fold weight (fraction total blade weight)	

19-2.1   Lift-Offset Rotor

For lift-offset rotors, the blade and hub weights can be calculated based on the methodology of reference 3. The blade weight is estimated based on the beam stiffness required to maintain the clearance s when the blade is loaded by the lift offset. The blade tip deflection is proportional to $\delta \propto PR^3/EI$, where EI is the bending stiffness. The beam loading is $P \propto n_z W_{SD} L/N_{\mathrm{blade}}$. With A_{xs} the blade cross-section area, the moment of inertia $I \propto A_{xs} t^2$. The criterion is $\delta = h - s$. Hence the blade weight is $W_{\mathrm{blade}} \propto \rho N_{\mathrm{blade}} R A_{xs} \propto (\rho/E) n_z W_{SD} L R^4/(t^2(h - s))$ with E the elastic modulus, and ρ here the material density. The hub weight is estimated based on the structure in upper and lower hub plates required to react a tensile force $F = C_{\mathrm{cent}} + M_{\mathrm{bend}}/(x/2)$ due to combined centrifugal force and bending moment at the root; where the hub plate separation x scales with the blade thickness t. The centrifugal force $C_{\mathrm{cent}} \propto (W_{\mathrm{blade}}/N_{\mathrm{blade}})V_{\mathrm{tip}}^2/R$. The bending moment $M_{\mathrm{bend}} \propto n_z W_{SD} R$. The limit tensile stress $\sigma \propto F/A_{xs}$ gives a criterion for the total hub arm area A_{xs}. The radius of the hub ℓ scales with the blade thickness t. Hence the hub weight is $W_{\mathrm{hub}} \propto \rho N_{\mathrm{blade}} \ell A_{xs} \propto (\rho/\sigma)\big(W_{\mathrm{blade}}V_{\mathrm{tip}}^2 t/R + K n_z W_{SD} R N_{\mathrm{blade}}\big)$ The distribution factor K is determined from the calibration weights. The inter-rotor shaft weight is estimated based on the structure to react the hub moment caused by lift offset. The hub moment $M \propto n_z W_{SD} L R$. The shaft diameter d scales with the blade thickness t. The shaft length ℓ scales with the rotor separation h. The ultimate bending stress $\sigma = M/(I/c)$ gives a criterion for the area moment $I/c \propto d^2 w$, hence for the shaft wall thickness w. Then $W_{\mathrm{shaft}} \propto \rho \ell d w \propto (\rho/\sigma) n_z W_{SD} L R h/t$ is the shaft weight.

The blade, hub-hinge, and inter-rotor shaft weights are:

$$w_{\mathrm{blade}} = N_{\mathrm{rotor}}\, 0.000083770\, wL\, R^3/(2(h - s)t_{.2R}^2) \qquad\qquad W_{\mathrm{blade}} = \chi_{\mathrm{blade}} w_{\mathrm{blade}}$$

$$w_{\mathrm{hub}} = N_{\mathrm{rotor}} \Big(0.17153\, w\, RN_{\mathrm{blade}} + 0.000010534(W_{\mathrm{blade}}/N_{\mathrm{rotor}})V_{\mathrm{tip}}^2 t_{.2R}/R\Big) \qquad W_{\mathrm{hub}} = \chi_{\mathrm{hub}} w_{\mathrm{hub}}$$

$$w_{\mathrm{shaft}} = N_{\mathrm{rotor}}\, 0.081304\, wL\, R^2 2h/t_{.2R} \qquad\qquad W_{\mathrm{shaft}} = \chi_{\mathrm{shaft}} w_{\mathrm{shaft}}$$

where $w = n_z W_{SD}/1000$, and

$$t_{.2R} = \tau_{.2R} c \left(\frac{0.8 + 0.2\lambda}{0.5 + 0.5\lambda}\right)$$

is the blade thickness at $20\%R$. These equations were developed for the coaxial rotor configuration ($N_{\mathrm{rotor}} = 2$), and calibrated to the XH-59A weights. The material factors (ρ/E and ρ/σ) are included in the technology factors χ. These weight terms can be used separately, with other hub and blade weight models. Parameters are defined in table 19-4, including units as used in these equations.

Table 19-4. Parameters for lift-offset rotor weight.

parameter	definition	units
N_{rotor}	number of rotors	
N_{blade}	number of blades per rotor	
W_{SD}	structural design gross weight	lb
n_z	design ultimate flight load factor at W_{SD}	g
λ	blade taper ratio (tip chord/root chord)	
$\tau_{.2R}$	blade airfoil thickness-to-chord ratio (at $20\%R$)	
R	rotor radius	ft
c	blade mean chord	ft
h	coaxial rotor separation (fraction rotor diameter)	
s	coaxial rotor tip clearance (fraction rotor diameter)	
L	lift offset (M_{roll}/TR)	
V_{tip}	rotor hover tip velocity	ft/sec

19–3   Empennage Group

The empennage group consists of: horizontal tail, vertical tail, and tail rotor. The tail plane weight consists of the basic structure and fold structure. The weight model depends on the aircraft configuration. The helicopter or compound model is AFDD82. The horizontal tail weight is:

tiltrotor or tiltwing $\qquad W_{ht} = \chi_{ht} S_{ht}(0.00395 S_{ht}^{0.2} V_{\mathrm{dive}} - 0.4885)$

helicopter or compound $\qquad W_{ht} = \chi_{ht} 0.7176 S_{ht}^{1.1881} A_{ht}^{0.3173}$

Based on 13 aircraft, the average error of the helicopter horizontal tail equation is 22.4% (figure 19-6). The vertical tail weight is:

tiltrotor or tiltwing $\qquad W_{vt} = \chi_{vt} S_{vt}(0.00395 S_{vt}^{0.2} V_{\mathrm{dive}} - 0.4885)$

helicopter or compound $\qquad W_{vt} = \chi_{vt} 1.0460 f_{\mathrm{tr}} S_{vt}^{0.9441} A_{vt}^{0.5332}$

where $f_{\mathrm{tr}} = 1.6311$ if the tail rotor is located on the vertical tail; 1.0 otherwise. Based on 12 aircraft, the average error of the helicopter vertical tail equation is 23.3% (figure 19-7). V_{dive} is the design dive speed, calculated or input; $V_{\mathrm{dive}} = 1.25 V_{\mathrm{max}}$, where V_{max} is the maximum speed at design gross weight and SLS conditions. The fold weight is a fraction of the basic weight: $W_{\mathrm{fold}} = \chi_{\mathrm{fold}} f_{\mathrm{fold}} W_{\mathrm{basic}}$, where $W_{\mathrm{basic}} = W_{ht}$ or W_{vt}.

The tail rotor weight is:

$$W_{tr} = \chi_{tr} 1.3778 R_{tr}^{0.0897} (P_{DS\mathrm{limit}} R / V_{\mathrm{tip}})^{0.8951}$$

Based on 19 aircraft, the average error of the helicopter tail rotor equation is 16.7% (figure 19-8). The tail rotor weight is calculated by the rotor component model. Rotor weight models can be used for the tail rotor. Parameters are defined in table 19-5, including units as used in these equations.

Table 19-5. Parameters for tail weight.

parameter	definition	units
S_{ht}	horizontal tail planform area	ft^2
S_{vt}	vertical tail planform area	ft^2
A_{ht}	horizontal tail aspect ratio	
A_{vt}	vertical tail aspect ratio	
V_{dive}	design dive speed at sea level	kts
R_{tr}	tail rotor radius	ft
$P_{DS\mathrm{limit}}$	drive system rated power	hp
R	main rotor radius in hover	ft
V_{tip}	main rotor hover tip velocity	ft/sec
f_{fold}	fold (fraction basic weight excluding fold)	

19–4   Fuselage Group

The fuselage group consists of: basic structure; wing and rotor fold/retraction; tail fold/tilt; and marinization, pressurization, and crashworthiness structure. The AFDD84 model is a universal body weight equation, used for tiltrotor and tiltwing as well as for helicopter configurations. The AFDD82 model is a helicopter body weight equation, not used for tiltrotor or tiltwing configuration.

For the AFDD84 (UNIV) model, the basic structure weight is

$$w_{\text{basic}} = 25.41 f_{LG\text{loc}} f_{LG\text{ret}} f_{\text{ramp}} \left(\frac{W_{MTO}}{1000} \right)^{0.4879} \left(\frac{n_z W_{SD}}{1000} \right)^{0.2075} S_{\text{body}}^{0.1676} \ell^{0.1512}$$

$$W_{\text{basic}} = \chi_{\text{basic}} w_{\text{basic}}$$

where $f_{LG\text{loc}} = 1.1627$ if the landing gear is located on the fuselage, and 1.0 otherwise; $f_{LG\text{ret}} = 1.1437$ if the landing gear is on the fuselage and retractable, and 1.0 otherwise; $f_{\text{ramp}} = 1.2749$ if there is a cargo ramp, and 1.0 otherwise. Based on 35 aircraft, the average error of the body equation is 6.5% (figure 19-9). The tail fold, wing and rotor fold, marinization, pressurization, and crashworthiness weights are:

$$w_{\text{tfold}} = f_{\text{tfold}} W_{\text{tail}} \qquad\qquad\qquad W_{\text{tfold}} = \chi_{\text{tfold}} w_{\text{tfold}}$$

$$w_{\text{wfold}} = f_{\text{wfold}} (W_{\text{wing}} + W_{\text{tip}}) \qquad\qquad W_{\text{wfold}} = \chi_{\text{wfold}} w_{\text{wfold}}$$

$$w_{\text{mar}} = f_{\text{mar}} W_{\text{basic}} \qquad\qquad\qquad W_{\text{mar}} = \chi_{\text{mar}} w_{\text{mar}}$$

$$w_{\text{press}} = f_{\text{press}} W_{\text{basic}} \qquad\qquad\qquad W_{\text{press}} = \chi_{\text{press}} w_{\text{press}}$$

$$w_{\text{cw}} = f_{\text{cw}} (W_{\text{basic}} + W_{\text{tfold}} + W_{\text{wfold}} + W_{\text{mar}} + W_{\text{press}}) \qquad W_{\text{cw}} = \chi_{\text{cw}} w_{\text{cw}}$$

Typically $f_{\text{tfold}} = 0.30$ for a folding tail, and $f_{\text{cw}} = 0.06$. For wing folding the weight on the wing tip (W_{tip}) is required (calculated as for the wing group). Parameters are defined in table 19-6, including units as used in these equations.

Table 19-6. Parameters for fuselage weight (AFDD84 model).

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
W_{SD}	structural design gross weight	lb
S_{body}	wetted area of body	ft²
R	main rotor radius	ft
n_z	design ultimate flight load factor at W_{SD}	g
ℓ	length of fuselage	ft
f_{tfold}	tail fold weight (fraction tail weight)	
f_{wfold}	wing and rotor fold weight (fraction wing/tip weight)	
W_{tail}	tail group weight	
$W_{\text{wing}} + W_{\text{tip}}$	wing group weight plus weight on wing tip	
f_{mar}	marinization weight (fraction basic body weight)	
f_{press}	pressurization (fraction basic body weight)	
f_{cw}	crashworthiness weight (fraction fuselage weight)	

For the AFDD82 (HELO) model, the basic structure weight is

$$w_{\text{basic}} = 5.896 f_{\text{ramp}} \left(\frac{W_{MTO}}{1000} \right)^{0.4908} n_z^{0.1323} S_{\text{body}}^{0.2544} \ell^{0.6100}$$

$$W_{\text{basic}} = \chi_{\text{basic}} w_{\text{basic}}$$

where $f_{\text{ramp}} = 1.3939$ if there is a cargo ramp, and 1.0 otherwise. Based on 30 aircraft, the average error of the body equation is 8.7% (figure 19-10). The tail fold, wing and rotor fold, marinization,

pressurization, and crashworthiness weights are:

$$w_{\text{tfold}} = f_{\text{tfold}}W_{\text{basic}} \qquad\qquad W_{\text{tfold}} = \chi_{\text{tfold}}w_{\text{tfold}}$$

$$w_{\text{wfold}} = f_{\text{wfold}}(W_{\text{basic}} + W_{\text{tfold}}) \qquad\qquad W_{\text{wfold}} = \chi_{\text{wfold}}w_{\text{wfold}}$$

$$w_{\text{mar}} = f_{\text{mar}}W_{\text{basic}} \qquad\qquad W_{\text{mar}} = \chi_{\text{mar}}w_{\text{mar}}$$

$$w_{\text{press}} = f_{\text{press}}W_{\text{basic}} \qquad\qquad W_{\text{press}} = \chi_{\text{press}}w_{\text{press}}$$

$$w_{\text{cw}} = f_{\text{cw}}(W_{\text{basic}} + W_{\text{tfold}} + W_{\text{wfold}} + W_{\text{mar}} + W_{\text{press}}) \qquad W_{\text{cw}} = \chi_{\text{cw}}w_{\text{cw}}$$

Typically $f_{\text{tfold}} = 0.05$ for a folding tail, and $f_{\text{cw}} = 0.06$. Parameters are defined in table 19-7, including units as used in these equations.

Table 19-7. Parameters for fuselage weight (AFDD82 model).

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
S_{body}	wetted area of body	ft^2
R	main rotor radius	ft
n_z	design ultimate flight load factor at W_{SD}	g
ℓ	length of fuselage	ft
f_{tfold}	tail fold weight (fraction basic structure)	
f_{wfold}	wing and rotor fold weight (fraction basic structure and tail fold)	
f_{mar}	marinization weight (fraction basic body weight)	
f_{press}	pressurization (fraction basic body weight)	
f_{cw}	crashworthiness weight (fraction fuselage weight)	

19–5   Alighting Gear Group

The alighting gear group consists of: basic structure, retraction, and crashworthiness structure. There are two models, parametric (AFDD82) and fractional. The basic landing gear weight is:

$$\text{parametric} \qquad w_{LG} = 0.4013W_{MTO}^{0.6662}N_{LG}^{0.5360}(W/S)^{0.1525}$$

$$\text{fractional} \qquad w_{LG} = f_{LG}W_{MTO}$$

and $W_{LG} = \chi_{LG}w_{LG}$. Typically $f_{LG} = 0.0325$ (fractional method). Based on 28 aircraft, the average error of the parametric equation is 8.4% (figure 19-11). The retraction and crashworthiness weights are:

$$w_{LG\text{ret}} = f_{LG\text{ret}}W_{LG} \qquad\qquad W_{LG\text{ret}} = \chi_{LG\text{ret}}w_{LG\text{ret}}$$

$$w_{LG\text{cw}} = f_{LG\text{cw}}(W_{LG} + W_{LG\text{ret}}) \qquad W_{LG\text{cw}} = \chi_{LG\text{cw}}w_{LG\text{cw}}$$

Typically $f_{LG\text{ret}} = 0.08$, and $f_{LG\text{cw}} = 0.14$. Parameters are defined in table 19-8, including units as used in these equations.

Table 19-8. Parameters for landing gear weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
f_{LG}	landing gear weight (fraction maximum takeoff weight)	
W/S	wing loading (1.0 for helicopter)	lb/ft^2
N_{LG}	number of landing gear assemblies	
$f_{LG\text{ret}}$	retraction weight (fraction basic weight)	
$f_{LG\text{cw}}$	crashworthiness weight (fraction basic and retraction weight)	

19–6 Engine Section or Nacelle Group and Air Induction Group

The engine section or nacelle group consists of: engine support structure, engine cowling, and pylon support structure. The weights (AFDD82 model) are:

$$W_{\text{supt}} = \chi_{\text{supt}} 0.0412 (1 - f_{\text{airind}}) (W_{\text{eng}}/N_{\text{eng}})^{1.1433} N_{\text{eng}}^{1.3762}$$

$$W_{\text{cowl}} = \chi_{\text{cowl}} 0.2315 S_{\text{nac}}^{1.3476}$$

$$W_{\text{pylon}} = \chi_{\text{pylon}} f_{\text{pylon}} W_{MTO}$$

Based on 12 aircraft, the average error of the engine support equation is 11.0% (figure 19-12). Based on 12 aircraft, the average error of the engine cowling equation is 17.9% (figure 19-13). The air induction group weight (AFDD82 model) is:

$$W_{\text{airind}} = \chi_{\text{airind}} 0.0412 f_{\text{airind}} (W_{\text{eng}}/N_{\text{eng}})^{1.1433} N_{\text{eng}}^{1.3762}$$

Typically $f_{\text{airind}} = 0.3$ (range 0.1 to 0.6). Based on 12 aircraft, the average error of the air induction equation is 11.0% (figure 19-14). Parameters are defined in table 19-9, including units as used in these equations.

Table 19-9. Parameters for engine section, nacelle, and air induction weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
W_{eng}	weight all main engines	lb
N_{eng}	number of main engines	
S_{nac}	wetted area of nacelles and pylon (less spinner)	ft^2
f_{airind}	air induction weight (fraction nacelle plus air induction)	
f_{pylon}	pylon support structure weight (fraction W_{MTO})	

19–7 Propulsion Group

The propulsion group consists of the engine system, fuel system, and drive system.

19-7.1 Engine System

The engine system consists of the main engines, the engine exhaust system, and the engine accessories. The engine system weights are:

$$W_{\text{eng}} = \chi_{\text{eng}} N_{\text{eng}} W_{\text{one eng}}$$

$$W_{\text{exh}} = \chi_{\text{exh}} N_{\text{eng}} (K_{0\text{exh}} + K_{1\text{exh}} P)$$

$$W_{\text{acc}} = \chi_{\text{acc}} 2.0088 f_{\text{lub}} (W_{\text{eng}}/N_{\text{eng}})^{0.5919} N_{\text{eng}}^{0.7858}$$

where $f_{\text{lub}} = 1.4799$ if the accessory weight includes the lubrication system weight, 1.0 if the lubrication system weight is in the engine weight. The exhaust system weight is per engine, including any IR suppressor. The accessory weight equation is the AFDD82 model. Based on 16 aircraft, the average

error of the accessories equation is 11.5% (figure 19-15). The engine system weight $W_{ES} = W_{\text{eng}} + W_{\text{exh}} + W_{\text{acc}}$. Parameters are defined in table 19-10, including units as used in these equations.

Table 19-10. Parameters for engine system weight.

parameter	definition	units
N_{eng}	number of main engines	
P	installed takeoff power (SLS static, specified rating) per engine	hp
$K_{0\text{exh}}, K_{1\text{exh}}$	engine exhaust weight vs. power, constants	

19-7.2   Propeller/Fan Installation

The auxiliary propulsion or propeller weight is:

$$\text{AFDD82} \qquad W_{at} = \chi_{at} 0.0809484 N_{at} T_{at}^{1.04771} (T_{at}/A_{at})^{-0.07821}$$

$$\text{AFDD10} \qquad W_{at} = \chi_{at} 9.90350 N_{at} P_{at}^{0.91996} N_{\text{blade}}^{-0.48578} \Omega_{\text{prop}}^{-0.45904} D_{\text{prop}}^{0.15690} f_m$$

where T_{at} is at maximum speed, design gross weight, and SLS conditions, calculated or input. The material factor $f_m = 1$ for composite construction; 1.20 for wood; 1.31 for aluminum spar; 1.44 for aluminum construction. Based on 16 aircraft, the average error of the AFDD10 equation is 10.5%. The propeller weight is calculated by the rotor component model. Rotor weight models can be used for the propeller. Parameters are defined in table 19-11, including units as used in these equations.

Table 19-11. Parameters for propeller weight.

parameter	definition	units
N_{at}	number of auxiliary thrusters	
T_{at}	thrust per propeller	lb
A_{at}	auxiliary thruster disk area	ft^2
P_{prop}	power per propeller	hp
N_{blade}	number of blades per propeller	
Ω_{prop}	propeller rotation speed	rpm
D	propeller diameter	ft

19-7.3   Fuel System

The fuel system consists of tanks and support structure (including fuel tanks, bladders, supporting structure, filler caps, tank covers, and filler material for void and ullage), and fuel plumbing (including fuel system weight not covered by tank weight). The fuel system weights (AFDD82 model) are:

$$W_{\text{tank}} = \chi_{\text{tank}} 0.4341 C_{\text{int}}^{0.7717} N_{\text{int}}^{0.5897} f_{\text{cw}} f_{\text{bt}}^{1.9491}$$

$$W_{\text{plumb}} = \chi_{\text{plumb}} \left[K_{0\text{plumb}} + K_{1\text{plumb}} (0.01 N_{\text{plumb}} + 0.06 N_{\text{eng}})(F/N_{\text{eng}})^{0.866} \right]$$

where $f_{\text{cw}} = 1.3131$ for ballistically survivable (UTTAS/AAH level) and 1.0 otherwise. The ballistic tolerance factor $f_{\text{bt}} = 1.0$ to 2.5. Based on 15 aircraft, the average error of the fuel tank equation is 4.6% (figure 19-16). The fuel flow rate F is calculated for the takeoff power rating at static SLS conditions. $K_{1\text{plumb}}$ is a crashworthiness and survivability factor; typically $K_{1\text{plumb}} = 2$. $K_{0\text{plumb}}$ is the sum of weights for auxiliary fuel (typically up to 120 lb), in-flight refueling (up to 150 lb), pressure refueling

(up to 150 lb), inerting system (up to 20 lb), etc.; typically $K_{0\text{plumb}} = 50$ to 250 lb. Parameters are defined in table 19-12, including units as used in these equations.

Table 19-12. Parameters for fuel system weight.

parameter	definition	units
N_{int}	number of internal fuel tanks	
C_{int}	internal fuel tank capacity	gallons
f_{bt}	ballistic tolerance factor	
N_{plumb}	total number of fuel tanks (internal and auxiliary) for plumbing	
N_{eng}	number of main engines	
$K_{0\text{plumb}}, K_{1\text{plumb}}$	plumbing weight, constants	
F	fuel flow rate	lb/hr

19-7.4   Drive System

The drive system consists of gear boxes and rotor shafts, drive shafts, and rotor brake. This distribution of drive system weights is based on the following functional definitions. Gearboxes are parts of the drive system that transmit power by gear trains, and the structure that encloses them. Rotor shafts are the structure (typically a shaft) that transmits power to the rotor. Drive shafts are the structure (typically a shaft) that transmits power in the propulsion system, but not directly to the rotor or by a gear train. The rotor brake weight encompasses components that can prevent the rotor from freely turning. The gear box and rotor shaft weights for the AFDD83 model are:

$$w_{gbrs} = 57.72 P_{DS\text{limit}}^{0.8195} f_Q^{0.0680} N_{\text{gb}}^{0.0663} (\Omega_{\text{eng}}/1000)^{0.0369}/\Omega_{\text{rotor}}^{0.6379}$$

$$W_{gb} = \chi_{gb}(1 - f_{rs})w_{gbrs}$$

$$W_{rs} = \chi_{rs}f_{rs}w_{gbrs}$$

Based on 30 aircraft, the average error of the gear box and rotor shaft equation is 7.7% (figure 19-17). The gear box and rotor shaft weights for the AFDD00 model are:

$$w_{gbrs} = 95.7634 N_{\text{rotor}}^{0.38553} P_{DS\text{limit}}^{0.78137} \Omega_{\text{eng}}^{0.09899}/\Omega_{\text{rotor}}^{0.80686}$$

$$W_{gb} = \chi_{gb}(1 - f_{rs})w_{gbrs}$$

$$W_{rs} = \chi_{rs}f_{rs}w_{gbrs}$$

Based on 52 aircraft, the average error of the gear box and rotor shaft equation is 8.6% (figure 19-18). Typically $f_{rs} = 0.13$ (range 0.06 to 0.20). Parameters are defined in table 19-13, including units as used in these equations.

The drive shaft (AFDD82 model) and rotor brake weights are:

$$W_{ds} = \chi_{ds}1.166 Q_{DS\text{limit}}^{0.3828} x_{\text{hub}}^{1.0455} N_{ds}^{0.3909}(0.01f_P)^{0.2693}$$

$$W_{rb} = \chi_{rb}0.000871 W_{\text{blade}}(0.01V_{\text{tip}})^2$$

where $f_P = f_Q\Omega_{\text{other}}/\Omega_{\text{main}}$. Based on 28 aircraft, the average error of the drive shaft equation is 16.0% (figure 19-19). Based on 23 aircraft, the average error of the rotor brake equation is 25.1% (figure 19-20). The clutch weight in the weight statement is associated with an auxiliary power unit, and is a fixed input

value. The conventional rotor drive system clutch and free wheeling device weights are included in the gear box and rotor shaft weight equations. Parameters are defined in table 19-14, including units as used in these equations.

Typically $f_P = f_Q = 60\%$ for twin main rotors (tandem, coaxial, and tiltrotor); for a single main rotor and tail rotor, $f_Q = 3\%$ and $f_P = 15\%$ (18% for 2-bladed rotors).

Table 19-13. Parameters for drive system weight.

parameter	definition	units
$P_{DS\text{limit}}$	drive system rated power	hp
N_{rotor}	number of main rotors	
N_{gb}	number of gear boxes	
Ω_{rotor}	main rotor rotation speed	rpm
Ω_{eng}	engine output speed	rpm
f_Q	second (main or tail) rotor rated torque	%
	(fraction of total drive system rated torque)	
f_{rs}	rotor shaft weight (fraction gear box and rotor shaft)	

Table 19-14. Parameters for drive shaft and rotor brake weight.

parameter	definition	units
$Q_{DS\text{limit}}$	$P_{DS\text{limit}}/\Omega_{\text{rotor}}$	hp/rpm
N_{ds}	number of intermediate drive shafts	
x_{hub}	length of drive shaft between rotors	ft
f_P	second (main or tail) rotor rated power	%
	(fraction of total drive system rated power)	
V_{tip}	main rotor tip speed	ft/sec

19–8   Flight Controls Group

The flight controls group consists of cockpit controls, automatic flight control system, and system controls. W_{cc} and W_{afcs} weights are fixed (input). System controls consist of fixed wing flight controls, rotary wing flight controls, and conversion (rotor tilt) flight controls. The weight equations model separately non-boosted controls (which do not see aerodynamic surface or rotor loads), boost mechanisms (actuators), and boosted controls (which are affected by aerodynamic surface or rotor loads). The load path goes from pilot, to cockpit controls, to non-boosted controls, to boost mechanisms, to boosted controls, and finally to the component.

Fixed wing flight controls consist of non-boosted flight controls and flight control boost mechanisms. The weights are:

$$\text{full controls} \qquad w = 0.91000 W_{MTO}^{0.6}$$
$$\text{only stabilizer controls} \qquad w = 0.01735 W_{MTO}^{0.64345} S_{ht}^{0.40952}$$

and then

$$W_{FWnb} = \chi_{FWnb} f_{FWnb}\, w$$
$$W_{FWmb} = \chi_{FWmb} (1 - f_{FWnb})\, w$$

For a helicopter, the stabilizer control equation is used. Parameters are defined in table 19-15, including units as used in these equations.

Table 19-15. Parameters for fixed wing flight control weight.

parameter	definition	units
S_{ht}	horizontal tail planform area	ft^2
W_{MTO}	maximum takeoff weight	lb
f_{FWnb}	fixed wing non-boosted weight (fraction total fixed wing flight control weight)	

Rotary wing flight controls consist of non-boosted flight controls, flight control boost mechanisms, and boosted flight controls. The non-boosted flight control weight (AFDD82 model) is:

$$\text{fraction method} \qquad W_{RWnb} = \chi_{RWnb} f_{RWnb}(1 - f_{RW\text{hyd}})w_{fc}$$

$$\text{parametric method} \qquad W_{RWnb} = \chi_{RWnb} 2.1785 f_{nbsv} W_{MTO}^{0.3999} N_{\text{rotor}}^{1.3855}$$

where $f_{nbsv} = 1.8984$ for ballistically survivable (UTTAS/AAH level); 1.0 otherwise. The parametric method assumes the rotor flight controls are boosted and computes the weight of the non-boosted portion up to the control actuators. Based on 20 aircraft, the average error of the non-boosted flight controls equation is 10.4% (figure 19-21). The flight control boost mechanism weight and boosted flight control weight (AFDD82 model) are:

$$w_{fc} = 0.2873 f_{mbsv}(N_{\text{rotor}} N_{\text{blade}})^{0.6257} c^{1.3286}(0.01V_{\text{tip}})^{2.1129} f_{RW\text{red}}^{0.8942}$$

$$W_{RWmb} = \chi_{RWmb}(1 - f_{RW\text{hyd}})w_{fc}$$

$$W_{RWb} = \chi_{RWb} 0.02324 f_{bsv}(N_{\text{rotor}} N_{\text{blade}})^{1.0042} N_{\text{rotor}}^{0.1155} c^{2.2296}(0.01V_{\text{tip}})^{3.1877}$$

where $f_{mbsv} = 1.3029$ and $f_{bsv} = 1.1171$ for ballistically survivable (UTTAS/AAH level); 1.0 otherwise; and $f_{RW\text{red}} = 1.0$ to 3.0. Typically $f_{RWnb} = 0.6$ (range 0.3 to 1.8); $f_{RW\text{hyd}} = 0.4$. Based on 21 aircraft, the average error of the boost mechanisms equation is 6.5% (figure 19-22). Based on 20 aircraft, the average error of the boosted flight controls equation is 9.7% (figure 19-23). Parameters are defined in table 19-16, including units as used in these equations.

Table 19-16. Parameters for rotary wing flight control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
N_{rotor}	number of main rotors	
N_{blade}	number of blades per rotor	
c	rotor mean blade chord	ft
V_{tip}	rotor hover tip velocity	ft/sec
f_{RWnb}	rotary wing non-boosted weight (fraction boost mechanisms weight)	
$f_{RW\text{hyd}}$	rotary wing hydraulics weight (fraction hydraulics plus boost mechanisms weight)	
$f_{RW\text{red}}$	flight control hydraulic system redundancy factor	

The conversion controls consist of non-boosted tilt controls, and tilt control boost mechanisms; they are used only for tilting rotor configurations. The weights are:

$$w_{CVmb} = f_{CVmb} W_{MTO} \qquad W_{CVmb} = \chi_{CVmb} w_{CVmb}$$
$$w_{CVnb} = f_{CVnb} W_{CVmb} \qquad W_{CVnb} = \chi_{CVnb} w_{CVnb}$$

Parameters are defined in table 19-17, including units as used in these equations.

Table 19-17. Parameters for conversion control weight.

parameter	definition	units
W_{MTO}	maximum takeoff weight	lb
f_{CVnb}	conversion non-boosted weight (fraction boost mechanisms weight)	
f_{CVmb}	conversion boost mechanisms weight (fraction maximum takeoff weight)	

19–9   Hydraulic Group

The hydraulic group consists of hydraulics for fixed wing flight controls, rotary wing flight controls, conversion (rotor tilt) flight controls, and equipment. The hydraulic weight for equipment, W_{EQhud}, is fixed (input). The weights (AFDD82 model) are

$$W_{FW\text{hyd}} = \chi_{FW\text{hyd}} f_{FW\text{hyd}} W_{FWmb}$$
$$W_{RW\text{hyd}} = \chi_{RW\text{hyd}} f_{RW\text{hyd}} w_{fc}$$
$$W_{CV\text{hyd}} = \chi_{CF\text{hyd}} f_{CV\text{hyd}} W_{CVmb}$$

Typically $f_{RW\text{hyd}} = 0.4$. Parameters are defined in table 19-18, including units as used in these equations.

Table 19-18. Parameters for hydraulic group weight.

parameter	definition	units
$f_{FW\text{hyd}}$	fixed wing hydraulics weight (fraction boost mechanisms weight)	
$f_{RW\text{hyd}}$	rotary wing hydraulics weight	
	(fraction hydraulics plus boost mechanisms weight)	
$f_{CV\text{hyd}}$	conversion hydraulics weight (fraction boost mechanisms weight)	

19–10   Anti-Icing Group

The anti-icing group consists of the anti-ice system. The electrical system for anti-icing is part of the electrical group. The weights are obtained from the sum over all rotors, all wings, and all engines:

$$W_{DI\text{elect}} = \chi_{DI\text{elect}} \sum k_{\text{elec}} A_{\text{blade}}$$
$$W_{DI\text{sys}} = \chi_{DI\text{sys}} \left(\sum k_{\text{rotor}} A_{\text{blade}} + \sum k_{\text{wing}} \ell_{\text{wing}} + k_{\text{air}} \sum W_{\text{eng}} \right)$$

Parameters are defined in table 19-19, including units as used in these equations.

Table 19-19. Parameters for anti-icing group weight.

parameter	definition	units
A_{blade}	total blade area of rotor, from geometric solidity	ft^2 or m^2
ℓ_{wing}	wing length (wing span less fuselage width)	ft or m
k_{elect}	electrical system weight factor	
k_{rotor}	rotor deice system weight factor	
k_{wing}	wing deice system weight factor	
k_{air}	engine air intake deice system weight factor	

19–11   Other Systems and Equipment

The following weights are fixed (input) in this model: auxiliary power group; instruments group; pneumatic group; electrical group (aircraft); avionics group (mission equipment); armament group (armament provisions and armor); furnishing and equipment group; environmental control group; and load and handling group. Typical fixed weights are given in table 19-20, based on medium to heavy helicopters and tiltrotors.

Table 19-20. Other systems and equipment weight.

group	typical weight (lb)
SYSTEMS AND EQUIPMENT	
flight controls group	
cockpit controls	100–125
automatic flight control system	35–200
flight control electronics, mechanical	35–100
flight control electronics, fly-by-wire	250
auxiliary power group	130–300
instruments group	150-250
hydraulic group	
equipment	50–300
electrical group	400–1000
avionics group (mission equipment)	400–1500
furnishings & equipment group	600–1000
crew only	100–200
environmental control group	50–250
anti-icing group	50–300
load & handling group	
internal	200–400
external	150–300
FIXED USEFUL LOAD	
crew	500–800
fluids (oil, unusable fuel)	50–150

19–12 Folding Weight

Folding weights are calculated in a number of groups: wing W_{fold} (including extensions), rotor W_{fold}, tail W_{fold}, fuselage W_{tfold} and W_{wfold}. These are the total weights for folding and the impact of folding on the group. A fraction f_{foldkit} of these weights can be in a kit, hence optionally removable. Thus of the total folding weight, the fraction f_{foldkit} is a kit weight in the fixed useful load of the weight statement, while the remainder is kept in the wing, rotor, or fuselage group weight.

19–13 Parametric Weight Correlation

Table 19-21 summarizes the satistics of the parametric weight estimation equations. Figure 19-24 shows the error of the calculated weight for the sum of all parametric weight, accounting on average for 42% of the empty weight. This sum is composed of the structural group (based on the AFDD00 equation for rotor blade and hub weights, and the AFDD84 equation for body weight), the propulsion group (based on the AFDD00 equation for drive system weight), and the flight controls group. Based on 42 aircraft, the average error of the sum of all parametric weight is 5.3%. The corresponding average error is 6.1% for the structural group (8.6% for the rotor group alone), 10.9% for the propulsion group, and 8.7% for the flight controls group.

Table 19-21. Statistics of parametric weight equations.

group		number of aircraft	average error (%)
wing		25	3.4
rotor blade	AFDD82	37	7.7
rotor hub	AFDD82	37	10.2
rotor blade	AFDD00	51	7.9
rotor hub	AFDD00	51	12.2
horizontal tail		13	22.4
vertical tail		12	23.3
tail rotor		19	16.7
fuselage	AFDD82	30	8.7
fuselage	AFDD84	35	6.5
alighting gear		28	8.4
engine support		12	11.0
engine cowling		12	17.9
air induction		12	11.0
accessory		16	11.5
propeller	AFDD10	16	10.5
fuel tank		15	4.6
gear box + rotor shaft	AFDD83	30	7.7
gear box + rotor shaft	AFDD00	52	8.6
drive shaft		28	16.0
rotor brake		23	25.1
rotary wing flight controls	non-boosted	20	10.4
rotary wing flight controls	boost mechanisms	21	6.5
rotary wing flight controls	boosted	20	9.7

19–14   References

1) Chappell, D., and Peyran, R. "Methodology for Estimating Wing Weights for Conceptual Tilt-Rotor and Tilt-Wing Aircraft." SAWE Paper No. 2107, Category No. 23, May 1992.

2) Chappell, D.P. "Tilt-rotor Aircraft Wing Design." ASRO-PDT-83-1, 1983.

3) "Weight Trend Estimation for the Rotor Blade Group, Rotor Hub Group, and Upper Rotor Shaft of the ABC Aircraft." ASRO-PDT-83-2, 1983.
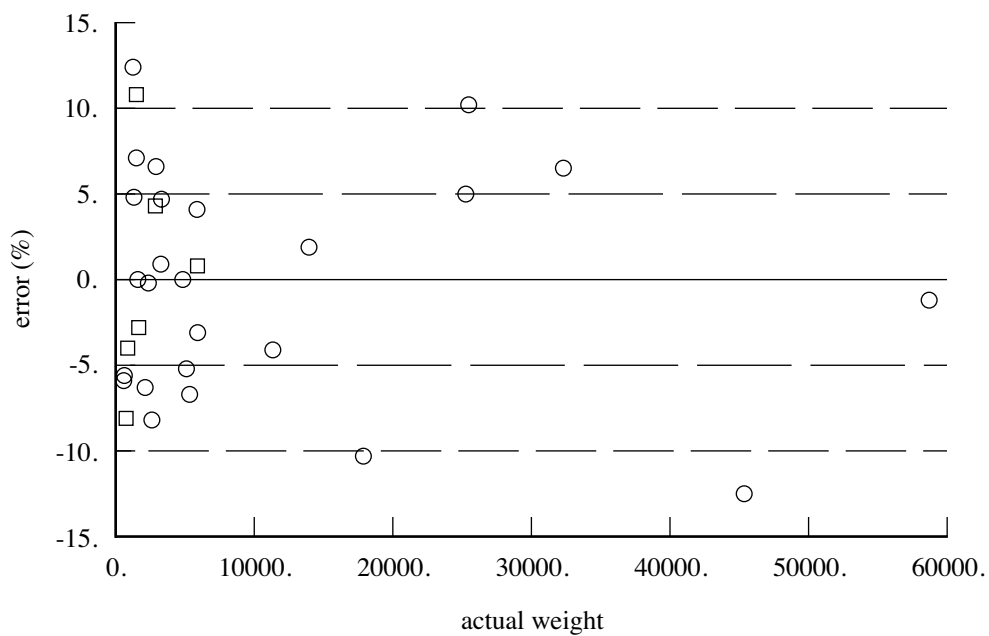
Figure 19-1. Wing group (AFDD93).
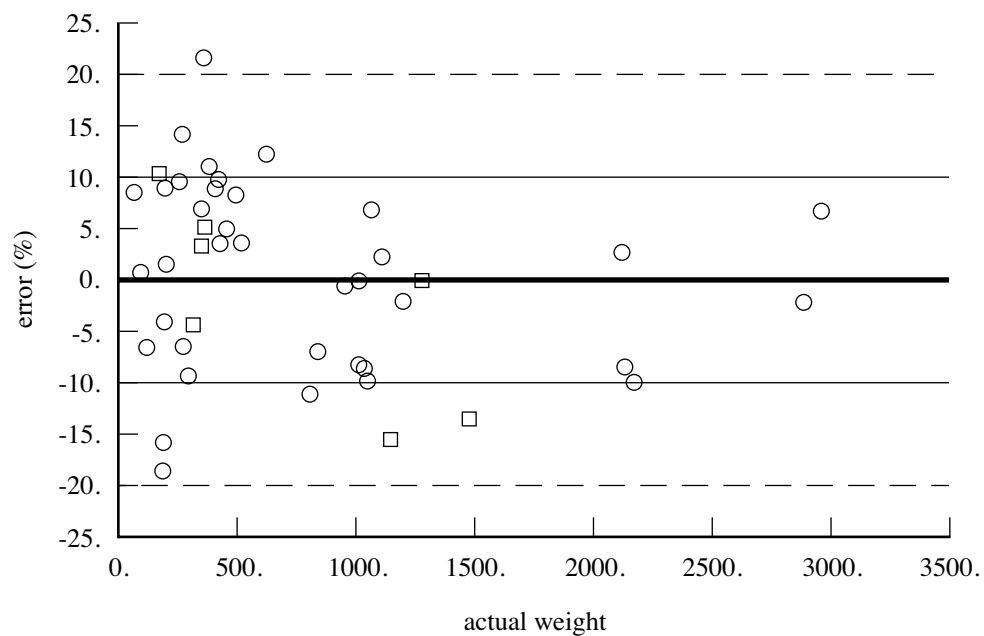
Figure 19-2. Rotor group, blade weight (AFDD82).
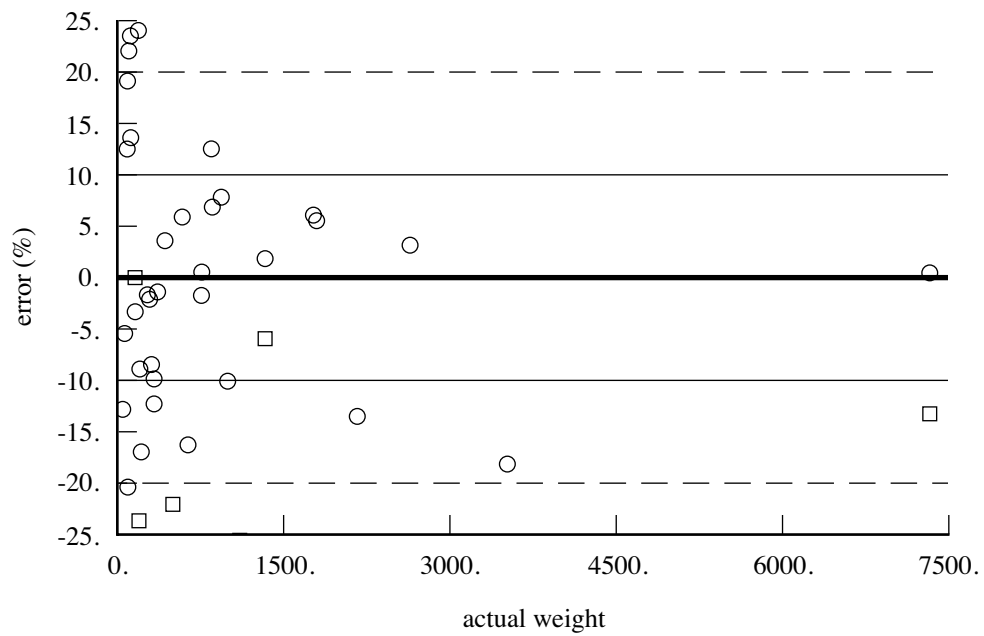
Figure 19-3. Rotor group, hub weight (AFDD82).
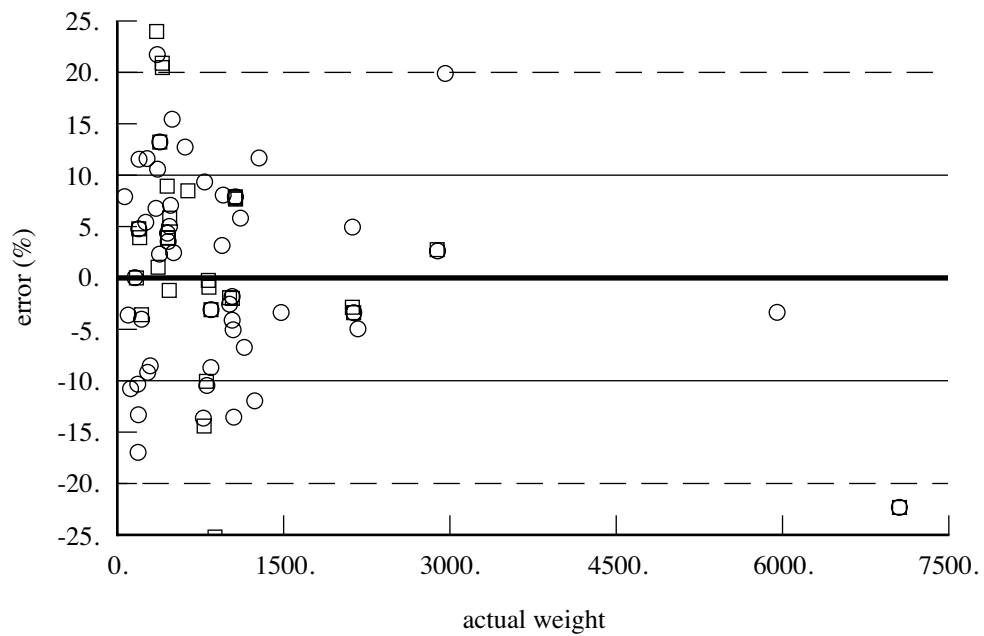
Figure 19-4. Rotor group, blade weight (AFDD00).
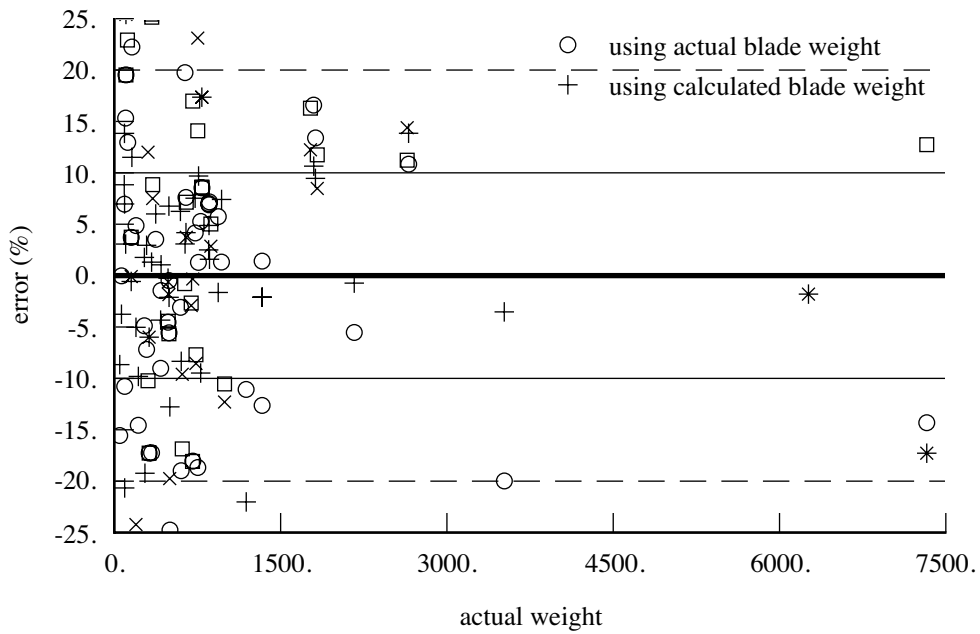
Figure 19-5. Rotor group, hub weight (AFDD00).
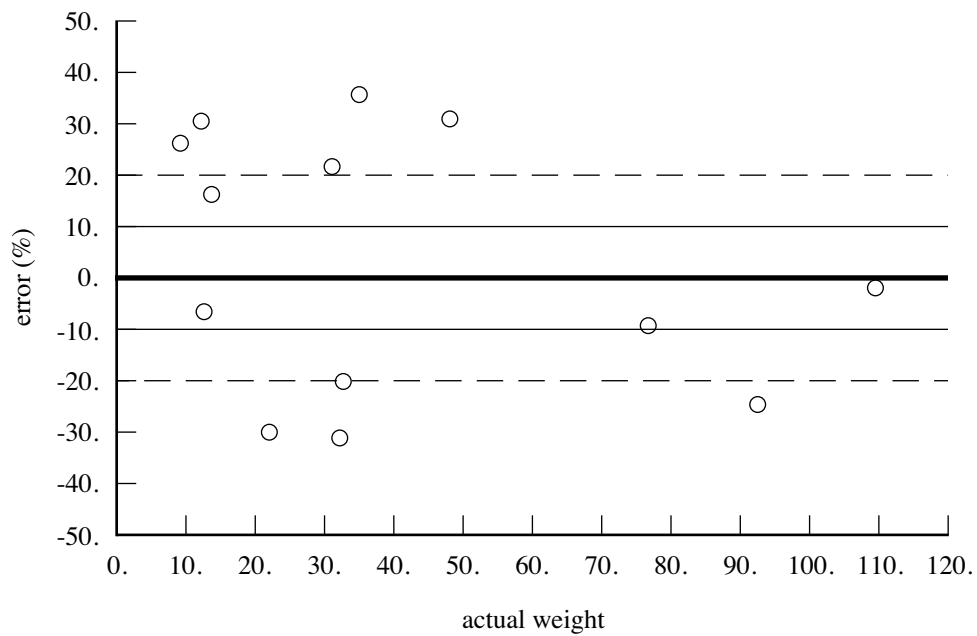
Figure 19-6. Empennage group, horizontal tail weight (AFDD82).
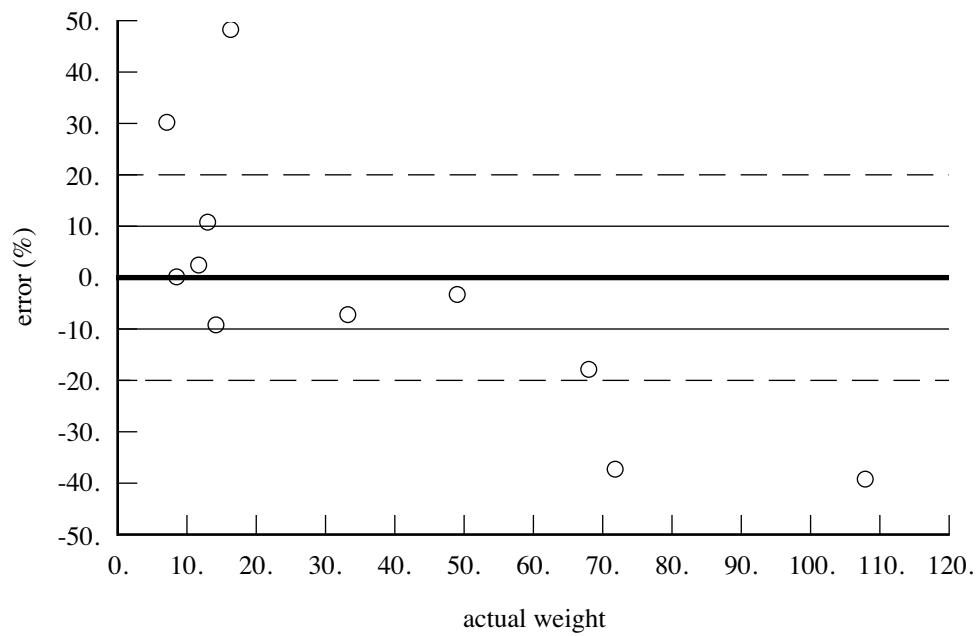
Figure 19-7. Empennage group, vertical tail weight (AFDD82).
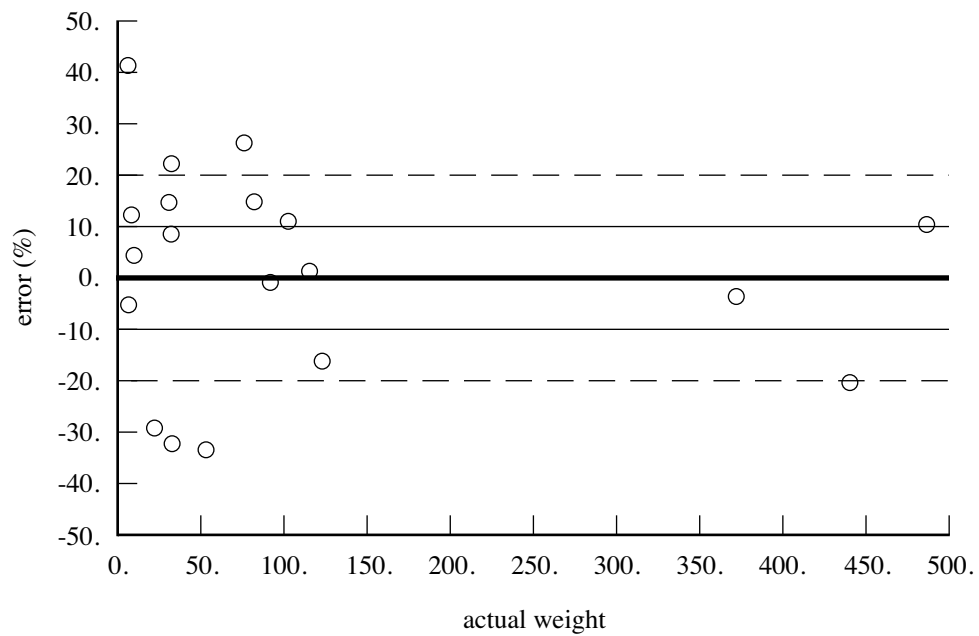
Figure 19-8. Empennage group, tail rotor weight (AFDD82).
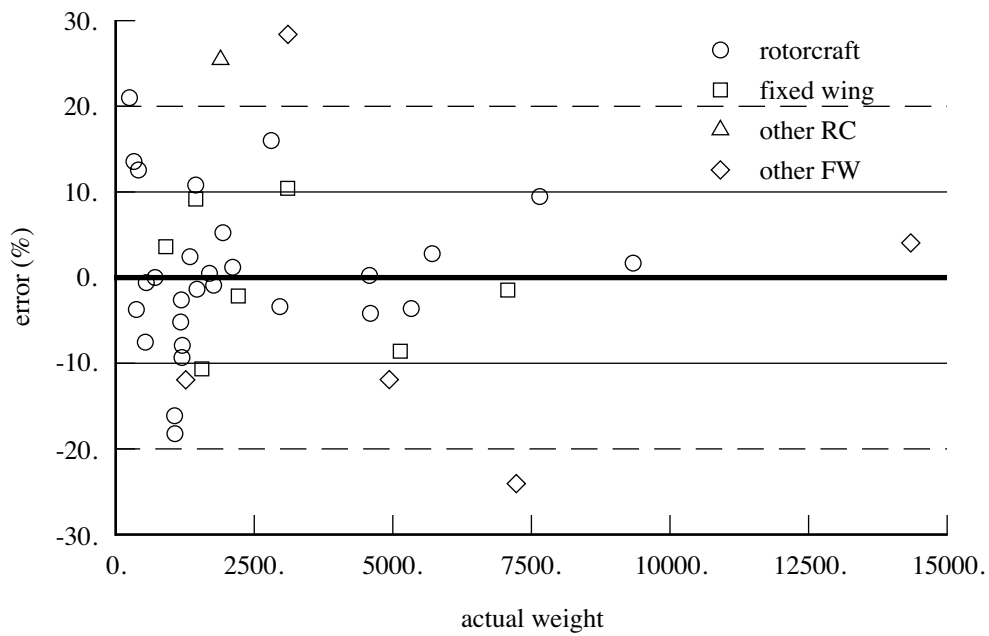
Figure 19-9. Fuselage group, fuselage weight (AFDD84).
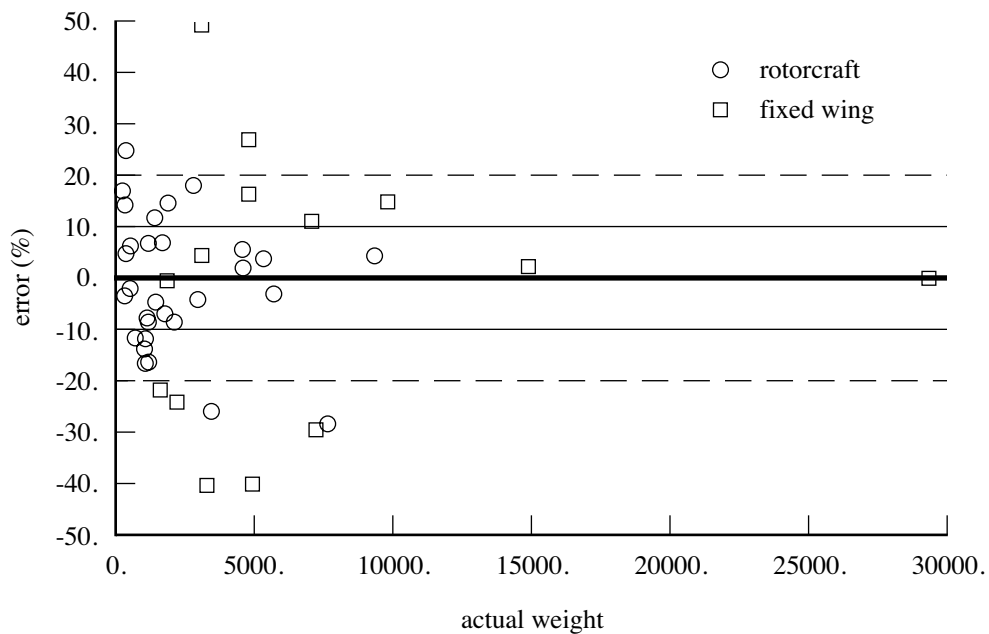
Figure 19-10. Fuselage group, fuselage weight (AFDD82).
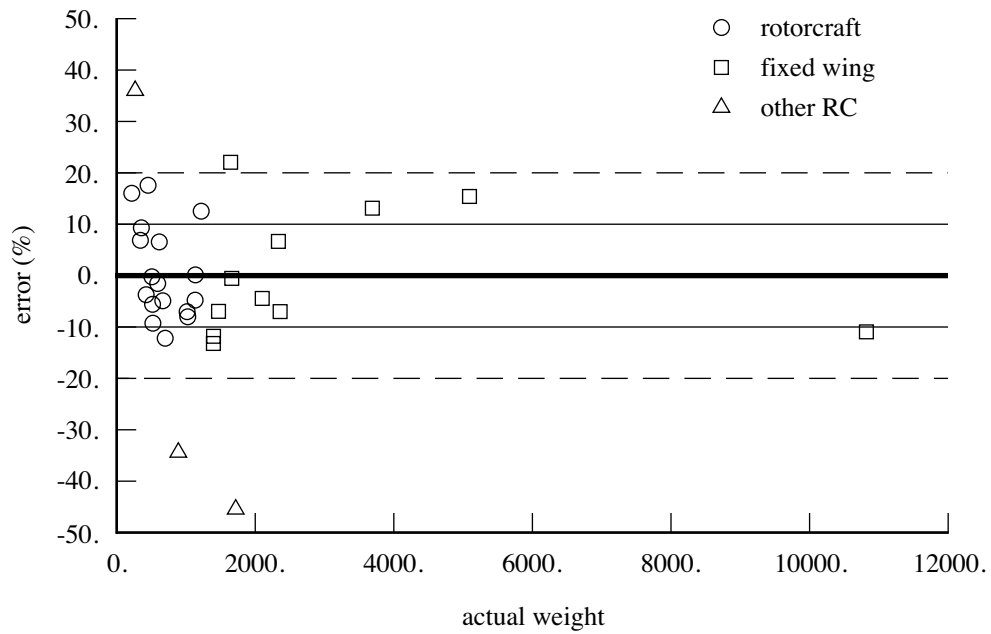
Figure 19-11. Alighting gear group, landing gear weight (AFDD82).
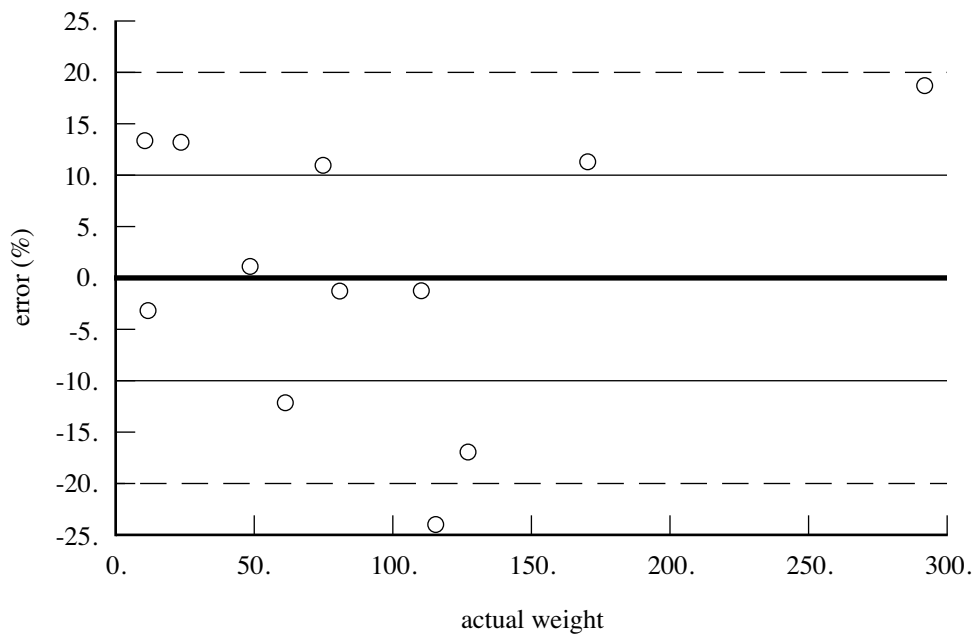
Figure 19-12. Engine section or nacelle group, engine support weight (AFDD82).
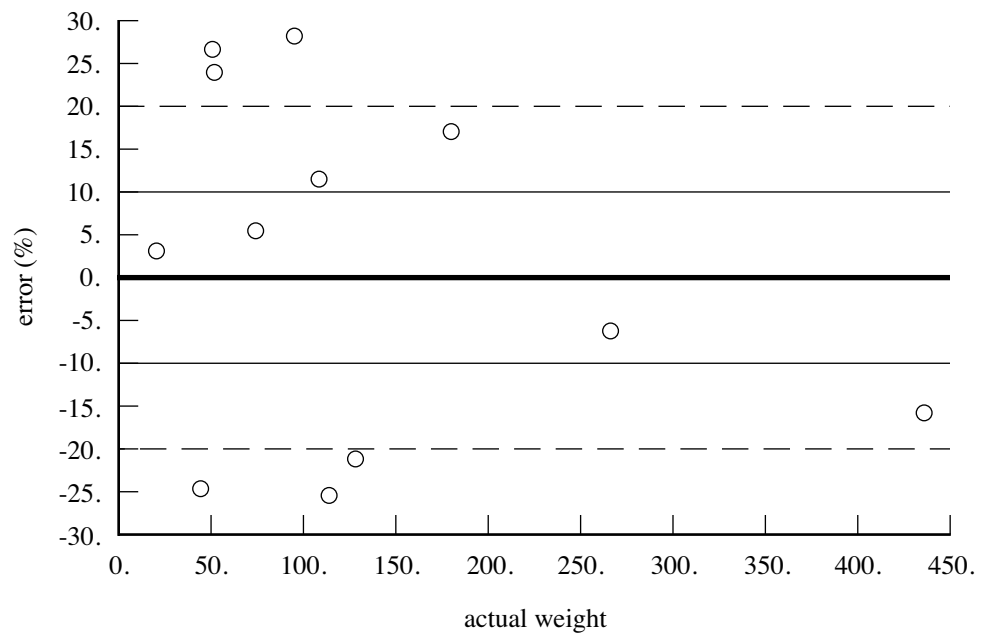
Figure 19-13. Engine section or nacelle group, cowling weight (AFDD82).
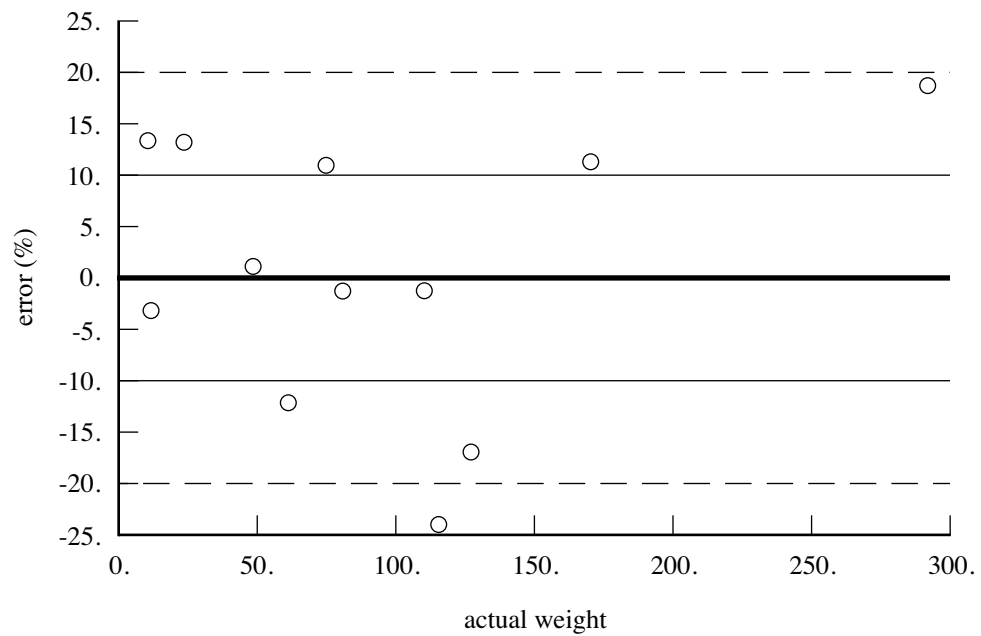
Figure 19-14. Air induction group, air induction weight (AFDD82).
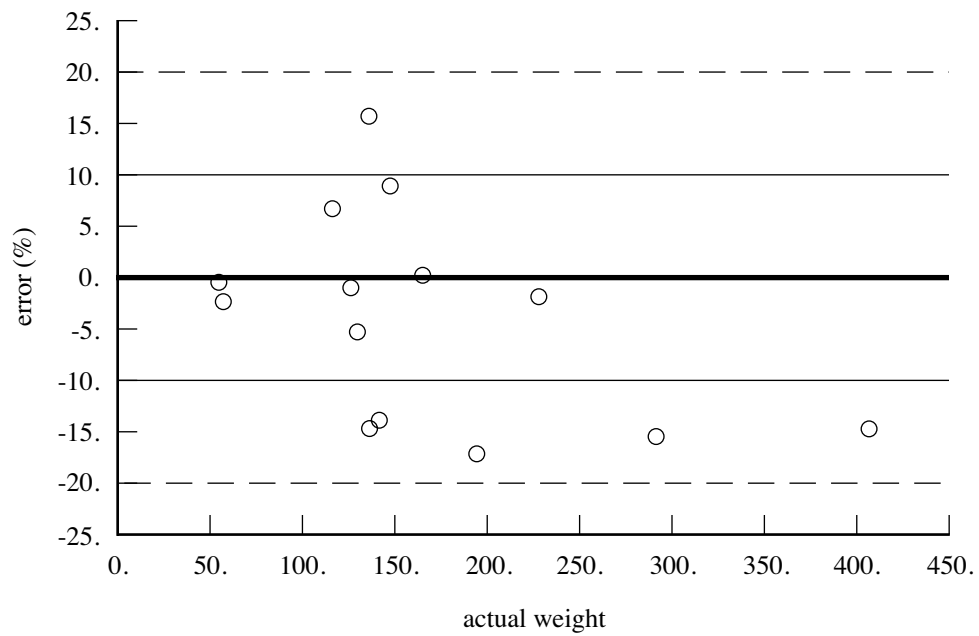
Figure 19-15. Propulsion group, accessories weight (AFDD82).
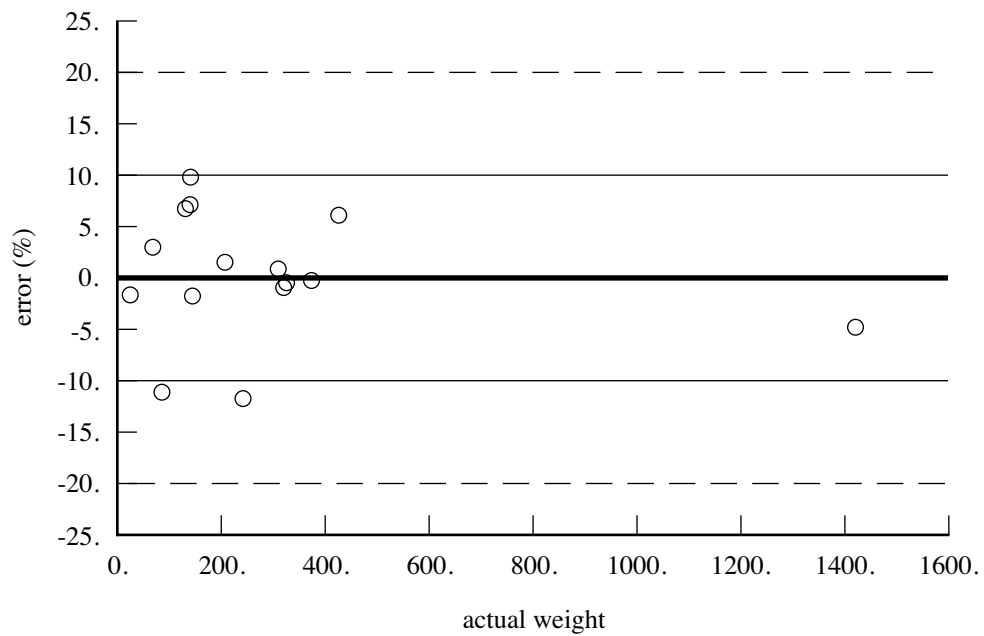
Figure 19-16. Propulsion group, fuel tank weight (AFDD82).
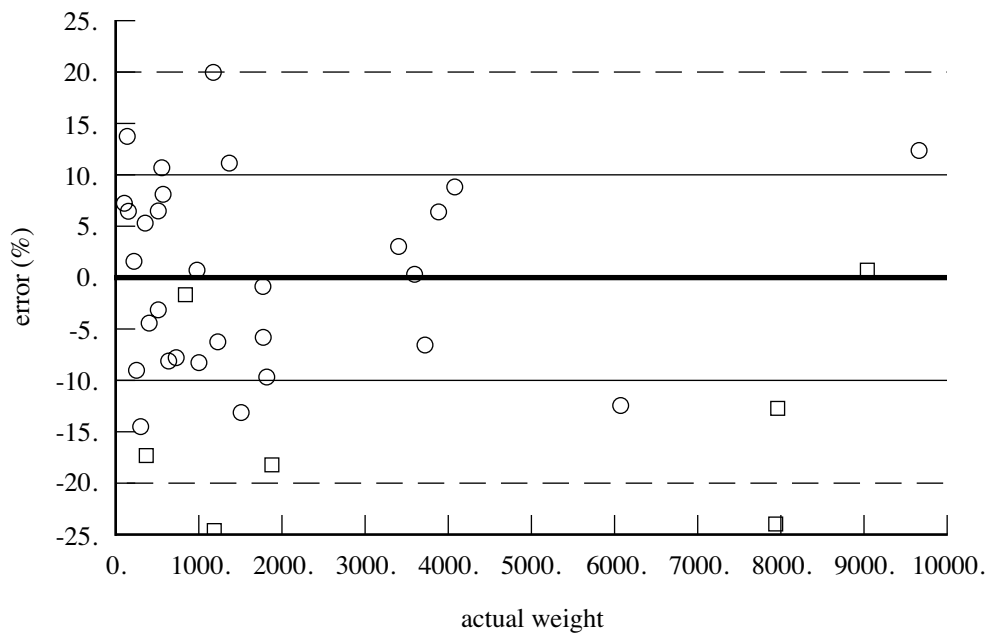
Figure 19-17. Propulsion group, gear box and rotor shaft weight (AFDD83).
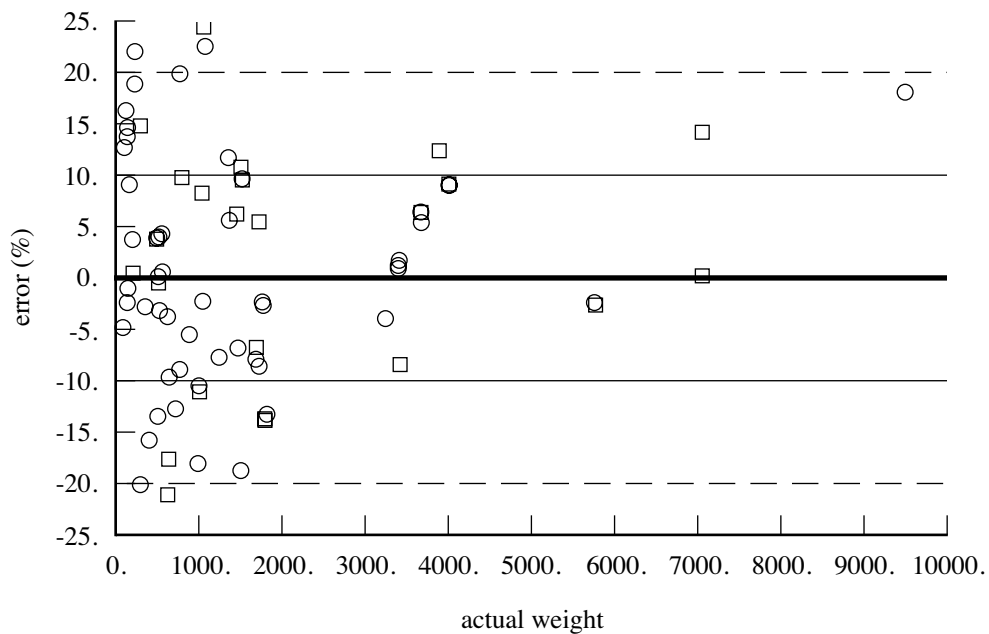
Figure 19-18. Propulsion group, gear box and rotor shaft weight (AFDD00).
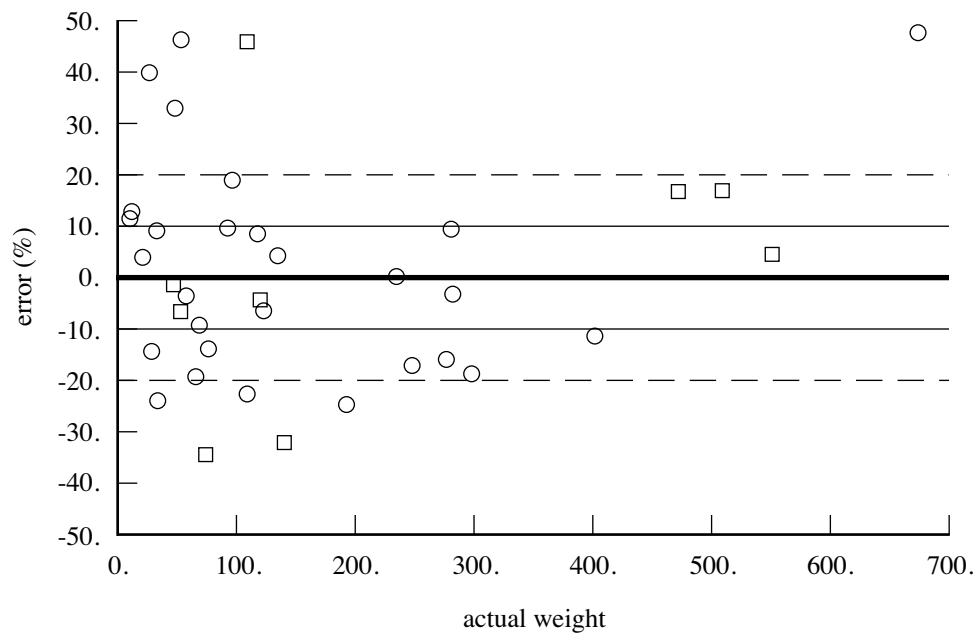
Figure 19-19. Propulsion group, drive shaft weight (AFDD82).
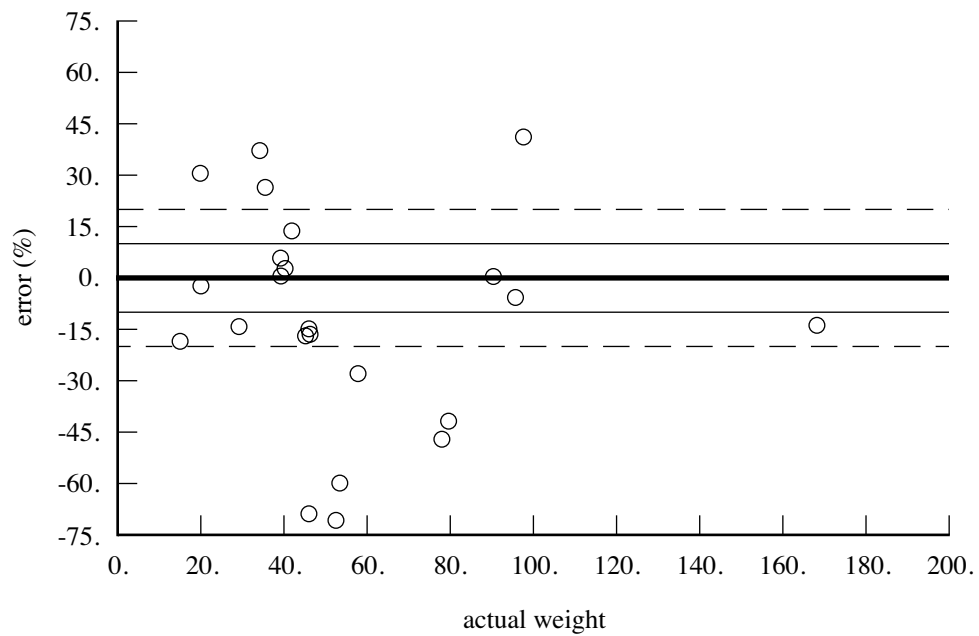
Figure 19-20. Propulsion group, rotor brake weight.
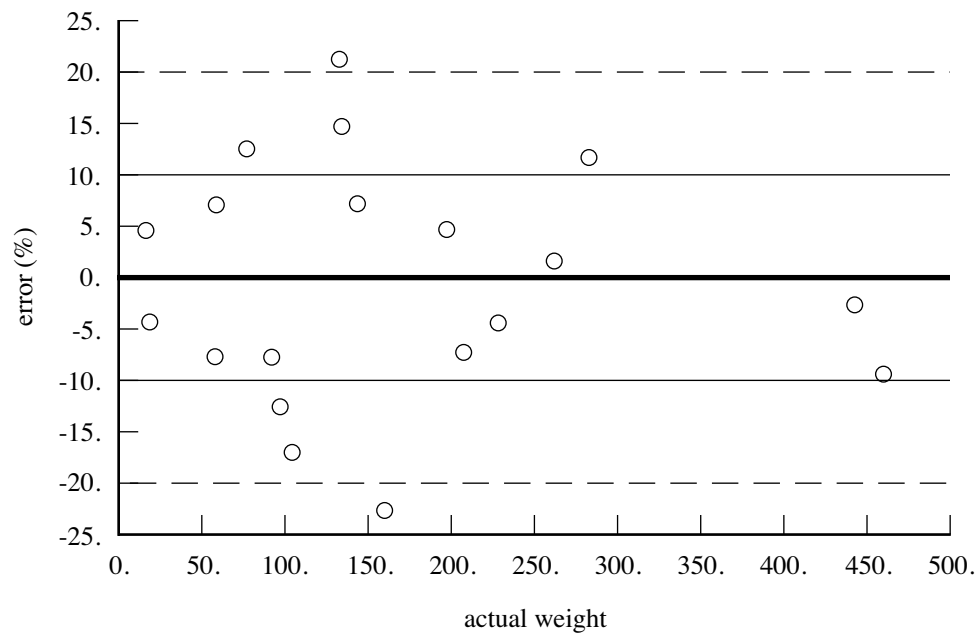
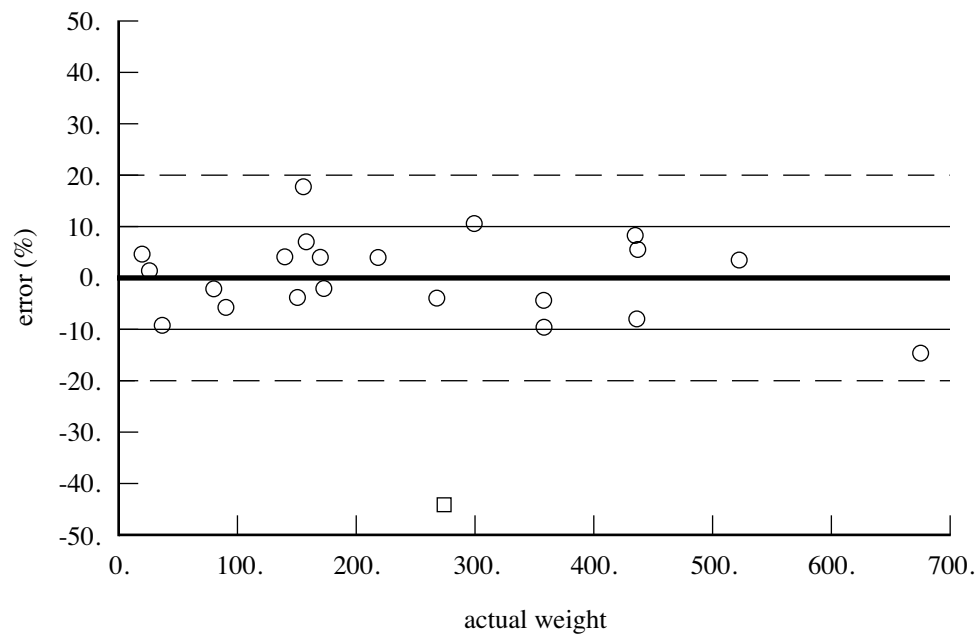Figure 19-21. Flight controls group, rotor non-boosted control weight (AFDD82).

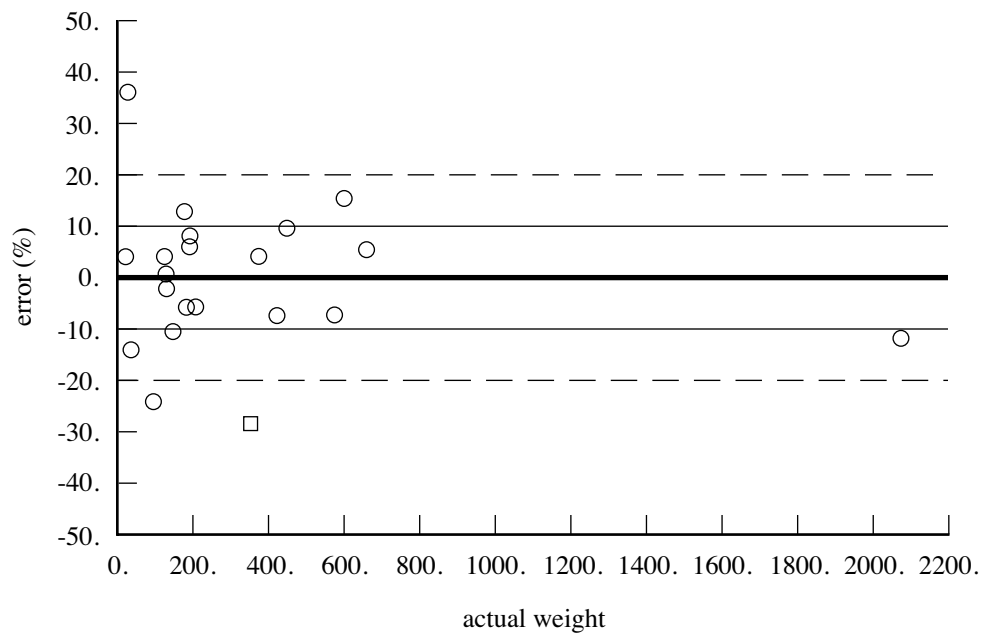Figure 19-22. Flight controls group, rotor boost mechanisms weight (AFDD82).

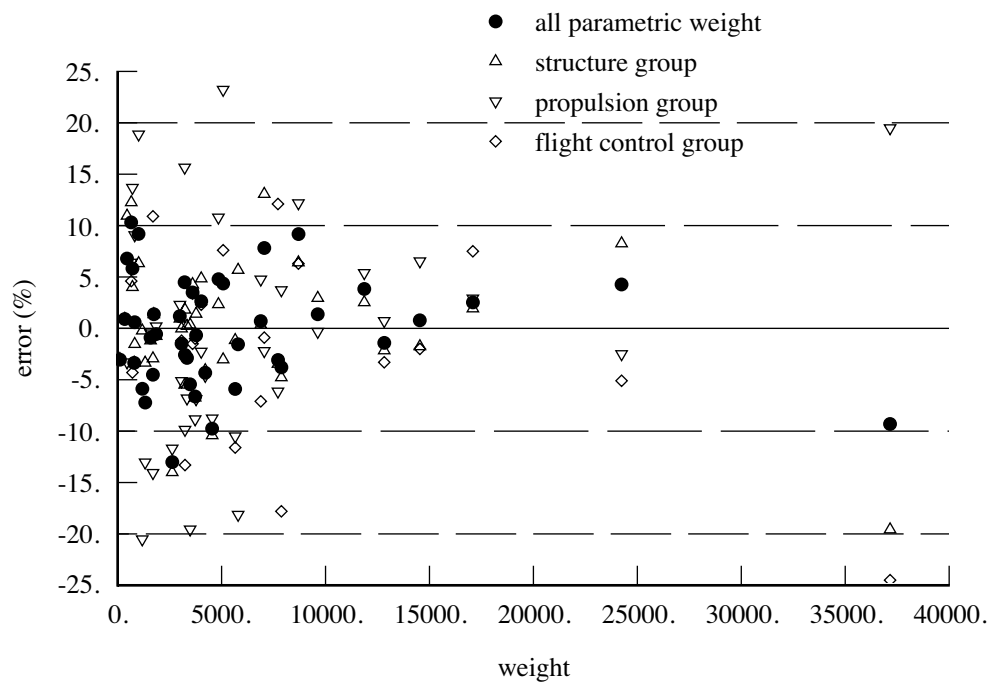Figure 19-23. Flight controls group, rotor boosted control weight (AFDD82).

Figure 19-24. Sum of all parametric weight.

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